

Bayesian Uncertainty Quantification and Propagation in Large-Order Finite Element Models using CMS Techniques

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ABSTRACT

Bayesian inference is used for quantifying and calibrating uncertainty models in structural dynamics based on vibration measurements, as well as propagating these modelling uncertainties in structural dynamics simulations to achieve updated robust predictions of system performance, reliability and safety. The Bayesian tools for identifying system and uncertainty models as well as performing robust prediction analyses are Laplace methods of asymptotic approximation and/or more accurate stochastic simulation algorithms (e.g. MCMC). These tools involve solving optimization problems, generating samples for tracing and then populating the important uncertainty region in the parameter space, as well as evaluating integrals over high-dimensional spaces of the uncertain model parameters. They require a moderate to very large number of repeated system analyses to be performed. Consequently, the computational demands depend highly on the number of system analyses and the time required for performing a system analysis. Component mode synthesis (CMS) techniques are integrated with Bayesian techniques to efficiently handle large-order models of hundreds of thousands or millions degrees of freedom and localized nonlinear actions activated during system operation. Fast and accurate CMS techniques are proposed, consistent with the finite element (FE) model parameterization, to achieve drastic reductions in computational effort.

Keywords: *Bayesian inference, uncertainty quantification, dynamics, component mode synthesis.*

1 INTRODUCTION

1.1 Bayesian Model Updating

Bayesian techniques are used for quantifying and calibrating uncertainty models in structural dynamics based on vibration measurements, as well as propagating these modeling uncertainties in structural dynamics simulations to achieve updated robust predictions of system performance, reliability and safety (Papadimitriou et al. 2001). The Bayesian model identification techniques are next presented using experimentally estimated modal properties. Let $D = \{\hat{\omega}_r, \hat{\varphi}_r \in R^{N_0}, r = 1, \dots, n\}$ be the estimated modal frequencies $\hat{\omega}_r$ and modeshape components $\hat{\varphi}_r$ at N_0 measured DOFs, where n is the number of observed modes. Consider a parameterized FE model class $M^{(m)}$ and let $\underline{\theta}^{(m)} \in R^{N_\theta^{(m)}}$ be the free parameters of the model class, where $N_\theta^{(m)}$ is the number of parameters in $\underline{\theta}^{(m)}$. Let $\Pi(\underline{\theta}^{(m)}; M^{(m)}) = \{\omega_r(\underline{\theta}^{(m)}; M^{(m)}), \varphi_r(\underline{\theta}^{(m)}; M^{(m)}) \in R^{N_0}\}$ be the predictions of the modal frequencies

and modeshapes from a particular model in the model class $M^{(m)}$, with $\underline{\varphi}_r \equiv \underline{\varphi}_r(\underline{\theta}^{(m)}; M^{(m)}) = L\underline{\phi}_r(\underline{\theta}^{(m)}; M^{(m)})$, where $\underline{\phi}_r(\underline{\theta}^{(m)}; M^{(m)})$ the complete modeshape and $L \in R^{N_0 \times N}$ selects the N_0 measured DOFs from the N DOFs of the FE model.

In Bayesian inference the probability distribution of the model parameters $\underline{\theta}^{(m)}$ is updated based on the available measurements. The formulation starts by building a probabilistic model that characterizes the discrepancy between the model predictions $\Pi(\underline{\theta}^{(m)}; M^{(m)})$ obtained from a particular value of the model parameters $\underline{\theta}^{(m)}$ and the corresponding data D . Let $M^{(e)}$ be a family of probability model classes for the discrepancy terms, that depend on a set of prediction error parameters $\underline{\theta}^{(e)}$. The Bayesian approach to model calibration deals with updating the values of the parameter set $\underline{\theta} = (\underline{\theta}^{(m)}, \underline{\theta}^{(e)})$ associated with the structural model parameters and the prediction error parameters. A probability distribution $\pi(\underline{\theta} | M)$ is assigned a priori to incorporate subjective prior information on the uncertainty in the values of these parameters, where $M = \{M^{(m)}, M^{(e)}\}$ includes the structural and prediction error model classes. The updated distribution $p(\underline{\theta} | D, M)$ of the set $\underline{\theta}$, given the data D and the model class M , results from Bayes theorem as follows

$$p(\underline{\theta} | D, M) = \frac{p(D | \underline{\theta}, M) \pi(\underline{\theta} | M)}{p(D | M)} \quad (1)$$

where $p(D | \underline{\theta}, M)$ is the likelihood of observing the data from the model class and $p(D | M)$ is the evidence of the model class M given by the multi-dimensional integral

$$p(M | D) = \int_{\Theta} p(D | \underline{\theta}, M) \pi(\underline{\theta} | M) d\underline{\theta} \quad (2)$$

over the space of the uncertain model parameters.

Assuming that the prediction errors are independent Gaussian zero-mean random variables with variance σ^2 , the likelihood is readily obtained in the form (Christodoulou and Papadimitriou 2007)

$$p(D | \underline{\theta}, M) = \frac{1}{(\sqrt{2\pi}\sigma)^{N_0(n+1)}} \exp \left[-\frac{N_0(n+1)}{2\sigma^2} J(\underline{\theta}^{(m)}; M^{(m)}) \right] \quad (3)$$

where $J(\underline{\theta}^{(m)}; M^{(m)})$ given by

$$J(\underline{\theta}^{(m)}; M^{(m)}) = \frac{1}{n} \sum_{r=1}^n \frac{[\omega_r(\underline{\theta}^{(m)}; M^{(m)}) - \hat{\omega}_r]^2}{\hat{\omega}_r^2} + \frac{1}{n} \sum_{r=1}^n \frac{\|\alpha_r \underline{\varphi}_r(\underline{\theta}^{(m)}; M^{(m)}) - \hat{\underline{\varphi}}_r\|^2}{\|\hat{\underline{\varphi}}_r\|^2} \quad (4)$$

represents the measure of fit between the experimentally obtained modal data and the modal data predicted by a particular model in the class $M^{(m)}$, and $\|\cdot\|$ is the usual Euclidian norm.

1.2 Bayesian Model Selection

The Bayesian probabilistic framework can also be used to compare two or more competing model classes and select the optimal model class based on the available data. Consider a family $M_{Fam} = \{M_i, i=1, \dots, \mu\}$, of μ alternative, competing, parameterized FE and prediction error model classes, and let $\underline{\theta}_i \in R^{N_{\theta_i}}$ be the free parameters of the model class M_i . The posterior probabilities $P(M_i | D)$ of the various model classes given the data D is (Beck and Yuen 2004)

$$P(M_i | D) = \frac{p(D | M_i) P(M_i)}{p(D | M_{Fam})} \quad (5)$$

where $P(M_i)$ is the prior probability and $p(D | M_i)$ is the evidence of the model class M_i . The optimal model class M_{best} is selected as the one that maximizes $P(M_i | D)$ given by (5).

2 BAYESIAN TOOLS

The Bayesian tools for identifying uncertainty models and performing robust prediction analyses are Laplace methods of asymptotic approximation and stochastic simulation algorithms.

2.1 Asymptotic Approximations

For large enough number of experimental data, the posterior distribution of the model parameters in (1) can be asymptotically approximated by a Gaussian distribution (Beck and Katafygiotis 1998)

$$p(\underline{\theta} | D, M) \approx \frac{|h(\hat{\underline{\theta}})|^{1/2}}{(2\pi)^{N_\theta/2}} \exp\left[-\frac{1}{2}(\underline{\theta} - \hat{\underline{\theta}})^T h(\hat{\underline{\theta}})(\underline{\theta} - \hat{\underline{\theta}})\right] \quad (6)$$

centered at the most probable value $\hat{\underline{\theta}}$ of the model parameters with covariance equal to the inverse of the Hessian $h(\theta)$ of the function $g(\underline{\theta}; M) = (NN_0/2)[\sigma^{-2}J(\underline{\theta}; M) + \ln \sigma^2] - \ln \pi(\underline{\theta} | M)$ evaluated at the most probable value $\hat{\underline{\theta}}$. Such approximation requires the computation of the most probable value $\hat{\underline{\theta}}$ and the Hessian $h(\hat{\underline{\theta}})$. The asymptotic expression (6) is approximate. Moreover, even for large number of experimental data, it may fail to give a good representation of the posterior probability distribution in the case of multimodal distributions. In addition, the asymptotic approximation fails to provide acceptable estimates for un-identifiable cases manifested for relatively large number of model parameters in relation to the information contained in the data.

For model selection, an asymptotic approximation based on Laplace's method is also used to give an estimate of the integral in (2) appearing in (5) (Papadimitriou & Katafygiotis 2004):

$$p(D | M_i) \approx c_0 (2\pi)^{N_\theta/2} \frac{\pi_{\theta}(\hat{\underline{\theta}}_i | M_i) [J_i(\hat{\underline{\theta}}_i)]^{-N_j}}{\sqrt{\det h_i(\hat{\underline{\theta}}_i)}} \quad (7)$$

where $\hat{\underline{\theta}}_i$ minimizes the function $g(\underline{\theta}; M_i)$ and $h_i(\hat{\underline{\theta}}_i)$ is the Hessian of $g(\underline{\theta}; M_i)$ evaluated at $\hat{\underline{\theta}}_i$.

2.2 Stochastic Simulation Algorithms

For more accurate estimates, one should use stochastic simulation algorithms (e.g. MCMC, Transitional MCMC – TMCMC, Delayed Rejection Adaptive Metropolis - DRAM) to generate samples that populate the posterior pdf in (1) and then evaluate the integral (2). Among the stochastic simulation algorithms available, the transitional MCMC algorithm (Ching & Chen 2007) is one of the most promising algorithms for selecting the most probable model as well as finding and populating with samples the importance region of interest of the posterior pdf, even in the unidentifiable cases and multi-modal posterior probability distributions. In addition, the TMCMC method yields an estimate of the evidence in (2) of the model class M_i based on the samples generated by the algorithm. The samples $\underline{\theta}^{(j)}, j=1, \dots, N$ generated at the final stage of the algorithm can further be used for estimating the probability integrals (Papadimitriou et al 2001) encountered in robust prediction of various performance quantities of interest.

2.3 Computational Issues

The asymptotic approximations and the stochastic simulation algorithms, involve solving optimization problems, generating samples for tracing and then populating the important region in the uncertain parameter space, as well as evaluating integrals over high-dimensional spaces of the uncertain model parameters. They require a moderate to very large number of repeated system analyses to be performed over the space of uncertain parameters. Consequently, the computational demands depend highly on the number of system analyses and the time required for performing a

system analysis. The proposed Bayesian techniques require a large number of FE model simulations to be carried out which imposes severe computational limitations on the application of the technique. For FE models involving hundreds of thousands or even million degrees of freedom and localized nonlinear actions activated during system operation these computational demands for repeatedly solving the large-scale eigen-problems and the gradient of the eigensolutions may be excessive.

The objective of this work is to examine the conditions under which substantial reductions in the computational effort can be achieved using dynamic reduction techniques such as CMS. Dividing the structure into components and reducing the number of physical coordinates to a much smaller number of generalized coordinates certainly alleviates part of the computational effort. However, in each iteration one needs to re-compute the eigen-problem and the interface constrained modes for each component. This procedure is usually a very time consuming operation and computationally more expensive than solving directly the original matrices for the eigenvalues and the eigenvectors. It is shown in this study that for certain parameterization schemes for which the mass and stiffness matrices of a component depend linearly on only one of the free model parameters to be updated, often encountered in FE model updating formulations, the repeated solutions of the component eigen-problems are avoided, reducing substantially the computational demands in FE model updating formulations, without compromising the solution accuracy.

3 INTEGRATION OF COMPONENT MODE SYNTHESIS TECHNIQUES

In CMS techniques (Craig & Bampton 1965), a structure is divided into several components. For each component, the unconstrained DOFs are partitioned into the boundary DOFs, denoted by the subscript b and the internal DOFs, denoted by the subscript i . The boundary DOFs of a component are common with the boundary DOFs of adjacent components, while the internal DOFs of a component are not shared with any adjacent component. The stiffness and mass matrices of a component s are $K^{(s)} \in \mathbb{R}^{n^{(s)} \times n^{(s)}}$ and $M^{(s)} \in \mathbb{R}^{n^{(s)} \times n^{(s)}}$. Without loss of generality, we limit the formulation to stiffness matrices that depend linearly on the model parameters $\underline{\theta}$ and constant mass

matrices, i.e. $K(\underline{\theta}) = K_0 + \sum_{j=1}^{N_\theta} K_{,j} \theta_j$, $M(\underline{\theta}) = M_0$, where M_0 , K_0 and $K_{,j}$, $j=1, \dots, N_\theta$, are constant matrices. The linear representation implies a similar representation at component level, i.e.

$$K^{(s)} = K_0^{(s)} + \sum_{j=1}^{N_\theta} K_{,j}^{(s)} \theta_j \quad (8)$$

and $M^{(s)} = M_0^{(s)}$. Consider the case for which the stiffness matrix of a component is proportional to a single parameter in $\underline{\theta}$. Let S_j be the set of components that depends on the j -th variable θ_j in the parameter set $\underline{\theta}$. Due to (8), the stiffness matrix of the component in $s_j \in S_j$ take the form

$$K^{(s_j)} = \bar{K}^{(s_j)} \theta_j \quad (9)$$

It can be shown that the matrix $\Lambda_{kk}^{(s_j)}$ of the kept eigenvalues and the matrix of the eigenvectors Φ_{ik} of the component fixed-interface modes are given with respect to the parameter θ_j in the form

$$\Lambda_{kk}^{(s_j)} = \bar{\Lambda}_{kk}^{(s_j)} \theta_j \quad \text{and} \quad \Phi_{ik} = \bar{\Phi}_{ik}^{(s_j)} \quad (10)$$

where the matrices $\bar{\Lambda}_{kk}^{(s_j)}$ and $\bar{\Phi}_{ik}^{(s_j)}$ are solutions of the following eigen-problem

$$\bar{K}_{ii}^{(s_j)} \bar{\Phi}_{ik}^{(s_j)} = \bar{M}_{ii}^{(s_j)} \bar{\Phi}_{ik}^{(s_j)} \bar{\Lambda}_{kk}^{(s_j)} \quad (11)$$

which is independent of the values of θ_j . The subscript k denotes the kept modes of a component. Also the constrained modes, given by

$$\Psi_{ib}^{(s_j)} = -[K_{ii}^{(s_j)}]^{-1} K_{ib}^{(s_j)} = -[\bar{K}_{ii}^{(s_j)}]^{-1} \bar{K}_{ib}^{(s_j)} \quad (12)$$

are constant independent of the values of the parameter θ_j . It should be noted that only a single component analysis is required to estimate the fixed-interface and constrained modes, independent of the values of θ_j . The component's mass and stiffness matrices are $\hat{M}^{(s)} = \Psi^{(s)T} M^{(s)} \Psi^{(s)}$ and $\hat{K}^{(s)} = \Psi^{(s)T} K^{(s)} \Psi^{(s)}$. It is straightforward to verify that $\hat{K}^{(s_j)} = \hat{K}^{(s_j)} \theta_j$, where $\hat{K}^{(s_j)}$ is a constant matrix given by $\hat{K}_{kk}^{(s_j)} = \bar{K}_{kk}^{(s_j)}$, $\hat{K}_{kb}^{(s_j)} = \hat{K}_{bk}^{(s_j)T} = 0_{kb}^{(s_j)}$ and $\hat{K}_{bb}^{(s_j)} = \bar{K}_{bb}^{(s_j)} - [\bar{K}_{ii}^{(s_j)}]^{-1} \bar{K}_{ib}^{(s_j)} \bar{K}_{ib}^{(s_j)}$, independent of the model parameters $\underline{\theta}$. Also, using the fact that $M^{(s)} = M_0^{(s)}$ is constant, the reduced matrix $\hat{M}^{(s)} = \Psi^{(s)T} M_0^{(s)} \Psi^{(s)} \equiv \hat{M}_0^{(s)}$ is also constant.

The assembled Craig-Bampton stiffness matrix $\hat{K}^{CB} \in \mathbb{R}^{n_q \times n_q}$ and mass matrix $\hat{M}^{CB} \in \mathbb{R}^{n_q \times n_q}$ for the reduced set of independent generalized coordinates $\underline{q}(t)$ is

$$\hat{K}^{CB} = \mathbb{F}[\hat{K}^{(1)}, \dots, \hat{K}^{(N_s)}] = \sum_{s=1}^{N_s} \mathbb{F}_s[\hat{K}^{(s)}] \quad (13)$$

$$\hat{M}^{CB} = \mathbb{F}[\hat{M}^{(1)}, \dots, \hat{M}^{(N_s)}] = \sum_{s=1}^{N_s} \mathbb{F}_s[\hat{M}^{(s)}] \quad (14)$$

For N matrices $A_1 \in \mathbb{R}^{n_1 \times n_1}, \dots, A_N \in \mathbb{R}^{n_N \times n_N}$, the mathematical operators $\mathbb{F}[M_1, \dots, M_N]$ and $\mathbb{F}_s[A_s]$ are defined as follows $\mathbb{F}[A_1, \dots, A_N] = S^T \text{blockdiag}(A_1, \dots, A_N) S$, where $\text{blockdiag}(A_1, \dots, A_N)$ is a block diagonal matrix having as diagonal blocks the matrices (A_1, \dots, A_N) and $\mathbb{F}_s[A_s] = \mathbb{F}[0_{n_1}, \dots, 0_{n_{s-1}}, A_s, 0_{n_{s+1}}, \dots, 0_{n_N}]$ where $0_i \in \mathbb{R}^{i \times i}$ denotes a matrix of zeroes.

Introduce the index set $\Sigma = \{s_1, \dots, s_{N_\theta}\}$ to contain the structural components that depend on a parameter in the set $\underline{\theta}$. Then the set $\bar{\Sigma} = \{1, \dots, N_s\} - \Sigma$ contains the component numbers that their properties are constant, independent of the values of the parameter set $\underline{\theta}$. Using the aforementioned analysis, the stiffness matrix of the reduced system admits the representation

$$\hat{K}^{CB} = \hat{K}_0^{CB} + \sum_{i=1}^{N_\theta} \hat{K}_{,j}^{CB} \theta_j \quad (15)$$

and $\hat{M}^{CB} = \hat{M}_0^{CB}$, where \hat{K}_0^{CB} and $\hat{K}_{,j}^{CB}$ are assembled from the component stiffness matrices by

$$\hat{K}_0^{CB} = \sum_{s \in \bar{\Sigma}} \mathbb{F}_s[\hat{K}^{(s)}] \quad \text{and} \quad \hat{K}_{,j}^{CB} = \sum_{s_j \in S_j} \mathbb{F}_{s_j}[\hat{K}^{(s_j)}] \quad (16)$$

The sum in the second of (16) considers that more than one components $s_j \in S_j$ depend on θ_j .

Solving the eigen-problem associated with the reduced matrices \hat{M}^{CB} and \hat{K}^{CB}

$$\hat{K}^{CB} Q = \hat{M}^{CB} Q \Lambda \quad (17)$$

one obtains the retained modal frequencies in $\Lambda = \text{diag}(\omega_i^2) \in \mathbb{R}^{N_k \times N_k}$ and the corresponding mode shapes $Q \in \mathbb{R}^{n_q \times N_k}$ of the reduced system, while the physical mode shapes are recovered as follows

$$\Phi = \hat{S} \Psi S Q = L Q \quad \text{or} \quad \phi_r = L \hat{q}_r \quad (18)$$

where $\hat{S} \in \mathbb{R}^{N_0 \times n_p}$ maps the generalized coordinates of each structural component to the physical coordinates of the structure, $Q = [\hat{q}_1, \dots, \hat{q}_m]$ is the matrix of mode shapes for the reduced system, $\Psi = \text{blockdiag}[\Psi^{(1)}, \dots, \Psi^{(N_s)}] \in \mathbb{R}^{n_p \times n_q}$, and $L = \hat{S} \Psi S$ is constant, independent of the parameters $\underline{\theta}$.

The matrices \hat{K}_0^{CB} and $\hat{K}_{,j}^{CB}$ in (15) are independent of the values of $\underline{\theta}$. In order to save computational time, these constant matrices are computed and assembled once and, therefore, there is no need this computation to be repeated during the iterations involved in optimization and

stochastic simulation algorithms. At each iteration step involved in model updating for which the value of the parameter set $\underline{\theta}$ changes, this procedure saves significant computational time since it avoids (a) re-computing the fixed-interface and constrained modes, and (b) assembling the reduced matrices from these components.

It should be noted that the modal frequency and mode shape residuals have the same exactly form as in (4) with $\underline{\phi}_r(\underline{\theta})$ replaced by $\underline{\hat{q}}_r(\underline{\theta})$ and L replaced by $L = \hat{S}\Psi S$. Available Bayesian uncertainty quantification and propagation software can thus be readily used to handle the parameter estimation using the reduced mass and stiffness matrices by just replacing the original eigenvalue problem with the eigenvalue problem (18) of the reduced system matrices and also replacing the matrix L of zeros and ones by the constant matrix $L = \hat{S}\Psi S$.

It should be pointed out that the significant savings arising partly from the reduction of the size of the eigenvalue problem in the CMS technique and partly from the fact that the estimation of the component fixed-interface modes and the constrained interface modes need not to be repeated for each iteration involved in the optimization. The computational savings depend on the size of the reduced system. This size is controlled by the number of fixed interface modes needed to describe the deformation of the component as well as the number of interface DOFs for each component. However, the number of interface DOFs may be large compared to the number of the fixed interface modes. The interface DOFs may control the size of the reduced mass and stiffness matrices. Further reduction in the generalized coordinates can be achieved by replacing the interface DOFs by a reduced number of constraint interface modes formed by a reduced basis. Selecting the reduced basis to be constant, independent of $\underline{\theta}$, the formulation significantly simplifies. The reduced basis can be kept constant at each iteration involved in the optimization algorithm or updated every few iterations in order to improve convergence and maintain accuracy.

4 APPLICATION

The computational efficiency and accuracy of the CMS technique for FE model updating is demonstrated using simulated data from the Metsovo bridge. Detailed FE models are created using 3-dimensional tetrahedron quadratic Lagrange FEs to model the whole bridge. An extra coarse mesh is chosen to predict the lowest 20 modal frequencies and mode shapes of the bridge. The model has 97,636 FEs and 563,586 DOFs. For demonstration purposes, the bridge is divided into fifteen physical components with eight interfaces between components as shown in Figure 1. Each deck component consists of several 4-5m deck sections. The tallest pier also consists of several sections. The size of the elements in the extra coarse mesh is the maximum possible one that can be considered, with typical element length of the order of the thickness of the deck cross-section.

The cut-off frequency ω_c is introduced to be the highest modal frequency that is of interest in FE model updating. In this study the cut-off frequency is selected to be equal to the 20th modal frequency of the nominal model. i.e. $\omega_c = 4.55$ Hz. The effectiveness of the CMS technique as a function of the number of modes retained for each component is next evaluated. For each component it is selected to retain all modes that have frequency less than $\omega_{\max} = \rho\omega_c$, where the ρ values affect computational efficiency and accuracy of the CMS technique. Representative ρ values range from 2 to 10. The total number of internal DOFs per component before the model reduction is applied are shown in Figure 2a. The number of modes retained per components for various ρ values is also given in Figure 2a. For the case $\rho = 8$, a total of 276 internal modes are retained for all 15 components. The total number of DOFs of the reduced model is 8,325 which consist of 276 fixed interface generalized coordinates and 8,049 constraint interface DOFs for all components. It is clear that a two orders of magnitude reduction in the number of DOFs is achieved using CMS.

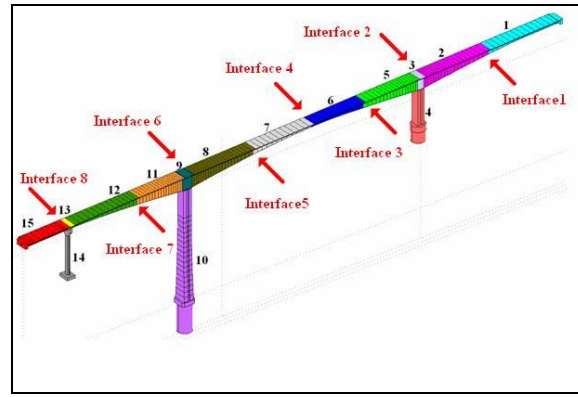


Figure 1- Components of FE model of Metsovo bridge.

Figure 2b shows the fractional error between the modal frequencies computed using the complete FE model and the ones computed using the CMS technique as a function of the mode number for $\rho = 2, 5$ and 8 . It is seen that the error for the lowest 20 modes fall below 10^{-5} for $\rho = 8$, 10^{-4} for $\rho = 5$ and 10^{-3} for $\rho = 2$. A very good accuracy is achieved for the case of $\rho = 2$.

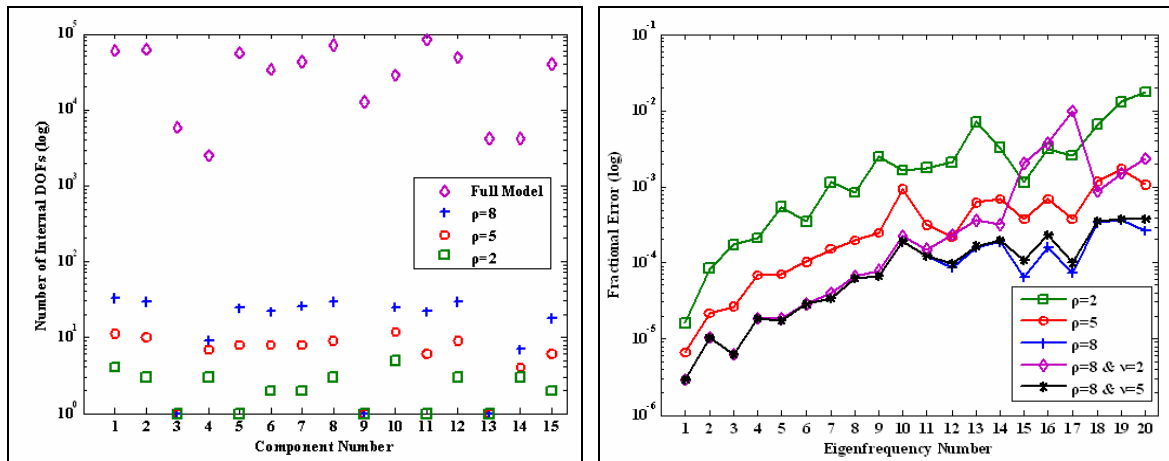


Figure 2- (a) Number of DOFs per component of FE model; (b) Fractional modal frequency error between predictions of the full and reduced model.

It is thus obvious that a large number of generalized coordinates for the reduced system arises from the interface DOFs. A further reduction in the number of generalized coordinates for the reduced system can be achieved by retaining only a fraction of the constrained interface modes. For each interface, it is selected to retain all modes that have frequency less than $\omega_{\max} = \nu\omega_c$, ν is user and problem dependent. Results are shown in Figure 2b for $\nu = 2$ and 5 . It can be seen that the fractional error for the lowest 20 modes of the structure fall below 10^{-3} for $\nu = 5$. The number of modes retained for different ν values is given in Table 1. In particular, the value of $\nu = 5$ and $\rho = 5$ gives accurate results and the number of retained interfaces modes for all interfaces is 54. The reduced system has 155 DOFs from which 101 generalized coordinates are fixed-interface modes for all components and the rest 54 generalized coordinates are constrained interface modes. The number of generalized coordinates is drastically reduced.

Table 1- Number of internal and boundary DOFs.

Total DOFs	Original	Reduced $\nu = 8$ & $\rho = 8$	Reduced $\nu = 5$ & $\rho = 5$	Reduced $\nu = 2$ & $\rho = 2$
Boundary	8,049	84	54	31
Internal	554,052	276	101	35
Total	562,101	360	155	66

The computational time needed to estimate the lowest 20 modal properties using CMS with $\rho \leq 8$ is five times less than the time required to solve the complete FE model. Reducing the constrained interface modes ($\nu \leq 5$), the computational time reduces by three to four orders of magnitude. It is thus obvious that CMS is expected to drastically reduce the computational effort in Bayesian uncertainty quantification and propagation framework without sacrificing in accuracy.

5 CONCLUSIONS

Component mode synthesis methods were presented to substantially reduce the computational effort required in the Bayesian tools for uncertainty quantification and propagation in structural dynamics. Exploiting certain schemes often encountered in finite element model parameterization, the mass and stiffness matrices of the reduced system are shown to depend linearly on the model parameters with the mass and stiffness sensitivity matrices to be assembled once and to remain constant during the iteration process. The only time consuming operation left is associated with the solution of the eigen-problem of the reduced system, avoiding the expensive estimation of the component eigen-problems at each iteration. The methodology is particularly efficient for large-scale finite element models where the solution of the component eigen-problem may be a computationally demanding operation. Further computational savings can be achieved by adopting surrogate models to substantially speed-up computations, and parallel computing algorithms to efficiently distribute the computations in available GPUs and multi-core CPUs.

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