THE EFFECT OF THE GEOMETRIC IMPFECTIONS ON THE ROTATIONAL CAPACITY OF STEEL BEAMS AT ELEVATED TEMPERATURES

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Abstract. *The main objective of this paper is to propose advanced three-dimensional models based on the use of shell elements that can be used for the simulation of the structural steel behaviour under fire conditions. The problem is handled through coupled thermo-mechanical analysis in the context of the finite element method. The basic target is to ensure that the developed numerical models can describe adequately the complex behaviour of structural steel in elevated temperatures. For this purpose, an experimental study, available in the literature, is used for comparison. The three – dimensional numerical model that is proposed in the current study, is developed using the non-linear finite element code MSC Marc. The first step is to simulate the behaviour of steel I-beams studied experimentally in [1] and compare the test results with the corresponding values from the numerical analysis. Based on the proposed numerical models, which are verified through the results contained in the literature, moment – rotation curves for various members in elevated temperatures are obtained. These functions can be used for the analysis of frame structures under fire conditions through simpler software packages, utilizing beam finite elements with concentrated nonlinear behaviour.*

1 INTRODUCTION

The global plastic analysis of steel structures requires that at the plastic hinge locations, the cross sections of the members which contain the plastic hinge should have rotational capacity greater than the required at the plastic hinge position. According to Eurocode 3, this problem is handled through the classification of the cross sections. More specifically, sufficient rotation capacity may be assumed at the plastic hinge if the cross section of the member is of Class1. In the case of the fire design of steel structures, the classification of the cross sections is conducted in the same way as in room temperature, except that a factor of 0.85 is used for the calculation of ε i.e. it is:

$$\varepsilon = 0.85\left[235/f_y\right]^{0.5} \tag{1}$$

where f_y is the yield strength at 20°C.

As it is stated in Eurocode 3 – Part 1.2 [3] the reduction factor considers influences due to the increased temperature. This consideration could be conservative or not since it does not take into account several factors that affect the rotational capacity of steel members under fire conditions, as the lack of the strain hardening in the stress-strain relationship after the temperature of 400°C, the effect of the initial imperfections etc. These parameters may lead to a premature occurrence of local or lateral torsional buckling in the plastic range, therefore limiting the available rotational capacity.

In fact, nowadays, the experimental and numerical research concerning plastic local buckling and plastic lateral torsional buckling of steel beams under fire conditions is rather limited. According to R.B. Dharma and K.H. Tan [1] the cross section classification of Eurocode 3 – Part 1.2 [3], which is based on the cross-sectional dimensions and the material yield strength, is inadequate to address the ductility of beams at elevated temperatures. This study reveals that the rotational capacity of steel I-beams reduces at elevated temperatures. The results of the experimental program that is presented in [1] indicate the effect of the main parameters (web slenderness, the flange slenderness, the effective length) on the rotational capacity at elevated temperatures. More specifically it is indicated that the difference in shape of the moment rotation curves at elevated temperatures compared with respect to the corresponding curves at ambient temperature, is due to the introduction of a non-linear elastic segment in the stress strain relationship of steel at elevated temperatures. Also, according to the test results, the increased slenderness of the web and flange reduces the rotational capacity at elevated temperatures. Additionally, it is noticed that the reduction of the effective length provides greater rotational capacity and changes the failure mode from lateral torsional buckling to local buckling. The numerical models that are developed in [2] are validated against the test results and they are used for further research to quantify the inelastic behaviour of steel beams under fire conditions. Finally, a simple moment – rotation model is proposed that is useful for design purposes. This model takes into account all the experimental and numerical findings.

The objective of the present study is to propose an advanced three-dimensional numerical model for the evaluation of the real behaviour of steel beams under fire conditions. It must be noticed that the three-dimensional model is based on shell finite elements and takes into account the existing initial imperfections of the steel members. First, the numerical model is validated against the published experimental results that are presented in [1]. Then, parametric analyses are conducted with respect to the amplitude of the initial imperfections, in order to obtain moment – rotation curves for the steel beams at elevated temperatures. Finally, moment – rotation diagrams are proposed, that take into account the reduced rotational capacity of steel beams at elevated temperatures and the effect of the initial imperfections. These functions can be used for the global plastic analysis of frame structures under fire conditions through more simple, commercial software packages, utilizing beam finite elements.

2 DESCRIPTION OF THE PROBLEM

First, a two span continuous steel I-beam under uniform loading is considered. The objective of the study is to define the rotational capacity of the steel beam at the possible plastic hinge locations under fire conditions. Therefore, it is important to assess the fire behaviour of the beam at the hogging moment region i.e. at the internal supports. In order to simplify the problem, a simply supported beam is considered which is loaded at the mid-span. This beam can represent the part of a continuous beam close to the internal support, between the points where the bending moment diagram becomes zero (Figure 1). In the case of the simply supported beam the plastic hinge will be formed at mid-span, corresponding actually to the internal support of the continuous beam. It should be noticed that the beam has been designed so that it is able to reach the plastic moment capacity at elevated temperatures.

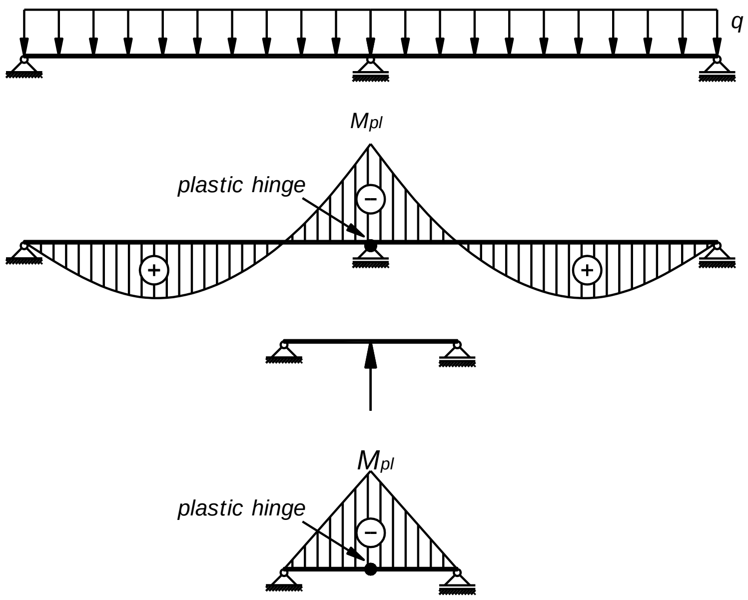

Figure 1. Simplification of the problem from a continuous beam to a simply supported beam

The structural system that is used in this study is referred to a typical beam specimen of the experimental study by R.B. Dharma and K.H. Tan [1]. More specifically, the steel I-beam S2-1 of [1] is chosen. All the geometrical dimensions and the material properties are considered according to this study. In detail, the total length of the simply supported beam is equal to 3.65m while the distance between the supports is 3.45m. Also, web stiffeners are used at the support and at the mid-span where the load is applied. The beam is laterally restrained at the position of supports at both ends and at the mid-span. Therefore, the effective length of the beam is equal to 1.725m. The cross-section dimensions and the structural system are illustrated in Figure 2. The material properties at room temperature are defined from tensile tests and they are presented in Table 1.

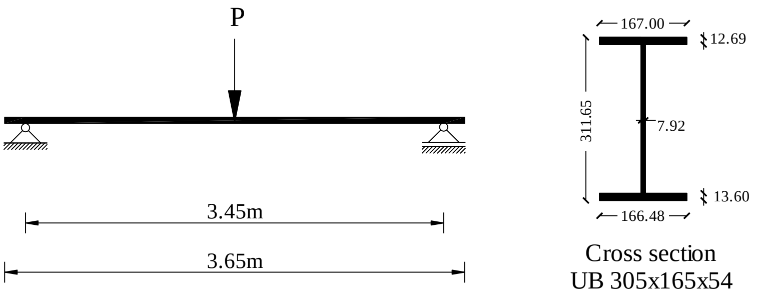

Figure 2. The structural system and the cross section (dimensions of the cross section in mm)

	Yield strength $f_{y,20}$ (Mpa)	Ultimate strength $f_{u,20}$ (Mpa)	Elastic modulus E_{20} (Mpa)
web	307.4	491.4	205253
flange	297.6	483.6	203677

Table 1 : Material properties at ambient conditions

It is stressed that the beam is laterally restrained only at the ends and at the mid-span and, therefore, the plastic lateral-torsional buckling is possible under certain conditions at elevated temperatures. Moreover, plastic local buckling may arise.

Taking into account the latter, the problem that is handled in this study is the evaluation of the rotational capacity of a simply supported steel I-beam under fire conditions. Specifically, the moment-rotation curves are obtained, for different temperatures, under the consideration that the temperature of the beam is uniform and constant. Additionally, various analyses are conducted taking into account different amplitudes of the initial geometrical imperfections.

It must be noticed that the beam is free to expand longitudinally, which means that the study does not take into account the effect of compressive forces. Moreover the assumption that the temperature is uniform simplifies the problem, since there is no thermal gradient at the cross section.

At this study the available rotational capacity r_a is calculated as the ratio between the inelastic rotation θ_a and the plastic rotation θ_{pl}, as it is indicated in Figure 3, i.e. $r_a = \dfrac{\theta_\alpha}{\theta_{pl}}$. Here θ_{pl} corresponds to the plastic moment resistance of the cross-section, defined as:

$$M_{pl,T} = f_{y,T} w_{pl} \tag{2}$$

where $f_{y,T}$ is the yield stress of the steel at temperature *T*.

The range of the rotation over which the plastic moment resistance of the cross section is retained is called θ_a . Notice that the characteristic point in which the elastic moment M_{el} is attained, in *not* used in this definition.

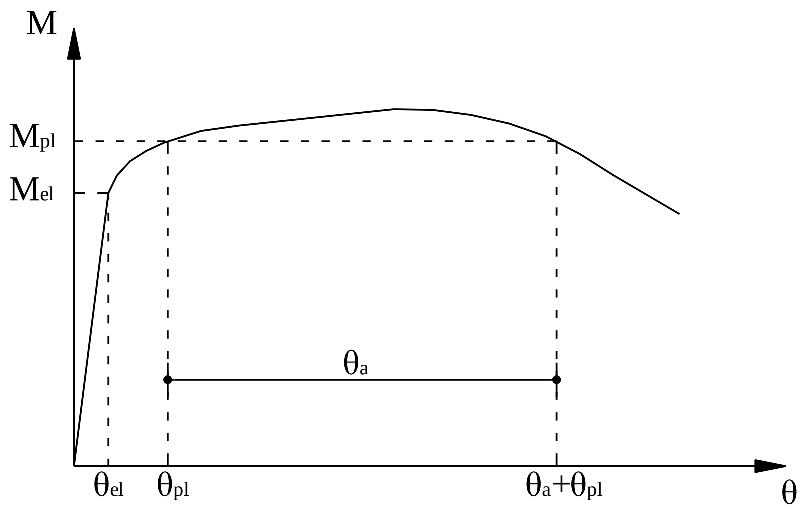

Figure 3. Definition of the rotational capacity

3 VALIDATION OF THE NUMERICAL MODEL

3.1 The numerical simulation

A detailed finite element model is proposed in order to evaluate the rotational capacity of the steel I-beam at elevated temperatures. The numerical analysis is carried out using the nonlinear finite element code MSC-MARC [4]. The three-dimensional numerical model (Figure 4) utilizes four-node, thick-shell elements with global displacements and rotations as degrees of freedom. Bilinear interpolation is used for the coordinates, displacements and rotations and the integration through the shell thickness is performed numerically using Simpson's rule. The numerical model takes into account the nonlinear elastic-plastic stress-strain relationship of steel in elevated temperatures. The yield stress, the proportionality limit and the elastic modulus are supposed to be temperature dependent according to Eurocode 3- Part1.2 [3].

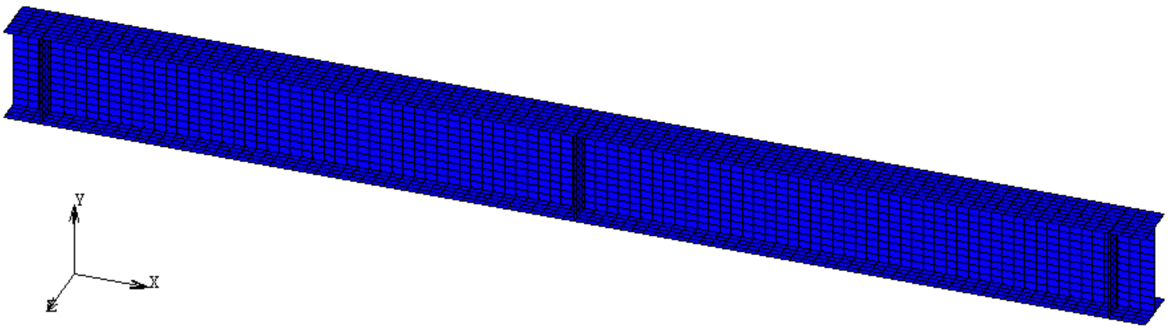

Figure 4. The numerical model

The Von Mises yield criterion is used in the numerical analysis. Additionally, the analysis takes into account the geometric non-linearity. It must be noticed that initial imperfections are incorporated in the geometry of the steel beam for a more realistic assessment of the behaviour. There are many different ways to introduce initial geometric imperfections in the structural members. A simple way in the context of finite element analysis is to extract the buckling eigenmodes and introduce them as imperfections with a fixed amplitude. More specifically, the normalized buckling mode is multiplied by a scale factor, leading to certain maximum amplitude and the resulting displacements are added to the initial coordinates of the structural member. For this case study two different eigenmodes are combined (see details in Figure 5). The first eigenmode is related with the local buckling along the upper flange of the beam (where compressive stresses arise under the considered loading) while the second buckling eigenmode used is the one corresponding to the lateral torsional buckling.

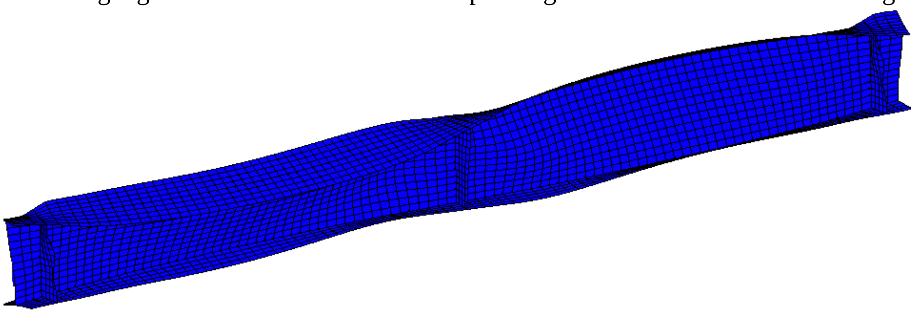

Figure 5a. Eigenmode corresponding to the lateral torsional buckling

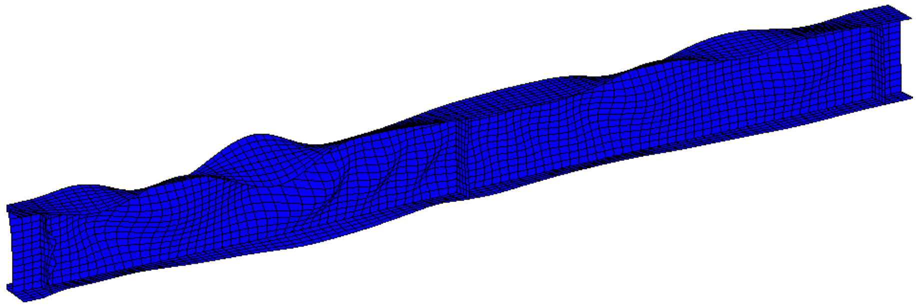

Figure 5b. Eigenmode corresponding to the local buckling of the upper flange

The numerical analysis has two different stages, following the test procedure that is described in [1]. At the first stage the steel beam is heated with a heating rate equal to 7°C/min until the desired temperature *T* is reached. It must be noticed that during the heating stage the temperature is supposed to be uniform along the member. At the second stage the temperature remains constant and the beam is submitted to loading at the mid-span until failure occurs.

The structural boundary conditions are described in Figure 6. At both supports the boundary conditions are applied in the middle node of the web. The vertical *y*- displacement and the rotation about the longitudinal-*x* axis (r_x) is restrained at both supports while the *x*-displacement is restrained only at the left support. The out-of-plane displacements are prevented at the location of the supports and at the mid-span.

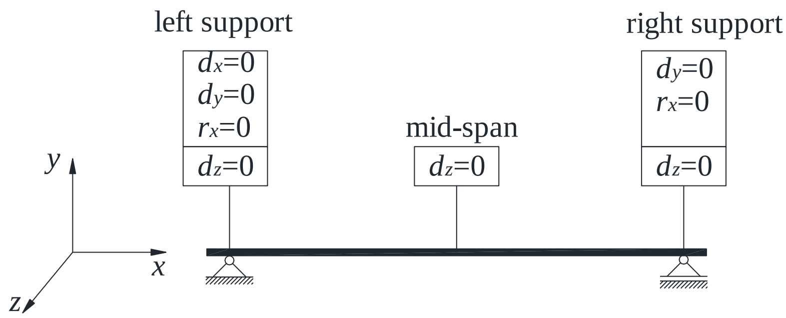

Figure 6. The boundary conditions

3.2 Comparison with the test results

The primary objective is to validate the numerical model against the published experimental results which are presented in the study of R.B. Dharma and K.H. Tan [1]. Due to the lack of details for the accurate profile of the initial imperfections an assumption has to be adopted. The amplitude of the initial imperfections that is used for the analysis is supposed to be 0.5mm for the buckling eigenmode which is related to the lateral torsional buckling and 2mm for the eigenmode which is related to the local buckling of the upper flange. These values are in accordance with the maximum values of the measured initial imperfections that are presented in [1].

For the calibration of the numerical model against the experimental results the material models proposed in Eurocode 3-Part 1.2 [3] were not followed regarding the ratio between the ultimate stress and the yield stress. This ratio, is limited by Eurocode to 1.25. Instead, the actual material properties for the specific steel used in the experiments were followed. More specifically, the tensile coupon tests, for both the web and the flange (Table 1), gave an ultimate strength approximately 1.6 times greater than the yield strength at room temperature. Another point were the suggestions of Eurocode were not completely followed was in the temperature above which the hardening of the material should be neglected. This temperature is defined to be 300°C. However, experimental results indicate that steel keeps a significant hardening even in the case of 400°C [5],[6]. For this reason and in order to obtain more realistic results, hardening was taken into account in the numerical model but with a reduced ratio between the ultimate to the yield stress. The specific value for this ratio was calculated here to have the value of 1.51, following the analogies yielded by the experimental results. Therefore, in the numerical analysis that is conducted for the calibration of the finite element model, the ultimate strength is supposed to be $f_{u,T} = 1.6 f_{y,T}$ for both the web and the flange, for temperatures ranging between 20^oC and 400^oC

and $f_{u,T} = 1.51 f_{y,T}$ for temperature the temperature of 410^oC.

Figure 7 illustrates the load-displacement curves obtained numerically and experimentally for the specimen S2-1. A very good agreement is obtained for the initial stiffness and for the maximum load of the system. Moreover, the softenign branch is well approximated by the numerical model. An insignificant difference is observed for the maximum strength of the steel beam that can be attributed to the lack of experimental data for the ultimate and the yield strength of steel at elevated temperatures. Also the slight incompatibility between the numerical and the test results at the unloading branch is due to the different profile of initial imperfections that is used in the numerical analysis. Additionally, the failure mode that results from the numerical analysis is very close to the experimental results, as it is presented at Figure 8. In both cases the failure is due to lateral – torsional buckling of the steel beam.

Taking into account the previous, it is considered that the numerical model can predict accurately the behaviour of the steel I-beam in elevated temperatures. Additionally, it is indicated that the shapes of the initial imperfections that were adopted can be used for parametric studies.

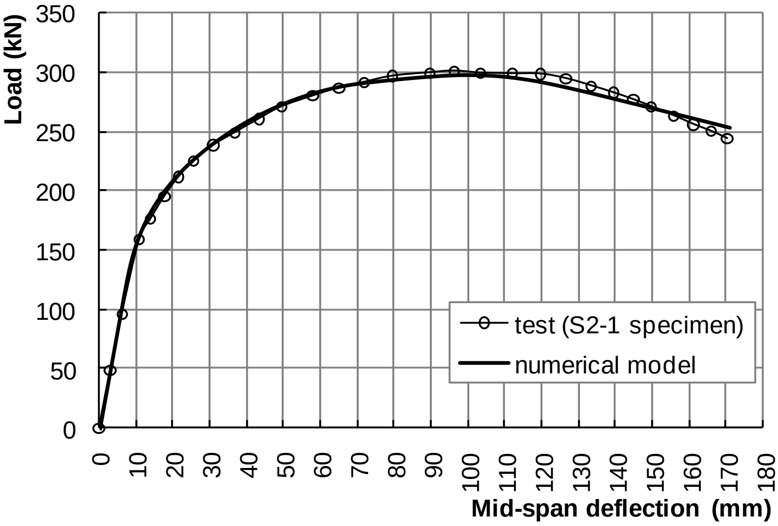

Figure 7. Comparison of the numerical analysis results with the test results for the specimen S2-1 at 415^{o}C

Figure 8a. Test results

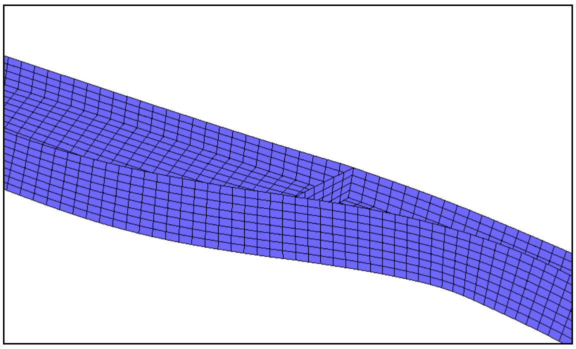

Figure 8b. Numerical analysis results

Figure 8. Deformed shape of the steel beam at the failure

4 PARAMETRIC ANALYSES

The numerical model which has been validated against experimental results is now used for the evaluation of the rotational capacity of the steel I-beam in elevated temperatures. Parametric analyses are conducted at various temperature levels in order to find out the effect of the initial imperfection on the available rotational capacity of the beams. In particular, the analyses are conducted for temperatures ranging between 100^{o}C and 900^{o}C. The amplitude of the initial imperfections is considered to be between 0.5mm and 5mm for both the local buckling and the lateral torsional buckling eigenmodes. The objective is to obtain moment – rotation curves that might be used for the elastic-plastic analysis of frame structures under fire conditions through simple software packages, utilizing beam finite elements with nonlinear behavior concentrated at the ends of the members. The results of the analysis could be used for the fire design of steel structures. Therefore the numerical models that are used for the parametric analyses adopt fully the stress-strain models proposed for various temperatures in Eurocode 3-Part 1.2 [3].

The results of the numerical analysis in terms of load – displacement curves for various temperatures levels are presented at Figure 8.

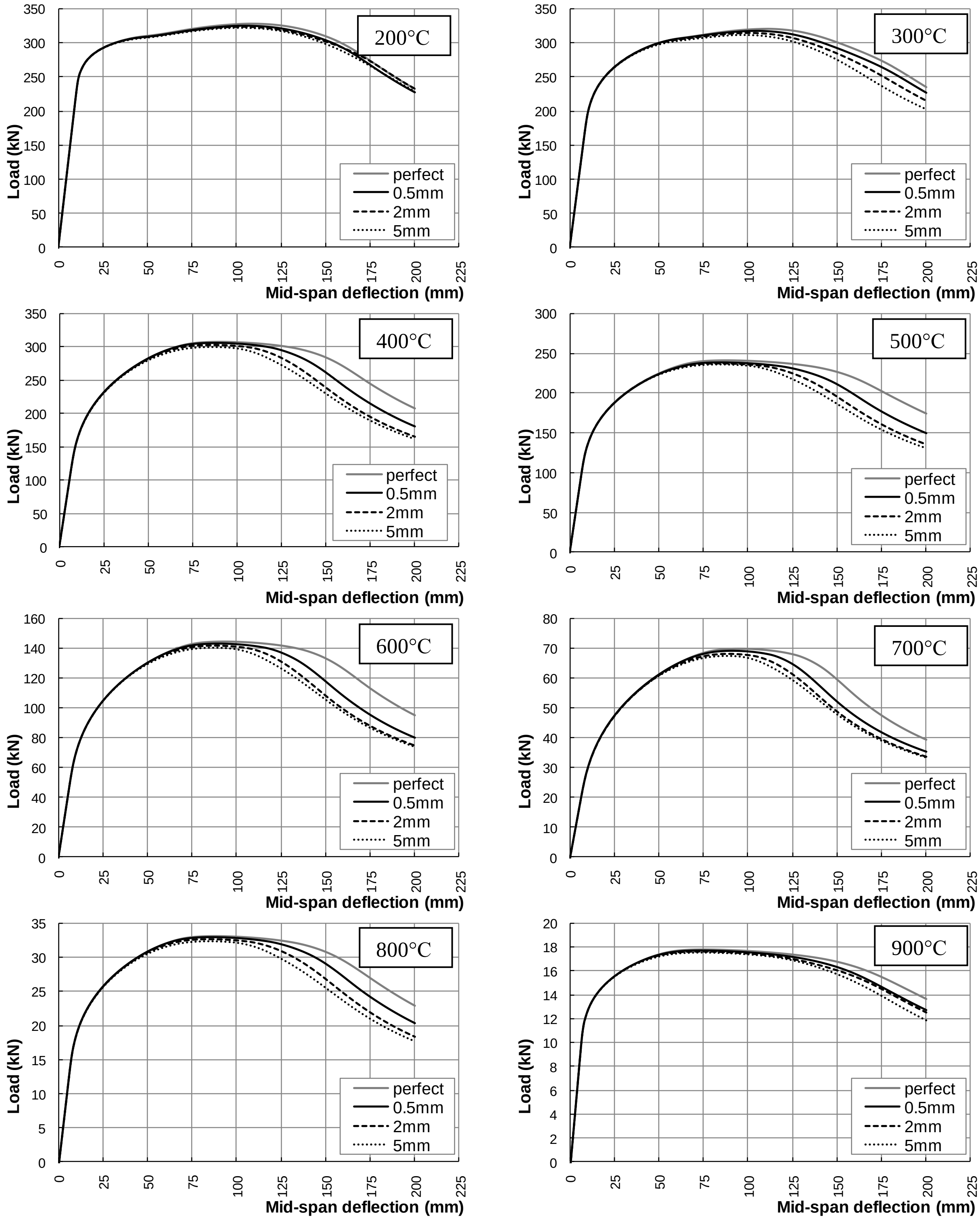

Figure 9. Load – displacement curves at various temperature levels

At every temperature level four different analyses are conducted. Initially, the steel beam is considered to be "perfect" i.e. the initial imperfections are not taken into account. Then, parametric analyses are conducted considering amplitudes of initial imperfections equal to 0.5mm, 2mm and 5mm. The imperfection amplitudes are assumed to be the same for both the local buckling and the lateral torsional buckling eigenmodes. The results of the numerical analyses indicate that the steel beam fails due to plastic lateral torsional buckling for all the temperature levels. It is observed that the initial imperfections do not affect the maximum load bearing capacity of the beam. This result holds for all the temperature ranges. For temperature values less than 300°C the incorporation of the initial imperfections to the geometry of the steel beam has a minor effect to the softening branch of the diagram. On the contrary, for temperature ranges between 400°C and 800°C, as the amplitude of the imperfection increases, the softening branch becomes steeper.

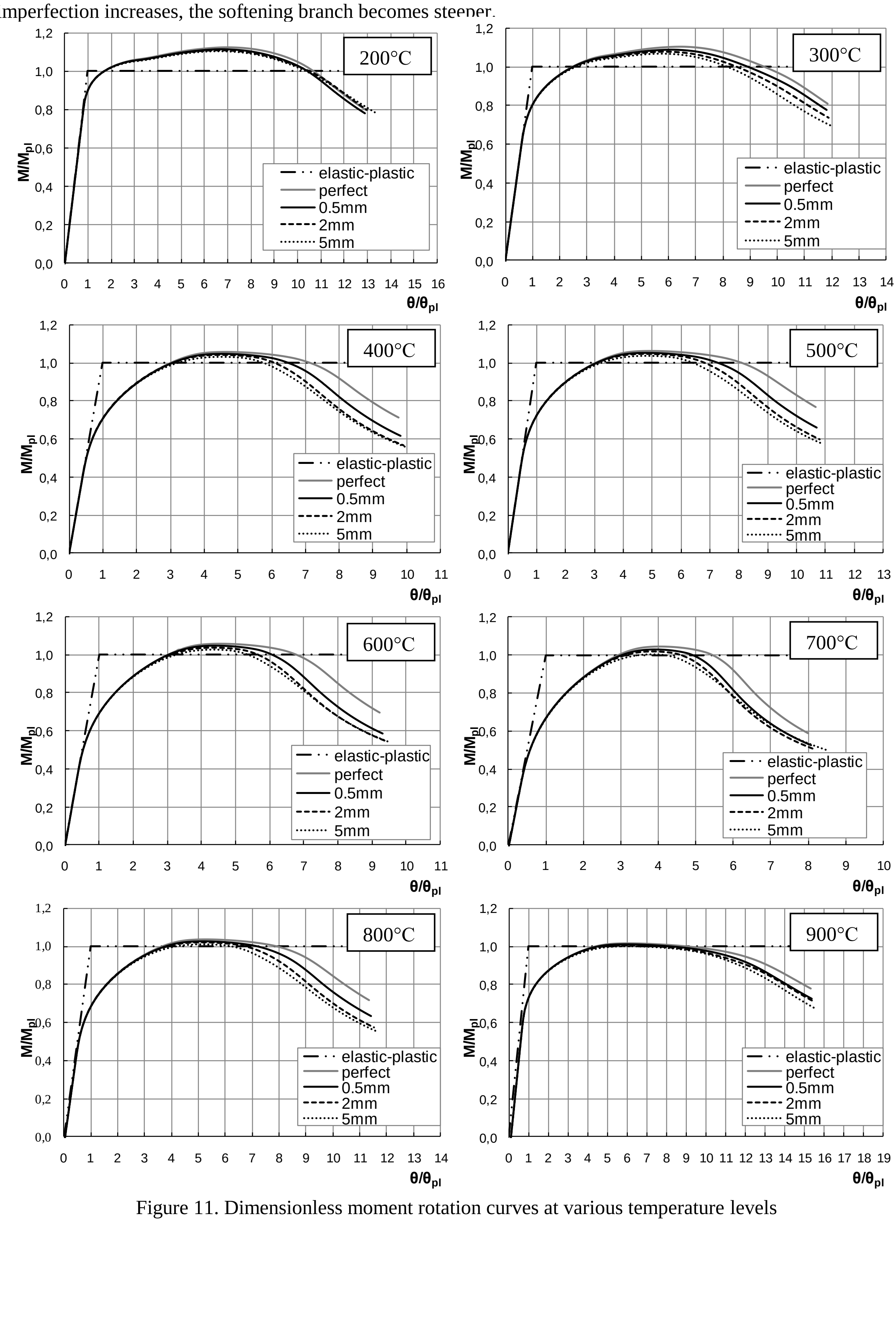

Figure 11. Dimensionless moment rotation curves at various temperature levels

The dimensionless moment-rotation curves at various temperature levels are illustrated in Figure 10. The idealized elastic- perfectly plastic diagram corresponds to the design plastic moment of the cross section that can be obtained from (2). It can be first noticed that for all the temperature levels the steel beam is able to reach the plastic moment resistance, since no local or lateral torsional buckling takes place in the elastic region. Despite the fact that according to Eurocode 3 [3] for temperatures ranging between 20°C and 400 °C the ultimate strength of the steel is $f_{u,T} = 1.25 f_{y,T}$, the ultimate moment that results from the numerical analysis is $M_{u,T} \approx 1.1 M_{pl,T}$. This can be attributed to the fact that geometric nonlinear phenomena arise as the deflection of the beam increases.
It is clear that for temperatures ranging between 400°*C* and 900°*C*, the rotational capacity of the steel I-beam is considerably reduced. Moreover, the available rotational capacity reduces as the amplitude of the initial imperfections becomes larger. This reduction is actually a consequence of the fact that the slope of the softening branch becomes steeper as the amplitude of the imperfections increases.

The values of the rotation capacity of the steel beam according to the results of the parametric numerical analyses are summarized in Table 2. It is noticed that both the initial imperfections and temperature have a significant effect on the available rotational capacity. Notice also that the rotational capacity seems to be increased for temperatures above 800°C with respect to the values obtained for lower temperatures (e.g. for 700°C).

	Amplitude of initial imperfections			
Temperature (°C)	*perfect*	*0.5mm*	*2mm*	*5mm*
100	10.55	10.48	10.47	10.77
200	9.01	8.76	8.82	8.63
300	7.02	6.45	6.00	5.58
400	4.13	3.48	2.96	2.53
500	5.00	4.20	3.64	3.17
600	3.71	2.96	2.48	2.03
700	2.47	1.91	1.44	0.80
800	4.23	3.52	2.86	2.17
900	4.75	3.87	2.90	1.69

Table 2. Available Rotational capacity of the steel beam at various temperature levels

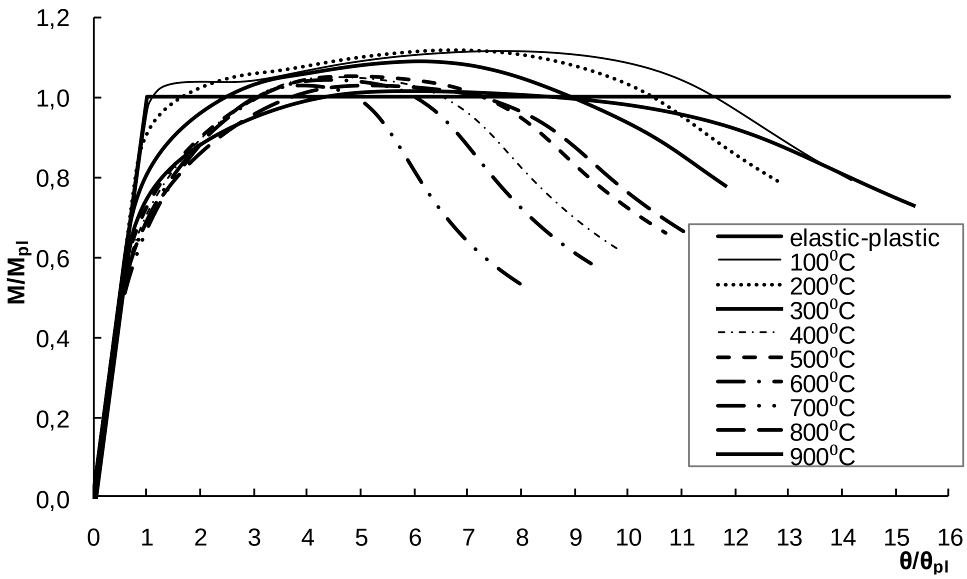

Figure 11. Dimensionless moment rotation curves at various temperature levels

5 CONCLUSIONS

A numerical model is proposed in order to evaluate the effect of the temperature and the amplitude of initial imperfections on the rotational capacity of steel I-beams at elevated temperatures. First, the model is validated against published experimental results. The results of the parametric analyses indicate that the rotational capacity is significantly reduced at temperatures greater than 400^{o}C. Moreover, for the same temperature range, as the amplitudes of the initial imperfections increase, a considerable reduction of the rotational capacity is noticed.

6 ACKNOWLEDGMENTS

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