New findings of the Transport Processes Lab on the primary instability of liquid film flows

Z. Cao[1] P.-K. Nguyen[1] A. Georgantaki[1] M. Vlachogiannis[1,2]
J. Vatteville[1] V. Bontozoglou[1]

[1]University of Thessaly, Department of Mechanical Engineering, GR-38334 Volos
[2]Technological Educational Institute of Larissa, GR-41100 Larissa

August 2010

Abstract

This work has been undertaken in the frame of Marie-Curie Initial Training Network “Multiflow”, which is investigating multiscale complex fluid flows and interfacial phenomena. We presently report the effect on liquid film flow of two apparently innocuous geometric constraints, a finite channel width and a periodically corrugated bottom wall. The work has an experimental and a computational /analytical component, and documents some unexpected finding: (i) The finite channel width introduces long-range transverse coherence of traveling disturbances, particularly so for high-surface tension liquids. As a consequence, the primary instability for film flow along a flat wall deviates in the order of 100% from the classical prediction. (ii) Steeply corrugated walls modify the above long-wave instability, but, most important, introduce at high enough inclination angles a new, oscillatory energy-transfer mechanism that results in an absolute instability at low *Re*.

Film flows at inclinations, θ, beyond a few tenths of a degree are first destabilized by an interfacial mode, whose streamwise scale is significantly larger than the mean film thickness. Classical theory[1] is based on long-wave expansion and results in a critical Reynols for the primary instability $Re_{cr} = \frac{5}{6} cot\,\theta$. Subsequent evolution is predicted to occur through non-linear interactions of saturated periodic waves, resulting in a train of solitary humps separated by flat streches of substrate.[2] The introduction of a corrugated wall is expected to modify the basis flow by triggering a steady deformation of the free surface.[3] This deformation attains large values for a range of *Re* (weak resonance).

The present experiments are performed in two facilities : one is a 3000 mm long by 800 mm wide inclined channel made of glass. The other is a 800 mm long by 250 mm wide channel made of Plexiglas. The width of the channels can be restricted by placing two plates of appropriate size along the sidewalls. The liquids used are water-glycerol solutions with 0-80 wt % in glycerol (1-30 times more viscous than water). The inclination angles tested cover the range of 3-45°. The spatio-temporal evolution of liquid film thickness is recorded by conductance probes and by a fluorescence imaging technique.

Computation/analysis is based on two approaches: The first is a finite-element solution of the full Navier-Stokes equations, which is used to describe accurately flow separation along steep corrugations and its interaction with free surface deformation. The second is an integral boundary layer equation,[4] which is derived by a weighted residual method and serves to defferentiate the temporal evolution of oscillatory instabilities into periodic and non-stationary/chaotic.

Considering first a channel with flat bottom and width, W, we document experimentally that the primary instability is postponed to higher Re as the channel becomes narrower. This unexpected finding occurs for widths that are orders of magnitude larger than the liquid film thickness. The deviation depends on the physical properties of the liquid, and is conveniently quantified by the Kapitza number, $Ka = \sigma/\rho g^{1/3}\nu^{4/3}$. The specific effect of Ka is shown in figure 1. Observations tend to the theoretical prediction for small Kapitza (Ka <100), but deviate progressively as Ka increases and eventually reach a plateau in the limit of high Kapitza (Ka >2000). The plateau values vary as $W^{-1/2}$.

Interpretation of the above behavior is facilitated by observing the evolution of disturbances beyond the stability threshold. Indeed, traveling waves are never two-dimensional, but attain a parabolic crestline shape, symmetric with respect to the channel centerplane (figure 2), with height diminishing towards the side-walls. The apex curvature of the parabola varies inversely with W and Re and increases with increasing Ka. These characteristics support an additional wave attenuation mechanism, which results from the decline with the approach to the side-walls of the capillary pressure below the wave hump. Thus, the dependence of the primary instability on Ka is interpreted as a competition between streamwise viscous dissipation and transverse capillary attenuation. The high-Ka plateau is reached when capillary forces dominate over viscous forces.

Moving next to a channel with periodically modified walls, we consider orthogonal corrugations with wavelength L=12 and 24 mm and height in the range B=0.4-1.6 mm. Corrugations are observed to postpone the long-wave instability to significantly higher Re (figure 3). Stabilization increases with the steepness of the corrugations (B/L), and is also affected by inclination angle, reaching maximum values in the range $\theta = 25 - 35°$. Beyond this inclination, the critical Re for the long-wave instability remains roughly constant or decreases slightly.

For the steepest wall (B/L=0.133) and at higher inclinations, a new instability appears at significantly lower Re (figure 4). It is a high-frequency oscillation of the free surface, which occurs irrespective of the imposition or not of a regular inlet disturbance. Thus, it has the appearance of an absolute instability. This finding is surprising, given that film flows are typically convectively unstable. The observed oscillation is roughly periodic in time, and thus can be satisfactory characterized by its frequency and amplitude. The dependence of these parameters on θ, Re and liquid properties is reported.

In an effort to understand the above behavior, and in particular the absolute instability, we examine computationally the detailed structure of steady film flow along steep corrugations. A key characteristic is the extensive flow separation over each corrugation trough and the interaction of the eddies with the free surface deformation. Multiple solutions are discovered over a parameter range, indicating the possibility of periodic switching from one to the other. Thus, the flow may be viewed as a series of weakly connected oscillators, which is susceptible to the "breather" instability known from similar models of solid-state systems.

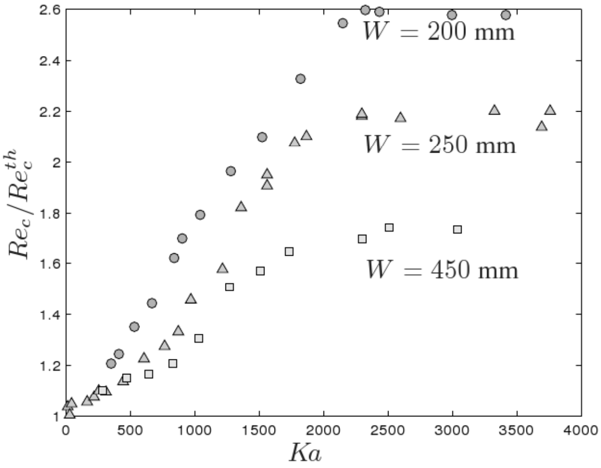

Figure 1: The ratio of experimental to theoretical critical Re as a function of Ka for a channel with a flat bottom and three different widths

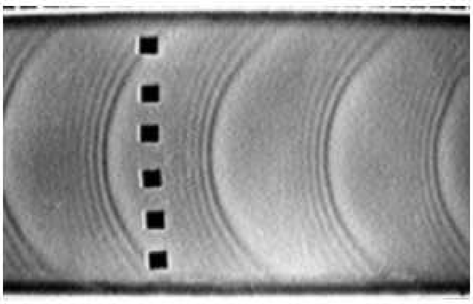

Figure 2: A train of fully-developed waves as viewed from below the channel (traveling direction from right to left)

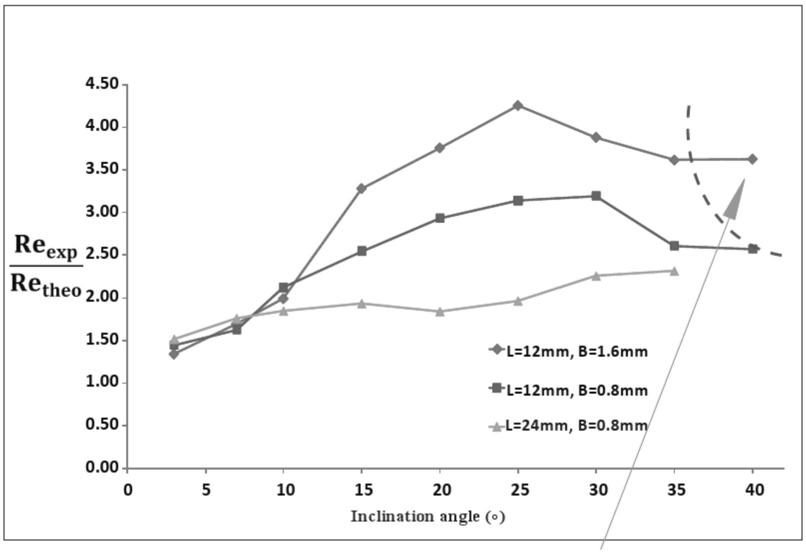

Figure 3: The effect of corrugation wavelength and height on the critical *Re*

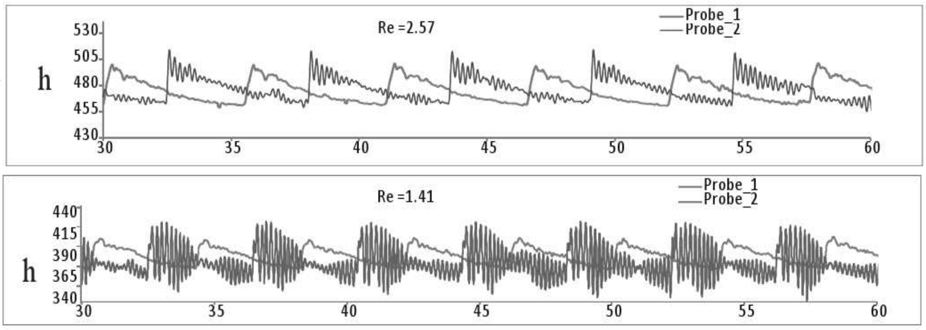

Figure 4: The oscillatory instability superimposed on a regular inlet disturbance

References

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