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PRETWISTED BEAMS IN AXIAL TENSION AND TORSION: AN ANALOGY WITH DIPOLAR GRADIENT ELASTICITY AND APPLICATIONS TO TEXTILE MATERIALS

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Abstract. *It is well known from St. Venant's torsion theory that when a torque is applied at the ends of a prismatic beam then the cross sections will, firstly, rotate about the centroid axis of the beam and, secondly, each cross section will warp in the longitudinal direction. Rotation is depicted through the angle of twist per unit length, while the warping is depicted through an appropriate warping function of the unrotated cross sections. In the present study we considered a prismatic beam with constant initial twist along its length and at the beam ends axial forces and torsional moments were applied. The governing equations of equilibrium and the boundary conditions are obtained using an energy variational statement. Focusing on the axial deformation, the results of the present study exhibit similarities with the results obtained from the analysis of prismatic beams subjected to axial tension using a dipolar gradient elasticity theory. The advantageous aspect of the present study is that the microstructural lengths emerge in a natural way from the geometrical characteristics of the beam cross section and the elastic material properties. The present results are extremely useful in modeling textile yarns with initial pretwist, as well in smart textiles where initial twist can be introduced deliberately.*

1 INTRODUCTION

Textiles are used in numerous advanced technological applications such as airbags, seat belts and body armor vests. The widespread usage of textile composites is mainly twofold. Primarily, their favorable mechanical properties classify them as very sufficient load carrying components. Secondarily, their low cost production and their easy handling make them very competitive structural materials.

The mechanics of textile composite materials can be addressed at three different scales: a) the macroscopic scale where textile is treated as an anisotropic, non-linear continuum medium, b) the mesoscopic scale where the overall mechanical behavior of the composite is characterized by the interactions between the yarns, c) the microscopic scale where interactions between the fibers inside yarns are taken into account. The present study focuses on the microscopic scale in the sense that the micromechanical parameters of the yarns are considered.

It is well known that most yarns are formed by the assemblage of a large number of fibers, usually some hundreds, which are pretwisted together about the longitudinal axis, Fig.1. In addition, the textile composite is subjected to tensile forces which tend to stretch the fiber in the longitudinal direction. So, the fiber's mechanical behavior is equivalent to a prismatic bar with initial twist subjected to an axial force at the same time.

Biot [1] was the first who mentioned that the torsional rigidity of a prismatic bar is increased when the bar is subjected to a tensile load. Chen [2] ascertains that, when a prismatic or a cylindrical thin walled bar possess an initial twist, the torsional rigidity of the bar is greater than the bar without initial twist. He concluded that the increase is due to the magnitude of the initial twist, the shape and the thickness of the thin wall cross section, as well as the material of the bar. The main goal of Chen's study was to estimate the torsional rigidity of steel thin wall prismatic bars and his findings were in good agreement with experimental results. The only handicap of his approach was that for a cyclic cross section he predicted an increase in torsional rigidity which is not true. The study of pretwisted prismatic bars intensified in the '70s, when the problems in helicopter blades came in the foreground (see for example [3, 4, 5]). Rosen [6] conducted a thorough report in which he verified the increase of torsional rigidity and he attributed it to the interference of the initial twist with the axial loading through the warping function of the cross section of the bar. This conclusion comes to recover the gap in Chen's theory, because it is in accordance with the well-known result that a cyclic cross section does not warp and generally it is not influenced by the twist of the beam.

2 KINEMATICS AND LINEAR STRAIN ANALYSIS

Consider a uniform bar of any cross section twisted by couples at the ends, Fig.2. An orthogonal coordinate system $Oxyz$ is adjusted at the center of an end cross section in such a way that the axis of the fiber coincides with the z axis. The displacement field of the cross section is defined as

$$u=-\frac{\partial\varphi(z)}{\partial z}zy \ , \ v=\frac{\partial\varphi(z)}{\partial z}zx \ , \ w(z)=w_1(z)-\frac{\partial\varphi(z)}{\partial z}\omega^*(x,y,z) \tag{1}$$

where u,v,w are the displacement components in Ox, Oy, Oz respectively, and w_1 is an additional displacement due to the application of the tension load. For a pretwisted bar with initial twist a_0 and length L, the Cartesian coordinates (ζ_1,ζ_2) are local to the cross section and are related to the global coordinates x,y,z as

$$\begin{aligned}\zeta_1 &= x\cos(a_0 z)+y\sin(a_0 z)\\ \zeta_2 &= -x\sin(a_0 z)+y\cos(a_0 z)\end{aligned} \tag{2}$$

since small deformation theory requires $|a_0 L| \ll 1$. From the assumed displacement field it is easy to evaluate the linear normal and shear strains in the cross sections:

$$\begin{aligned}&\varepsilon_{xx}=\frac{\partial u}{\partial x}=0 \ , \ \varepsilon_{yy}=\frac{\partial v}{\partial y}=0 \ , \ \varepsilon_{zz}=\frac{\partial w}{\partial z}=\frac{\partial w_1}{\partial z}-\frac{\partial^2\varphi}{\partial z^2}\omega^*-\frac{\partial\varphi}{\partial z}\frac{\partial\omega^*}{\partial z}\\ &\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=0 \ , \ \varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=-\frac{1}{2}\frac{\partial\varphi}{\partial z}\left(y+\frac{\partial\omega^*}{\partial x}\right) \ , \ \varepsilon_{yz}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=\frac{1}{2}\frac{\partial\varphi}{\partial z}\left(x-\frac{\partial\omega^*}{\partial y}\right)\end{aligned} \tag{3}$$

The function ω^* can be found from solving the classic Saint Venant problem in the (ζ_1,ζ_2) system [6]:

$$\frac{\partial^2\omega^*}{\partial\zeta_1^2}+\frac{\partial^2\omega^*}{\partial\zeta_2^2}=0 \tag{4}$$

with boundary conditions

$$-\frac{\partial\omega^*}{\partial n}=\zeta_2 n_{\zeta_1}-\zeta_1 n_{\zeta_2} \tag{5}$$

and

$$\iint\omega^* dxdy=0 \tag{6}$$

3 THE ELASTIC STRAIN ENERGY OF A PRETWISTED FIBER

The elastic strain energy stored in the fiber will be

$$U=\int_0^L\left[\iint\frac{E}{2}\varepsilon_{zz}^2 dxdy\right]dz+\int_0^L\left[\iint 2G\left(\varepsilon_{xz}^2+\varepsilon_{yz}^2\right)dxdy\right]dz \tag{7}$$

Substituting the strain components in the above equation we take

$$\begin{aligned}U=\frac{EA}{2}\int_0^L\Bigg\{&\left(\frac{\partial w_1}{\partial z}\right)^2+\frac{a_0^2K}{A}\left(\frac{\partial\varphi}{\partial z}\right)^2+\frac{J_\omega}{A}\left(\frac{\partial^2\varphi}{\partial z^2}\right)^2+\frac{2a_0R}{A}\frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial z^2}-\frac{2a_0S}{A}\frac{\partial w_1}{\partial z}\frac{\partial\varphi}{\partial z}\\ &+\frac{GJ}{EA}\left(\frac{\partial\varphi}{\partial z}\right)^2-\frac{2}{A}\frac{\partial^2\varphi}{\partial z^2}\frac{\partial w_1}{\partial z}\iint\omega^* dxdy\Bigg\}dz\end{aligned} \tag{8}$$

where in the last equation the following notations were introduced

$$A=\iint dxdy \tag{9}$$

$$a_0^2 K = \iint \left(\frac{\partial \omega^*}{\partial z} \right)^2 dxdy \tag{10}$$

$$a_0 R = \iint \omega^* \frac{\partial \omega^*}{\partial z} dxdy = \frac{1}{2} \iint \frac{\partial \left(\omega^*\right)^2}{\partial z} dxdy \tag{11}$$

$$a_0 S = \iint \frac{\partial \omega^*}{\partial z} dxdy \tag{12}$$

$$J_\omega = \iint \left(\omega^*\right)^2 dxdy = \ell^2 J \tag{13}$$

$$J = \iint \left[\left(y + \frac{\partial \omega^*}{\partial x} \right)^2 + \left(x - \frac{\partial \omega^*}{\partial y} \right)^2 \right] dxdy \tag{14}$$

where a_0 is the initial twist in the fiber, A, K, R, S are cross sectional constants, J_ω is the areal torsional constant of the cross section, ℓ is an internal length related to the geometry of the fiber cross section and J is the torsional constant of the cross section. Note that

$$A > 0 \quad , \quad K \geq 0 \quad , \quad J_\omega \geq 0 \quad , \quad S \leq 0 \quad , \quad J_p \geq J > 0 \tag{15}$$

Also, for cross sections with at least one axis of symmetry

$$\omega^* \left(\zeta_1 = 0, \zeta_2 = 0 \right) = 0 \tag{16}$$

4 CHARACTERISTIC EQUATIONS AND CORRESPONDING BOUNDARY CONDITIONS OF THE PROBLEM

The variation of the strain energy reads

$$\delta U = \int_0^L \left[\left(-EA \frac{\partial^2 w_1}{\partial z^2} + a_0 ES \frac{\partial^2 \varphi}{\partial z^2} \right) \delta w_1 \right] dz + \int_0^L \left\{ \left[\left(-a_0^2 EK - GJ \right) \frac{\partial^2 \varphi}{\partial z^2} + \ell^2 EJ \frac{\partial^4 \varphi}{\partial z^4} + a_0 ES \frac{\partial^2 w_1}{\partial z^2} \right] \delta \varphi \right\} dz$$
$$\left[\left(EA \frac{\partial w_1}{\partial z} - a_0 ES \frac{\partial \varphi}{\partial z} \right) \delta w_1 \right]_0^L + \left[\left(a_0^2 EK \frac{\partial \varphi}{\partial z} - \ell^2 EJ \frac{\partial^3 \varphi}{\partial z^3} - a_0 ES \frac{\partial w_1}{\partial z} + GJ \frac{\partial \varphi}{\partial z} \right) \delta \varphi \right]_0^L + \left[\left(\ell^2 EJ \frac{\partial^2 \varphi}{\partial z^2} + a_0 ER \frac{\partial \varphi}{\partial z} \right) \delta \frac{\partial \varphi}{\partial z} \right]_0^L \tag{17}$$

We suppose that there exists a distributed axial force p_z as mass force and a distributed torsional moment m_z as mass torsion such as

$$p_z = -\frac{\partial N}{\partial z} \quad , \quad m_z = -\frac{\partial T}{\partial z} \tag{18}$$

where N is the total axial force and T is the total torque at the cross sections of the fiber. The work done by the external loads is

$$\delta W = \int_0^L \left(p_z \delta w_1 + m_z \delta \varphi \right) dz + \left[N \delta w_1 \right]_0^L + \left[T \delta \varphi \right]_0^L + \left[-B \delta \frac{\partial \varphi}{\partial z} \right]_0^L \tag{19}$$

where B is the energy-conjugated quantity of $\delta(\partial\varphi/\partial z)$. The virtual work principal reads

$$\delta U - \delta W = 0 \Rightarrow \int_0^L \left[\left(-EA \frac{\partial^2 w_1}{\partial z^2} + a_0 ES \frac{\partial^2 \varphi}{\partial z^2} - p_z \right) \delta w_1 \right] dz$$
$$+ \int_0^L \left\{ \left[\left(-a_0^2 EK - GJ \right) \frac{\partial^2 \varphi}{\partial z^2} + \ell^2 EJ \frac{\partial^4 \varphi}{\partial z^4} + a_0 ES \frac{\partial^2 w_1}{\partial z^2} - m_z \right] \delta \varphi \right\} dz + \left[\left(EA \frac{\partial w_1}{\partial z} - a_0 ES \frac{\partial \varphi(z)}{\partial z} - N \right) \delta w_1 \right]_0^L$$
$$+ \left[\left(a_0^2 EK \frac{\partial \varphi}{\partial z} - \ell^2 EJ \frac{\partial^3 \varphi}{\partial z^3} - a_0 ES \frac{\partial w_1}{\partial z} + GJ \frac{\partial \varphi}{\partial z} - T \right) \delta \varphi \right]_0^L + \left[\left(\ell^2 EJ \frac{\partial^2 \varphi}{\partial z^2} + a_0 ER \frac{\partial \varphi}{\partial z} + B \right) \delta \frac{\partial \varphi}{\partial z} \right]_0^L = 0 \tag{20}$$

The last equation holds for all values of $\delta w_1, \delta\varphi$ so from the integrals one gets the equilibrium equations

$$-EA\frac{\partial^2 w_1}{\partial z^2}+a_0 ES\frac{\partial^2\varphi}{\partial z^2}-p_z=0 \tag{21}$$

$$\left(-a_0^2 EK-GJ\right)\frac{\partial^2\varphi}{\partial z^2}+\ell^2 EJ\frac{\partial^4\varphi}{\partial z^4}+a_0 ES\frac{\partial^2 w_1}{\partial z^2}-m_z=0 \tag{22}$$

and the energy-conjugated boundary conditions at the ends of the fiber $(z=0, z=L)$

$$\left[\left(EA\frac{\partial w_1}{\partial z}-a_0 ES\frac{\partial\varphi}{\partial z}-N\right)\delta w_1\right]_0^L=0 \tag{23}$$

$$\left[\left(a_0^2 EK\frac{\partial\varphi}{\partial z}-\ell^2 EJ\frac{\partial^3\varphi}{\partial z^3}-a_0 ES\frac{\partial w_1}{\partial z}+GJ\frac{\partial\varphi}{\partial z}-T\right)\delta\varphi\right]_0^L=0 \tag{24}$$

$$\left[\left(\ell^2 EJ\frac{\partial^2\varphi}{\partial z^2}+a_0 ER\frac{\partial\varphi}{\partial z}+B\right)\delta\frac{\partial\varphi}{\partial z}\right]_0^L=0 \tag{25}$$

5 CONSOLIDATION OF THE EQUATIONS AND THE 1D AXIAL MODEL

Differentiating equation (21) twice with respect to z we get

$$\frac{\partial^4\varphi}{\partial z^4}=\frac{1}{Ea_0 S}\frac{\partial^2 p_z}{\partial z^2}+\frac{A}{a_0 S}\frac{\partial^4 w_1}{\partial z^4} \tag{26}$$

Substituting equations (21) and (26) to equation (22) yields

$$\frac{\partial^2 w_1}{\partial z^2}-g^2\frac{\partial^4 w_1}{\partial z^4}+\frac{1}{AE}\left[-g^2 A^2\frac{\partial^2 p_z}{\partial z^2}+\left(\frac{a_0^2 EK+GJ}{JGc^2}\right)p_z+\frac{a_0 S}{Jc^2}\frac{E}{G}m_z\right]=0 \tag{27}$$

where in the last equation the following notation is used

$$c^2=\frac{J_{\text{eff}}}{J}=1+a_0^2\frac{E}{GJ}\left(K-\frac{S^2}{A}\right)\ ,\ c^2\geq 1 \tag{28}$$

$$g^2=\frac{E}{G}\frac{\ell^2}{c^2} \tag{29}$$

where J_{eff} is the effective torsional constant of the fiber with initial twist. The restriction $c^2>1$ stems from the fact that the condition $KA-S^2\geq 0$ is always true. Equation (28) shows that the torsional rigidity of the fiber is increased by the presence of an initial twist a_0 as has been also experimentally verified [2][6]. Also, equation (29) shows that the internal length g is defined by a natural way through E, G which are material properties, and ℓ, c which are geometrical constants of the cross section of the fiber. Equation (27) consist the governing equation of the problem regarding w_1. Combining boundary condition equations (23) and (24), in view of (21) stems

$$AE\left[\frac{\partial w_1}{\partial z}-g^2\frac{\partial^3 w_1}{\partial z^3}\right]=\mathcal{P} \tag{30}$$

where

$$\frac{a_0 SE}{c^2 GJ}T+g^2\frac{\partial p_z}{\partial z}+\frac{N}{c^2}\left(a_0^2\frac{E}{G}\frac{K}{J}+1\right)=\mathcal{P} \tag{31}$$

Combining boundary condition equations (23) and (25) in view of (21) one gets

$$AE\left[h\frac{\partial w_1}{\partial z}+g^2\frac{\partial^2 w_1}{\partial z^2}\right]=\mathcal{R} \tag{32}$$

where

$$h = \frac{a_0 R}{c^2 J} \frac{E}{G} \tag{33}$$

and

$$-\frac{\ell^2}{c^2} \frac{a_0 S}{GA} p_z + \frac{a_0 R}{c^2 GJA} N - \frac{a_0 S}{c^2 GJA} B = \mathcal{R} \tag{34}$$

The quantities $\mathcal{P}, \mathcal{R}$ can be thought as generalized force like quantities which can be illustrated by a concentrated/distributed torque, a concentrated/distributed axial force, a couple stress like force or a combination of all of them. The new problem is defined by equation (21) and the dynamic boundary condition (30) with the conjugate kinematical condition δw_1, and the dynamic condition (32) with the conjugate kinematical condition $\delta(\partial w_1/\partial z)$.

6 1-D STATIC DIPOLAR GRADIENT ELASTIC MODEL

Tsepoura et.al.[7] studied the problem of the response of a bar subjected to uniaxial loading using the linear dipolar gradient elasticity theory. In this context, the strain energy of the one-dimensional bar is defined as

$$U = \frac{1}{2} \cdot A \cdot \int_0^L \left[\tau \cdot \varepsilon + \mu \cdot \nabla \varepsilon \right] dz \tag{35}$$

where A is the cross sectional area of the bar, ε is the axial strain, $\nabla\varepsilon$ is the strain gradient and τ, μ are the Cauchy stress and couple stress, respectively. The constitutive equations are assumed to be

$$\tau = Eu' + \beta E u'' \tag{36}$$

$$\mu = \beta E u' + g^2 E u'' \tag{37}$$

where u is the axial displacement and primes denote differentiation with respect to longitudinal axis of the bar. The positive definiteness of the strain energy implies that

$$-g \le \beta \le g \ \left(g \ge 0 \right) \tag{38}$$

The variation of the strain energy of the bar is expressed as

$$\delta U = -\int_0^L AE\left(u'' - g^2 u^{IV}\right) \delta u dz + \left[AE\left(u' - g^2 u'''\right) \delta u \right]_0^L + \left[AE\left(g^2 u'' + \beta u'\right) \delta u' \right]_0^L \tag{39}$$

The variation of the work done by external forces is given by

$$\delta W = \int_0^L q \delta u dz + \left[\mathcal{P} \delta u \right]_0^L + \left[\mathcal{R} \delta u' \right]_0^L \tag{40}$$

where q are body forces, $\mathcal{P}$ are traction forces in the classical sense, and $\mathcal{R}$ are couple forces. The equilibrium of the dipolar gradient elastic bar implies that $\delta(U - W) = 0$ for every virtual kinematical quantity. From this condition stems the governing equation of the bar which reads

$$u''(z) - g^2 u^{IV} + \frac{1}{AE} q(z) = 0 \tag{41}$$

and the corresponding boundary conditions at the ends of bar $(z = 0, z = L)$

$$\left[\mathcal{P}(L) - AE\left[u'(L) - g^2 u'''(L) \right] \right] \delta u(L) - \left[\mathcal{P}(0) - AE\left[u'(0) - g^2 u'''(0) \right] \right] \delta u(0) = 0 \tag{42}$$

$$\left[\mathcal{R}(L) - AE\left[\beta u'(L) + g^2 u''(L) \right] \right] \delta u'(L) - \left[\mathcal{R}(0) - AE\left[\beta u'(0) + g^2 u''(0) \right] \right] \delta u'(0) = 0 \tag{43}$$

Comparison of equations(41) and (27),(42) and (30),(43) and (32), results to transparent similarities indicating the straightforward analogy between the dipolar gradient elastic theory with the present study. The internal material length h of the present study corresponds to the internal length β in [7].

7 PROBLEM SOLUTION

Assuming that $p_z = m_z = 0$(to simplify the problem) the governing equation (27) becomes

$$g^2 \frac{\partial^4 w_1}{\partial z^4} - \frac{\partial^2 w_1}{\partial z^2} = 0 \tag{44}$$

the latter equation has a solution of the form

$$w_1(z) = c_1 e^{\frac{z}{g}} + c_2 e^{\frac{-z}{g}} + c_3 z + c_4 \tag{45}$$

where c_1, c_2, c_3, c_4 are integration constants to be determined from the boundary conditions. Assuming that the fiber at one end is built –in, then is reasonable to state the displacement at that point to be zero, i.e.

$$w_1\big|_{z=0} = 0 \Rightarrow c_1 + c_2 + c_4 = 0 \tag{46}$$

The applied axial force on the other end of the fiber causes a specific strain, locally at that end and not along the whole length of the fiber, which implies

$$\left.\frac{dw_1}{dz}\right|_{z=L} = \varepsilon_1 \Rightarrow \frac{c_1}{g} e^{\frac{L}{g}} - \frac{c_2}{g} e^{\frac{-L}{g}} + c_3 = \varepsilon_1 \Rightarrow c_1 e^{\frac{L}{g}} - c_2 e^{\frac{-L}{g}} + g c_3 = g\varepsilon_1 \tag{47}$$

Supposing that on the free end of the fiber$(z = L)$, the condition $\mathcal{P}(z = L) = \mathcal{P}_1$ holds, then it would be

$$AE \left.\frac{\partial w_1}{\partial z}\right|_{z=L} - AEg^2 \left.\frac{\partial^3 w_1}{\partial z^3}\right|_{z=L} = \mathcal{P}_1 \Rightarrow \frac{c_1}{g} e^{\frac{L}{g}} - \frac{c_2}{g} e^{\frac{-L}{g}} + c_3 - \frac{c_1}{g} e^{\frac{L}{g}} + \frac{c_2}{g} e^{\frac{-L}{g}} = \frac{\mathcal{P}_1}{AE} \Rightarrow c_3 = \frac{\mathcal{P}_1}{AE} \tag{48}$$

Also, assuming that in the built –in end of the fiber $\mathcal{R}(z = 0) = 0$, then

$$AE\left[h\frac{\partial w_1}{\partial z} + g^2 \frac{\partial^2 w_1}{\partial z^2}\right] = 0 \Rightarrow h\left.\frac{\partial w_1}{\partial z}\right|_{z=0} + g^2 \left.\frac{\partial^2 w_1}{\partial z^2}\right|_{z=0} = 0 \Rightarrow (\lambda+1)c_1 - (\lambda-1)c_2 = \frac{-h\mathcal{P}_1}{AE} \tag{49}$$

where $\lambda = h/g$ is an auxiliary parameter. Equations $(46) - (49)$ form a system of four equations with four unknowns, the solution of which gives

$$\begin{gathered} c_1 = \frac{\left[\frac{-\lambda \mathcal{P}_1 L}{AE} + (1-\lambda)e^{L/g}\left(\varepsilon_1 L - \frac{\mathcal{P}_1 L}{AE}\right)\right]}{\frac{L}{g}\left[1+\lambda+e^{2L/g}(1-\lambda)\right]} \;,\; c_2 = \frac{(\lambda+1)e^{L/g}\left(-\varepsilon_1 L + \frac{\mathcal{P}_1 L}{AE}\right) - e^{2L/g}\frac{\lambda \mathcal{P}_1 L}{AE}}{\frac{L}{g}\left[1+\lambda+e^{2L/g}(1-\lambda)\right]} \\ c_3 = \frac{\mathcal{P}_1}{AE} \qquad ,\; c_4 = \frac{\lambda\left[\varepsilon_1 L - \frac{\mathcal{P}_1 L}{AE} + \frac{\mathcal{P}_1 L}{AE} cosh\left(\frac{L}{g}\right)\right]}{\frac{g}{L}\left[cosh\left(\frac{L}{g}\right) - \lambda sinh\left(\frac{L}{g}\right)\right]} \end{gathered} \tag{50}$$

In the case where $h = 0(\lambda = 0)$ the displacement field becomes

$$w^d = \frac{w_1(z)}{\frac{\mathcal{P}_1 L}{\mathrm{AE}}} = \left[\frac{e^{L/g} S_1}{\frac{L}{g}\left(1+e^{2L/g}\right)}\right] e^{\frac{z}{L}\left(\frac{L}{g}\right)} - \left[\frac{e^{L/g} S_1}{\frac{L}{g}\left(1+e^{2L/g}\right)}\right] e^{\frac{-z}{L}\left(\frac{L}{g}\right)} + \frac{z}{L} \tag{51}$$

where

$$S_1 = \frac{AE}{\mathcal{P}_1}\varepsilon_1 - 1 = \frac{N}{\mathcal{P}_1} - 1 \tag{52}$$

For the strain distribution along the fiber would be

$$\varepsilon(z)=\frac{dw_1(z)}{dz}=\frac{e^{\frac{L}{g}}\left(\varepsilon_1-\frac{\mathcal{P}_1}{AE}\right)}{1+e^{\frac{2L}{g}}}e^{\frac{z}{L}\left(\frac{L}{g}\right)}-\frac{e^{\frac{L}{g}}\left(\varepsilon_1-\frac{\mathcal{P}_1}{AE}\right)}{1+e^{\frac{2L}{g}}}e^{\frac{-z}{L}\left(\frac{L}{g}\right)}+\frac{\mathcal{P}_1}{AE} \tag{53}$$

where S_1 is given by equation(52). Plots of the displacement and strain field versus the ratio z/L, for $\lambda = 0$, and various values of S_1 are given in Fig.$3(a) - 3(b)$ and Fig. $4(a) - 4(b)$.

8 FIGURES

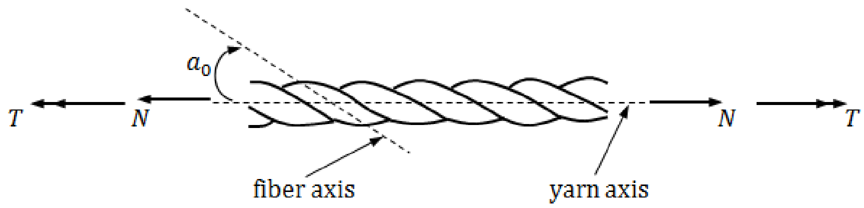

Figure 1. The angle of pretwist of the fiber is the angle between the fiber and the yarn axis.

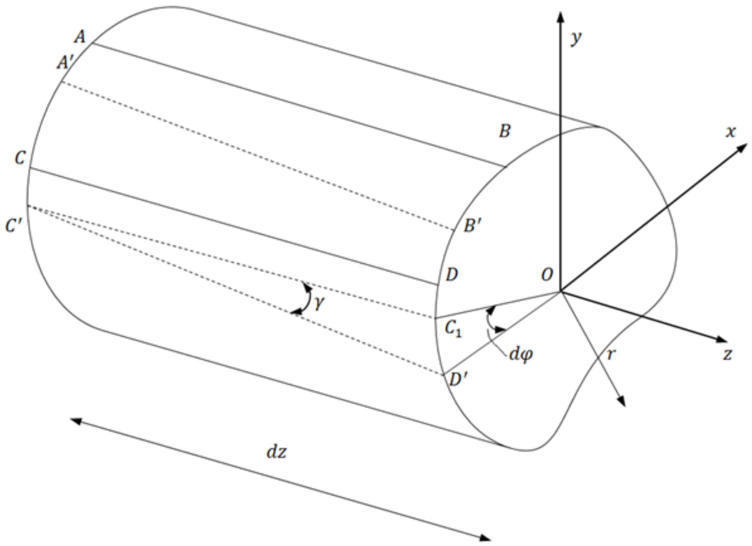

Figure 2. The rotation angle per unit length by couples applied at the ends.

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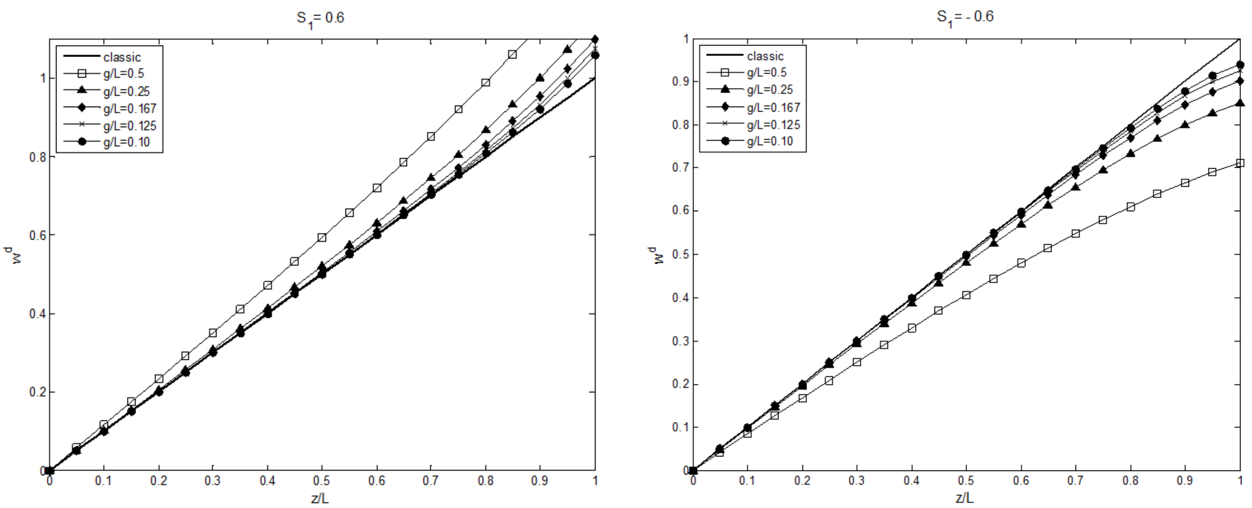
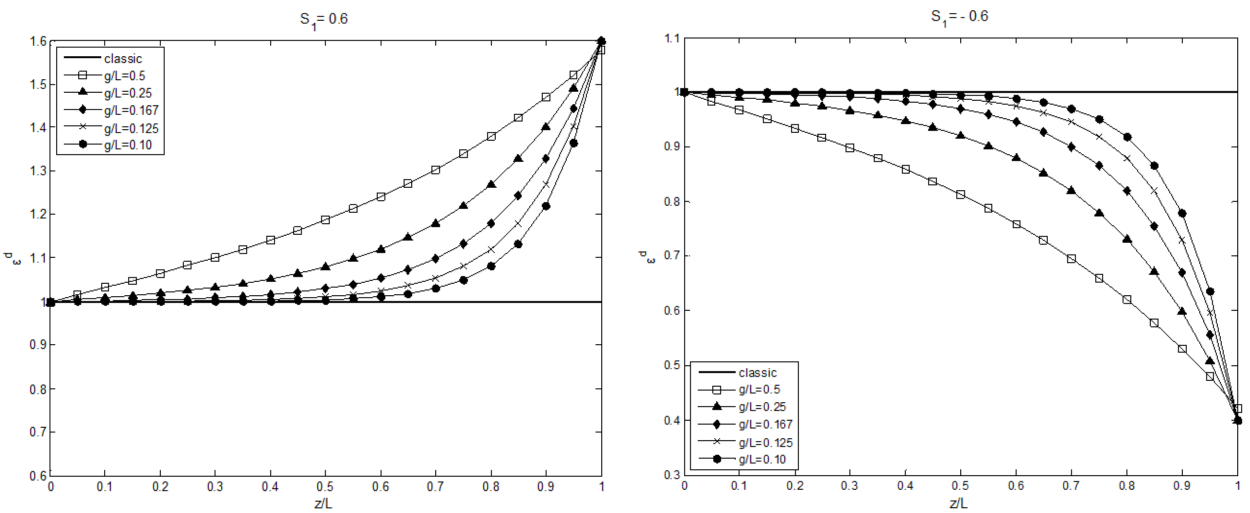


(a) (b)

Figure 3. Displacement field variation along the fiber axis for various values of $\boldsymbol{z/L}$, $(\boldsymbol{h=0})$.



(a) (b)

Figure 4. Strain field variation along the fiber versus $\boldsymbol{z/L}$ for various of $\boldsymbol{S_1}$ and $\boldsymbol{h=0}$.

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