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Uncertainty in optimal pollution levels: modelling and evaluating the benefit area

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This paper identifies the optimal pollution level under the assumptions of linear, quadratic and exponential damage and abatement cost functions and investigates analytically the certain restrictions that the existence of this optimal level requires. The evaluation of the benefit area is discussed and the mathematical formulation provides the appropriate methods for that to be calculated. The positive, at least from a theoretical point of view, is that both the quadratic and the exponential case obey the same form for evaluating the benefit area. These benefit area estimations can be used as indexes between different rival policies, and depending on the environmental problem, the policy that produces the maximum area will be the beneficial policy.

Keywords: benefit area; damage cost; abatement cost; pollution

1. Introduction

Rationality in the formulation and applicability of environmental policies depends on careful consideration of their consequences on nature and on society. For this reason it is important to quantify the costs and benefits in the most accurate way. However, the validity of any cost benefit analysis (hereafter CBA) is ambiguous as the results may have large uncertainties. Uncertainty is present in all environmental problems and this underscores the need for thoughtful policy design and evaluation. There may be uncertainty in the underlying physical or ecological processes, as well as about the economic consequences of the change in environmental quality.

These sources of uncertainty and their impact on policy formulation may be represented by the non-linear nature of the damage and abatement cost functions. Damage or external costs can be estimated by an analysis of the chain of pollution emissions, their dispersion and 'transportation' (in cases of transboundary pollution such as the acid rain problem), their effect measured among others with a dose-response function and their final (if feasible) monetary valuation. A similar picture is realised when referring to abatement costs, which may be less uncertain, compared to damage costs, but they are quite severe. The main problem in this case is related to technological change which may be essentially difficult to predict or sometimes even to characterise.[1]

Uncertainty is obvious not only in the parameters' estimation, but also in the choice of the appropriate model that 'fits' the problem. To make parameters' uncertainty clearer, we may think in terms of the fitted model assumed for the damage and abatement curves in a regression analysis that 'lies' between the upper and the lower bound of a 95%

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confidence interval. That is, there are two curves creating an interval of values for the fitted model and in this way uncertainty due to variation of the estimated coefficients.

As uncertainty may be due to the lack of appropriate abatement and damage cost data, here we apply a method of calibrating non-existing damage cost estimates relying on individual country abatement cost functions. In this way a 'calibrated' Benefit Area (BA^c) is estimated. Specifically, we try to identify the optimal pollution level under the assumptions of linear, quadratic and exponential abatement and damage cost functions. As far as the parameters are concerned, the first two are linear while the third is a non-linear function. That is, we consider the number of different model approximations of abatement and damage cost functions and in this way the assumed correct model eliminates uncertainty about curve fitting. The aim of this paper is to develop the appropriate theory whatever the model choice is.

The paper is structured as follows: section 2 discusses the background of the problem and reviews the relative existing literature. Section 3 identifies analytically the intersection of the marginal abatement cost curve (hereafter MAC) with the marginal damage cost curve (hereafter MD), in order to examine when and if an optimal pollution level exists. The existence of the intersection, despite the general belief, is not always true and the conditions are analytically examined here. In section 4 an empirical application for a sample of European countries, with different industrial structures, is presented. For these countries, the 'calibrated' Benefit Area (BA^c) is evaluated explicitly, provided there is an intersection of MD and MAC functions. The last section concludes the paper and comments on the policy implications related to this analysis providing evidence useful to researchers and policy makers.

2. Background to the problem

Abatement and damage cost functions are highly non-linear and the precise shapes of the functions are unknown. At the same time, environmental policies are related to significant irreversibilities, which usually interact in a very complex way with uncertainty. This complexity becomes worse if we think of the very long-term character of many environmental problems.

The damage cost function relates pollution and emissions of a specific pollutant. Damages are measured as the effect of these emissions on health, monuments, recreational activities, lakes, buildings etc. Efforts to measure the existence or other indirect use values are made with the help of contingent valuation and other methods (Freeman 1993; Grosclaude and Soguel 1994; Bjornstad and Kahn 1996; Von Blottnitz *et al*. 2006; Lavee and Becker 2009; Halkos and Jones 2012; Halkos and Matsiori 2012). As expected, the accurate measurement of damage is significant but also difficult due to many practical problems as presented in Georgiou *et al*. (1997), Barbier (1998) and Farmer *et al*. (2001). Ulph (2004) addressed the uncertainty in damage costs and the possible effects that may stem from resolving this uncertainty, leading countries to join an international environmental agreement.

Uncertainties in the functions of damage and control costs influence policy design in a number of ways. The first effect is in terms of the choice of the appropriate policy instrument. Weitzman (1974), in his seminal paper, showed that in the presence of uncertainty in cost functions, the instrument choice depends on the slopes of the curves. In certainty conditions either instrument will be equally effective but in uncertainty the choice is important and depends on the slopes of the marginal damage and abatement cost curves. In the case of steep marginal damage and flat marginal control cost curves, quantity-based instruments are more adequate, while in the case of steep marginal abatement and flat marginal damage cost curves a price-based instrument is to be chosen.

A number of studies have extended Weitzman's thesis and showed that in the case of uncertainty 'hybrid' policies of combining both instruments will dominate over the single instrument (Roberts and Spence 1976; Weitzman 1978; Pizer 2002; Jacoby and Ellerman 2004). It is worth mentioning that uncertainty may also affect the optimal timing of policy implementation if there are sunk costs in the implementation of that policy or the environmental damage from the lack of any policy is at least partly irreversible. The consequences of irreversibility have been studied extensively in the literature (Fisher and Hanemann 1990; Kolstad 1996; Ulph and Ulph 1997; Gollier, Bruno, and Treich 2000; Pindyck 2000, 2002).

As previously mentioned, damage and abatement cost functions seem to have a large curvature and in many cases are non-linear functions. A number of studies have tried to assess the cause and extent of the uncertainty over the benefits from a reduction in emissions.[2] Rabl, Spadaro, and van der Zwaan (2005) compared damage and abatement costs for a number of air pollutants. They distinguished between discrete and continuous policy choices. Setting a limit for sulphur dioxide emissions from power plants is an example of a continuous choice while the decision to demand a specific abatement method associated with a constant rate of emissions may be considered as a case of a discrete choice.

With regard to air pollution and in the case of GHGs, the first CBA was carried out by Nordhaus (1991). As found in Tol (2013) there are 16 studies and 17 estimates of the global welfare impacts of climate change (Nordhaus 1994a, 1994b, 2006, 2008, 2011; Fankhauser 1994, 1995; Tol 1995, 2002a, 2002b; Mendelsohn *et al*. 2000; Mendelsohn Schlesinger, and Williams 2000; Maddison 2003; Rehdanz and Maddison 2005; Maddison and Rehdanz 2011; Bosello, Eboli, and Pierfederici 2012). The welfare effect of doubling the atmospheric concentration of GHGs is relatively small (just a small percentage of GDP).

Tol (2013) presented a list of 75 studies with 588 estimates of the social cost of carbon emissions. He applied a kernel density estimator to the 588 observations expressed in 2010 US$ and pertaining to emissions in 2010.

Damage costs estimates can be also found among others in three well-known integrated assessment models (IAMs): the Dynamic Integrated Climate and Economy (DICE), the Policy Analysis of the Greenhouse Effect (PAGE), and the Climate Framework for Uncertainty, Negotiation and Distribution (FUND). For emission changes taking place in 2010, the value of the central social cost of carbon (hereafter SCC) is \$21/t of CO_2 emissions increasing to \$26/t of CO_2 in 2020 (Greenstone, Kopitsy, and Wolverton 2013).

Nordhaus (1994a) presented estimates of the percentage loss in gross world product, while Roughgarden and Schneider (1999) relying on Nordhaus' survey, together with other surveys, constructed confidence intervals for a damage function. Similarly Heal and Kriström (2002) and Pizer (2006) assessed uncertainty using subjective analysis and the opinions of experts. Specifically, Pizer (2003) modified the DICE model developed by Nordhaus (1994b), replacing the original quadratic relationship between damage and temperature change with a more complex function. The main conclusion from the empirical studies so far is that although there is a level of uncertainty, we are unable to quantify it. In terms of marginal damages of pollutants, Nordhaus (2008) presented a range of between \$6 and \$65/t carbon with a central estimate of \$27.

3. Determining the optimal level of pollution

Economic theory suggests that the optimal pollution level occurs when the marginal damage cost equals the marginal abatement cost. Graphically the optimal pollution level is presented in Figure 1 where the marginal abatement (MAC = g(z)) and the marginal damage (MD = φ(z)) are represented as typical mathematical cost functions. The point of intersection of the two curves, I = I(z_o, k_o), reflects the optimal level of pollution with k_0 corresponding to the optimum cost (benefit) and z_0 to the optimum damage restriction. It is assumed (and we will subsequently investigate the validity of this assumption) that the curves have an intersection and the area created by these curves (region AIB) is what we define as Benefit Area (see Kneese 1972, among others).

Halkos and Kitsos (2005) examined only three cases for the abatement cost function (linear, quadratic and exponential) and only linearity for the marginal damage cost function. We briefly review it so that the extensions become clearer, and cover all the possible cases in practice, especially the non-linearity nature of the damage cost function. Under these assumptions the extracted benefit areas were calculated. In the sequence of this paper we will examine how this crucial benefit area can be evaluated, providing an index, when different areas are investigated (such as countries or provinces), adopting different rival models and policies as they are expressed by the two curves under consideration.

Let A and B be the points of the intersection of the curves MD and MAC (see Figure 1) with the 'Y-axis'. Obviously we are restricted to positive values. For these points A = A(0, a) and β = β(0, b), the values of a and b are the constant terms of the

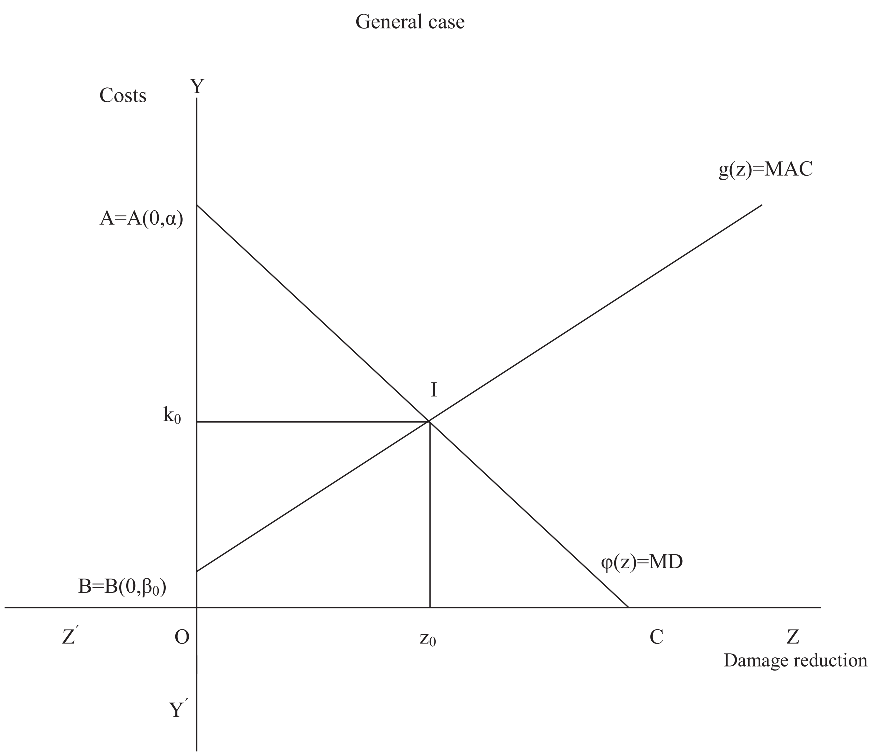

Figure 1. Graphical presentation of the optimal pollution level.

assumed curves that approach MD and MAC, respectively. For the linear case we have A = A(0, α) and $\beta = \beta(0, \beta_0)$ (see Figure 1) and it is assumed that $\alpha > \beta_0$ in Figure 1, provided that MAC is an increasing function and MD is a decreasing one.[3]

Specifically, we are now considering a number of cases in order to examine under what restrictions the two curves have an intersection, which is presented as $I = I(z_0, k_0)$. That is the equivalent mathematical problem: what the values are of the points z_0 and k_0 in order to have the optimal damage restriction and the corresponding value of the optimal cost, respectively. It is clear that, in principle, the intersection satisfies that $MAC(z_0) = MD(z_0) \Leftrightarrow g(z_0) = \varphi(z_0)$, with z_0 being the optimal restriction in damages. We are emphasising that the coefficients of the abatement cost functions (β_0, β_1, β_2) can be estimated by applying the OLS method to the appropriate dataset. Each assumption and case will now be examined in turn.

3.1. Case 1: MD and MAC functions are both linear

In the case of linearity of both MAC and MD the intersection $I = I(z_0, k_0)$ satisfies the following relationship: $\beta_0 + \beta_1 z_o = \alpha + \beta z_o$ and therefore z_0 can be evaluated as:

$$z_0 = -\frac{\beta_0 - \alpha}{\beta_1 - \beta} \tag{1}$$

Now we are asking for z_0 to be positive, i.e. to lie on the right half of $z'z$ axis as in Figure 1. If both g(z) and $\varphi(z)$ are linear the intersection exists at z_0 as in (1) if $\beta_1 > \beta$ as already $\alpha > \beta_0$. The corresponding optimal cost or benefit values should be equal for both curves.

$$k_0 = \phi\left(\frac{\beta_0 - \alpha}{\beta - \beta_1}\right) = \alpha + \beta\frac{\beta_0 - \alpha}{\beta - \beta_1} \quad \text{or} \quad k_0 = g\left(\frac{\beta_0 - \alpha}{\beta - \beta_1}\right) = \beta_0 + \beta_1\frac{\beta_0 - \alpha}{\beta - \beta_1}$$

This is true as their difference is zero. The benefit area, BA, is evaluated, in principle, through the following relation:

$$BA = (ABI) = (AIz_0 0) - (BIz_0 0) \tag{2}$$

where the parenthesis are the corresponding evaluated areas of Figure 1.

The benefit area for the case linear-linear (LL), BA_{LL}, can be evaluated as the area of the triangle ABI, namely:

$$BA_{LL} = (ABI) = \frac{(AB)(Ik_0)}{2} = \frac{(\alpha - \beta_0)(0z_0)}{2} = \frac{(\alpha - \beta_0)^2}{2(\beta_1 - \beta)} \tag{3}$$

Similarly, using Equation (2) the area can be evaluated by subtraction of the areas of the two trapezoidals leading up to Equation (3) (for details see Halkos and Kitsos 2005).

3.2. Case 2: MD linear and MAC quadratic functions

Let us consider now the case of a quadratic abatement cost function, which is the most likely case, i.e. $g(z) = MAC(z) = \beta_0 + \beta_1 z + \beta_2 z^2$. It is assumed that $b = MAC(0) = \beta_0 > 0$ and

$\frac{dg(z)}{dz} = \beta_1 + 2\beta_2 z > 0$, i.e. positive marginal abatement cost, that is $z > -\frac{\beta_1}{2\beta_2}$, which also means that the function g is increasing. The intersection point with the marginal damage can be evaluated as: $\mathrm{MAC}(z_0) = \mathrm{MD}(z_0) \Rightarrow \beta_0 + \beta_1 z_0 + \beta_2 z_o{}^2 = \alpha + \beta z_0$.

Recall that for the points A = A(0, a), B = B(0, b), and a = α and b = β_0, we assume that $\alpha > \beta_0$, therefore we have:

$$(\beta_0 - \mathrm{a}) + (\beta_1 - \beta)\mathrm{z}_0 + \beta_2 \mathrm{z}_0^2 = 0 \tag{4}$$

If we set $K = \beta_0 - \alpha$, $L = \beta_1 - \beta$ then Equation (4) becomes: $\beta_2 z_0{}^2 + Lz_0 + K = 0$, with roots:

$$z_0 = \frac{-L \pm \sqrt{D}}{2\beta_2}, \qquad \mathrm{D} = (\beta_1 - \beta)^2 - 4\beta_2(\beta_0 - \alpha) \geq 0 \tag{5}$$

The negative D has no economical meaning (as no real roots are evaluated) so a zero D leads from Equation (4) to a double or unique optimal restriction of damages of the form:

$$z_0 = -\frac{\beta_1 - \beta}{2\beta_2} \tag{6}$$

When assuming $\beta_1 < \beta$ the value of z_0 is positive, as β_2 has been assumed positive already. Thus the corresponding k_0 value, for the evaluated z_0 is:

$$\mathrm{k}_0 = \varphi(z_0) = \alpha - \beta\frac{\beta_1 - \beta}{2\beta_2} \quad \text{or}$$

$$k_0 = g(z_0) = \beta_0 + \beta_1\left[-\frac{\beta_1 - \beta}{2\beta_2}\right] + \beta_2\left[-\frac{\beta_1 - \beta}{2\beta_2}\right]^2$$

Under the assumptions of $\beta_0 - \alpha < 0$ and $\beta_2 > 0$ the quantity $-4\beta_2(\beta_0 - \alpha) > 0$ and therefore the value of the determinant is $D > 0$. This is true because the sum of the roots (equals to $2z_0$) is positive, while the product of the roots (equals to $[(\beta_0 - \alpha)/\beta_2]$) is negative. We are interested for at least a positive root z_0 in Equation (4), which under the assumption $\alpha > \beta_0$ can be evaluated only when $\beta_1 < \beta$ and eventually from Equation (5) we choose the positive z_0.

The corresponding benefit area for linear MD and quadratic MAC case, BA_{LQ}, is evaluated through the general form (2) subtracting from the trapezoidal AIz_o0 the area BIz_00, namely:

$$\mathrm{BA}_{\mathrm{LQ}} = \frac{(OA) + (Iz_0)}{2}(0z_0) - \int_0^{z_0} g(z)dz = \frac{\alpha + g(z_0)}{2} z_0 - [G(z_0) - G(0)] \tag{7}$$

with $G(z) = \beta_0 z + \beta_1 \dfrac{z^2}{2} + \beta_2 \dfrac{z^3}{3}$

which implies that G(0) = 0. So (7) is reduced to:

$$\mathrm{BA_{LQ}} = \frac{\alpha + g(z_0)}{2} z_0 - G(z_0),\ \ \mathrm{g(z) = MAC(z)} \tag{8}$$

The value of z_0 is as in Equation (6) and the assumptions $\beta > \beta_1$ and $\alpha > \beta_0$. That is as in Equation (3) a general form for the benefit area was produced when a linear marginal abatement cost φ(z) was examined.

3.3. *Case 3: MD linear and MAC exponential functions*

Let us consider the case of an exponential MAC function, i.e. MAC(z) = $\beta_0 \exp(\beta_1 z)$. In such a case b = MAC(0) = β_0 with a = α and the general line of thought for the intersection leads to:

$$\beta_0 e^{\beta_1 z_0} {=} a + \beta z_0 \Leftrightarrow \exp(\beta_1 z_0) = a^* + \beta^* z_0,\ \text{with}\ \ a^* = \frac{\alpha}{\beta_0}, \beta^* = \frac{\beta}{\beta_0}\ \text{with}\ \beta_0 \neq 0.$$

This results to (Halkos and Kitsos 2005):

$$\beta_1 \mathrm{z}_0 = \ln(\alpha^* + \beta^* \mathrm{z}_0) \Leftrightarrow z_0 = \frac{1}{\beta_1} \ln(\alpha^* + \beta^* z_0) = F(z_0) \tag{9}$$

Now Equation (9) is of the form z_0 = F(z_0), and can be only solved adopting numerical analysis techniques through the fixed-point theorem (see Ortega and Rheinbolt 1970; Halkos and Kitsos 2005 for details). The iteration is formed as:

$$z_{0,n+1} = \frac{1}{\beta_1} \ln(\alpha^* + \beta^* z_{0,n}) \quad n = 0, 1, 2 \tag{10}$$

converges to z_0, i.e. lim $z_{0,n+1} \to z_0 = F(z_0)$.[4] Specifically, the optimal restriction of damages level, z_0, in the exponential case of MAC only approximately can be evaluated and therefore the corresponding optimal cost or benefit level is approximately evaluated too.[5]

The corresponding benefit area (AIB) for the linear-exponential case, $\mathrm{BA_{LE}}$, is then evaluated through Equation (2) as:

$$\mathrm{BA_{LE}} = (\mathrm{AIz_0 0}) - \int_0^{z_0} g(z)dz = \frac{\alpha + g(z_0)}{2} z_0 - [G(z_0) - G(0)] \tag{11}$$

$$\text{with:}\quad G(z_0) - G(0) = \int_0^{z_0} \beta_0 e^{\beta_1 z} dz = \frac{\beta_0}{\beta_1}(e^{\beta_1 z_0} - 1) \tag{11.1}$$

i.e. G(0) = 1, while Equation (7) still holds, providing an index for benefit area for both quadratic and exponential cases, but with G(z_0) as in Equation (11.1) and z_0 approximated as in Equation (10).

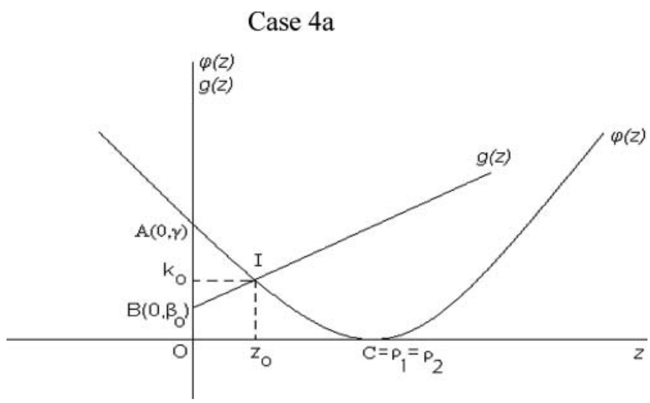

Figure 2. $C = C\left(-\frac{\beta}{2\alpha}, 0\right), \quad \alpha > 0.$

3.4. Case 4: MD quadratic and MAC linear functions

Let us now consider that:

$$\text{MAC} = g(z) = \beta_0 + \beta_1 z, \quad \beta_1 \neq 0 \quad \text{and} \quad \text{MD} = \varphi(z) = \alpha z^2 + \beta z + \gamma, \quad a > 0$$

The intersections of MD and MAC with the Y-axis are b = MAC(0) = β_o and a = MD(0) = γ (see Figures 2, 3 and 4). To ensure that an intersection between MAC and MD occurs we need the restriction $0 < \beta_0 < \gamma$. Considering $\alpha > 0$ three cases can be discussed, through the determinant of *φ(z),* say *D*, $D = \beta^2 - 4\alpha\gamma$. (a) $D = 0$ (see Figure 2), (b) $D > 0$ (see Figure 3), and $D < 0$. Note that the case $D < 0$ has no economical interest (due to the complex roots). Therefore the two cases are discussed below, while for the dual case $\alpha < 0$ see case 4c.

3.4.1. Case 4a: $\alpha > 0, D = \beta^2 - 4\alpha\gamma = 0$

In this case there is a double real root for MD*(z),* say $\rho = \rho_1 = \rho_2 = -\frac{\beta}{2\alpha}$. In principle a double root means: only one value ρ exists for the quadratic model, where the optimal level z_0 lies with$(0, \rho)$. When two roots are evaluated, $\rho_1 < \rho_2$ we are restricted to

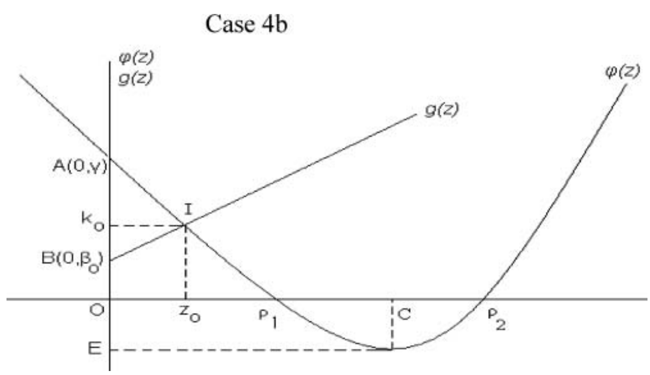

Figure 3. $C = C\left(-\frac{\beta}{2\alpha}, 0\right), \quad E = E\left(0, \varphi\left(-\frac{\beta}{2\alpha}\right)\right), \quad \varphi\left(-\frac{\beta}{2\alpha}\right) = \min \varphi(z), \quad \alpha > 0.$

investigate the interval $(0, \rho_1)$ the candidate interval where z_0 lies. We need the root $\rho > 0$ and hence is required that $\beta < 0$. To identify the optimal pollution level point $\mathrm{I}(z_0, k_0)$ the evaluation of point z_0 is the one for which:

$$\mathrm{MD}(z_0) = \mathrm{MAC}(z_0) \Leftrightarrow \alpha z_0^2 + (\beta - \beta_1)z_0 + (\gamma - \beta_0) = 0 \tag{12}$$

In order for Equation (12) to provide the unique (double) solution we need the quantity the determinant of Equation (12) $D = (\beta - \beta_1)^2 - 4\alpha(\gamma - \beta_0)$ to be zero, i.e. $D = 0$ which is equivalent to:

$$z_0 = \frac{\beta_1 - \beta}{2\alpha} \tag{13}$$

As z_0 is positive and $\alpha > 0$ we conclude that $\beta_1 > \beta$. So for the conditions are: $\alpha > 0$, $\beta_1 > \beta, 0 < \beta_0 < \gamma$ we can easily calculate

$$k_0 = \mathrm{MAC}(z_0) = \beta_0 + \beta_1 \frac{\beta_1 - \beta}{2\alpha} > 0 \tag{14}$$

and therefore $\mathrm{I}(z_0, k_0)$ is well defined. The corresponding Benefit Area ($\mathrm{BA_{QL}}$) in this case is:

$$\mathrm{BA}_{QL} = \int_0^{z_0} (\varphi(z) - g(z))dz = \int_0^{z_0} (az^2 + (\beta - \beta_1)z + (\gamma - \beta_0))dz = \alpha\frac{z_0^3}{3} + (\beta - \beta_1)\frac{z_0^2}{2} + (\gamma - \beta_0)z_0 \tag{15}$$

3.4.2. Case 4b: $\alpha > 0, \quad D = \beta^2 - 4\alpha\gamma > 0$

In such a case for the two roots ρ_1, ρ_2 we have $|\rho_1| \neq |\rho_2|$, $\varphi(\rho_1) = \varphi(\rho_2) = 0$ and we suppose $0 < \rho_1 < \rho_2$, (see Figure 3). The fact that $D > 0$ is equivalent to $0 < a\gamma < \left(\frac{\beta}{2}\right)^2$, while the minimum value of the MD function is:

$$\varphi\left(-\frac{\beta}{2\alpha}\right) = \frac{4\alpha\gamma - \beta^2}{4\alpha}$$

Therefore we state:

Proposition 1: *The order for the roots $0 < \rho_1 < \rho_2$ and the value which provides the minimum of the MD function is true under the relation*:

$$\beta < 0 < \alpha\gamma < \left(\frac{\beta}{2}\right)^2 \tag{16}$$

The proof is shown in the Appendix.

We can then identify the point of intersection z_0: $\mathrm{MAC}(z_0) = \mathrm{MD}(z_0)$ as before. Therefore under Equation (16) and $\beta_1 > \beta_0$ we evaluate k_0 as in (14) and the Benefit Area $\mathrm{BA_{QL}}$ can be evaluated as in Equation (15).

Let us now consider the case $\alpha < 0$. Under this assumption the restriction $D = 0$ is not considered, as the values of $\varphi(z)$ have to be negative. Consider Figure 2, the graph of $\varphi(z)$

will be symmetric around the point C to this in Figure 2. Therefore we consider the following case.

3.4.3. Case 4c: $\alpha < 0, D = \beta^2 - 4\alpha\gamma > 0$

Under the assumption of Case 4c, the value $\varphi\left(-\frac{\beta}{2\alpha}\right) = \frac{4\alpha\gamma-\beta^2}{4\alpha}$ corresponds to the maximum value of $\varphi(z)$. We consider the situation where $\rho_1 < 0 < -\frac{\beta}{2\alpha} < \rho_2$ (see Figure 4) while the case $0 < \rho_1 < -\frac{\beta}{2\alpha} < \rho_2$ has no particular interest (it can be also considered as in case 4b).

Proposition 2: *For the case 4c as above holds:* $\rho_1 < 0 < -\frac{\beta}{2\alpha} < \rho_2$ *when* $\alpha\gamma < 0$.

The proof is shown in the Appendix.

The imposed assumption is equivalent to $\alpha\varphi(0) < 0 \Leftrightarrow \alpha\gamma < 0$ true as $\rho_1\rho_2 < 0$ $\alpha\varphi\left(-\frac{\beta}{2\alpha}\right) < 0 \Leftrightarrow \alpha\gamma < \left(\frac{\beta}{2}\right)^2$. Therefore the imposed restrictions are $\alpha\gamma < 0 < \left(\frac{\beta}{2}\right)^2$. Actually $\alpha\gamma < 0$. In case 4c it is now asked $\beta_0 < \gamma$ and $\beta_1 > 0$. To calculate z_0 we proceed as in Equation (12) and z_0 is evaluated as in Equation (13) with $\alpha < 0$, therefore $\beta_1 - \beta < 0$ i.e. $\beta_1 < \beta$. Thus for $\beta_1 < \beta, \alpha\gamma < 0$, the BA as in (15) is still valid.

Therefore the following analysis is now considered, extending case 4 of both MD and MAC to be quadratic functions.

3.5. *Case 5: MD and MAC functions both quadratic*

Let us consider that both curves, MAC and MD are of the second order as:

$$\begin{aligned} \mathrm{MAC} &= g(z) = \beta_0 + \beta_1 z + \beta_2 z^2, \quad \beta_2 \neq 0(\text{with determinant d}) \\ \mathrm{MD} &= \varphi(z) = \alpha z^2 + \beta z + \gamma, \quad a \neq 0(\text{with determinant D}) \end{aligned}$$

In this case the intersections of MAC and MD with the Y-axis are b = MAC(0) = β_0 and a = MD(0) = γ. The following two substances are investigated below.

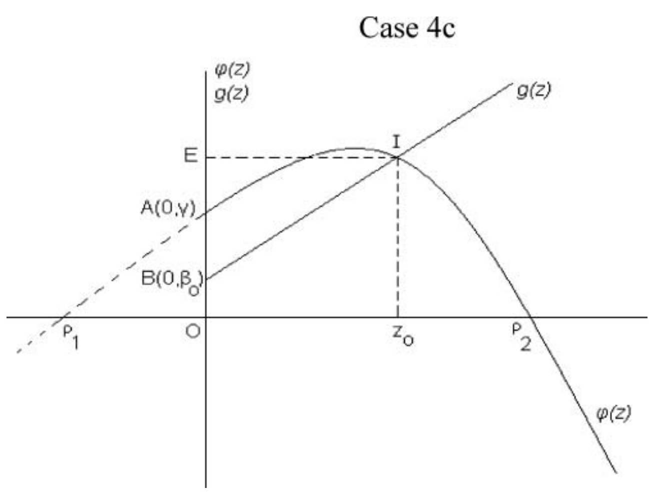

Figure 4. $C = C\left(-\frac{\beta}{2\alpha}, 0\right), \quad E = E\left(0, \varphi\left(-\frac{\beta}{2\alpha}\right)\right), \quad \varphi\left(-\frac{\beta}{2\alpha}\right) = \min\varphi(z), \quad \alpha < 0.$

3.5.1. Case 5a: $D = \beta^2 - 4\alpha\gamma = 0, a > 0$ and $d = \beta_1^2 - 4\beta_2\beta_0 \leq 0, \beta_2 > 0$, (see Figure 5).

For the identification of the optimal pollution level point $I(z_0, k_0)$ we need to estimate z_0 such that

$$\varphi(z_0) = g(z_0) \Leftrightarrow (\alpha - \beta_2)z_0^2 + (\beta - \beta_1)z_0 + (\gamma - \beta_0) = 0 \tag{17}$$

when the determinant of this equation δ is zero,

$$\delta = (\beta - \beta_1)^2 - 4(\alpha - \beta_2)(\gamma - \beta_0) = 0$$

the unique root of (17) equals $z_0 = -\dfrac{\beta - \beta_1}{2(\alpha - \beta_2)}$. (18)

Recall case 2 Equation (6). It is asked not only z_0 to be positive but to be less than the (double) root of MD (see Figure 5), i.e.

$$0 < z_0 = -\frac{\beta - \beta_1}{2(\alpha - \beta_2)} < -\frac{\beta}{2\alpha},$$

Therefore we have the restriction $\dfrac{\beta}{\alpha} < \dfrac{\beta - \beta_1}{\alpha - \beta_2}$.

Thus the corresponding to the optimal restriction in damage z_0, optimal cost k_0 point is:

$$k_0 = \varphi(z_0) = \alpha\frac{(\beta - \beta_1)^2}{4(\alpha - \beta_2)^2} - \beta_0\frac{\beta - \beta_1}{\alpha - \beta_2} + \gamma \tag{19}$$

Thus the corresponding Benefit Area, BA_{QQ}, can be evaluated as:

$$BA_{QQ} = \int_0^{z_0} (\varphi(z) - g(z))dz = (\alpha - \beta_2)\frac{z_0^3}{3} + (\beta - \beta_1)\frac{z_0^2}{2} + (\gamma - \beta_0)z_0 \tag{20}$$

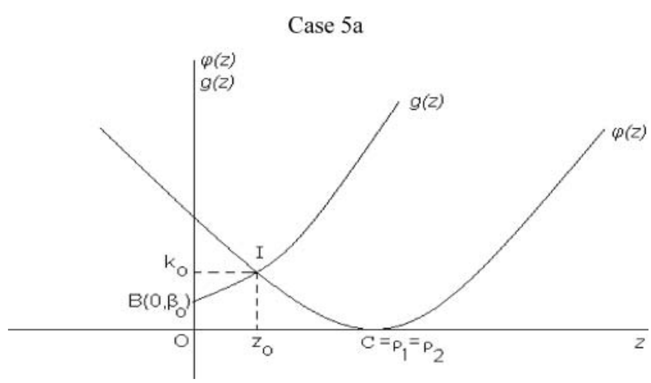

Figure 5. $C = \rho_1 = \rho_2 = C\left(-\frac{\beta}{2\alpha}, 0\right)$, $\varphi\left(-\frac{\beta}{2\alpha}\right) = 0$, $\alpha > 0$.

It is worth mentioning that in all cases investigated in this paper the MAC and MD are evaluated by using appropriate regression models for the former and by the use of calibrations for the latter. Therefore all parameter estimates are under error, implying that the Benefit Area evaluations are just approximations. The better fit modelling the less error in parameters and to Benefit Area eventually. This justifies why the appropriate model specification is essential in any empirical study.

Another interesting case is the following.

3.5.2. *Case 5b: $D = \beta^2 - 4\alpha\gamma > 0, a > 0$ and $d = \beta_1^2 - 4\beta_2\beta_0 \leq 0, \beta_2 > 0$ (see Figure 6).*

In this case we need the order of the points to be $0 < z_0 < \rho_1 < -\frac{\beta}{2\alpha} < \rho_2$. Recall also case 4b (see Figure 6). Therefore we need the following conditions $\alpha\varphi\left(-\frac{\beta}{2\alpha}\right) < 0$, which holds and $D > 0, \quad \alpha\varphi(z_0) > 0, \quad z_0 < -\dfrac{\beta}{2\alpha}$.

Eventually is needed to have the restrictions:

$$\alpha\varphi(z_0) > 0 \quad \text{and} \quad z_0 < -\frac{\beta}{2\alpha}.$$

The point z_0 is eventually as in Equation (18) and the corresponding k_0 as in Equation (19) so $\mathrm{I}(z_0, k_0)$ is identified. The BA is evaluated as in Equation (20).

3.6. *Case 6: MD quadratic and MAC exponential functions*

We now assume that MD is quadratic function and MAC is exponential, that is:

$$\mathrm{MAC}(z) = g(z) = \beta_0 e^{\beta_1 z}, \quad \beta_0 > 0$$

$$\mathrm{MD}(z) = \varphi(z) = \alpha z^2 + \beta z + \gamma, \quad a > 0$$

Here b = MAC(0) = β_0, and a = MD(0) = γ. In particular we are investigating the following sub-cases, which are within our target.

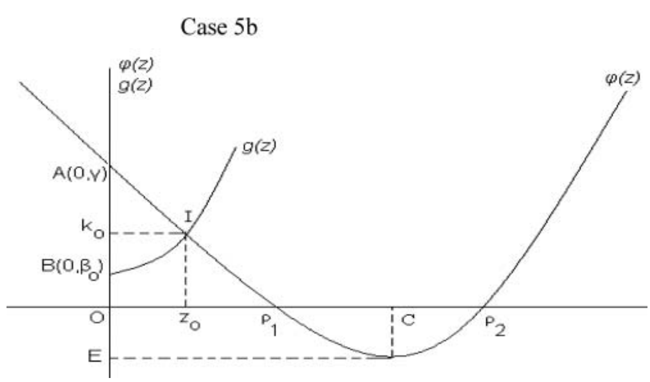

Figure 6. $C = C\left(-\dfrac{\beta}{2\alpha}, 0\right), \quad E = E\left(0, \varphi\left(-\dfrac{\beta}{2\alpha}\right)\right), \quad \varphi\left(-\dfrac{\beta}{2\alpha}\right) = \min\varphi(z), \quad \alpha > 0$

$$0 < z_0 < \rho_1 < -\frac{\beta}{2\alpha} < \rho_2.$$

3.6.1. Case 6a: For MD it is $D = \beta^2 - 4\alpha\gamma > 0$.

Evaluating I (z_0, k_0), z_0 has to obey to the relationship:

$$\varphi(z_0) = g(z_0) \Leftrightarrow \alpha z_0^2 + \beta z_0 + \gamma = \beta_0 e^{\beta_1 z_0} \Leftrightarrow \alpha z_0^2 + \beta z_0 + \gamma - \beta_0 e^{\beta_1 z_0} = 0 \tag{21}$$

Equation (21) is non-linear so we have to prove that there is one solution, which can be evaluated numerically.[6]

Proposition 3: *There is a value $z_0 \in (0, \rho_1)$ for the damage reduction where MD and MAC coincide, i.e. MD(z_0) = MAC(z_0).*

The proof is shown in the Appendix.

If we consider the equation $F(z) = \alpha(z - \rho_1)(z - \rho_2) - \beta_0 e^{\beta_1 z}$ then under the imposed restrictions $F(0) = \gamma - \beta_0 > 0$ as $\rho_1 \rho_2 = \frac{\gamma}{\alpha}$ and $F(\rho_1) = -\beta_0 e^{\beta_1 \rho_1} < 0$. Therefore as $F(0)F(\rho_1) < 0$ there exists a number $z_0 \in (0, \rho_1) : F(z_0) = 0$ and therefore Equation (21) is true that there exists a real solution (root) z_0 for it.

The evaluation of the root $z_0$of Equation (21) can be numerically calculated and one way is by adopting the Bisction method. The corresponding $k_0 = g(z_0) = \beta_0 e^{\beta_1 z_0}$ can be easily evaluated, as the corresponding Benefit Area (BA_{EQ}) is calculated as:

$$BA_{EQ} = \int_0^{z_0} (\varphi(z) - g(z))dz = \alpha \frac{z_0^3}{3} + \beta \frac{z_0^2}{2} + \gamma z_0 - \frac{\beta_0}{\beta_1}(e^{\beta_1 z_0} - 1) \tag{22}$$

The last case is to have both MAC and MD functions exponential as follows.

3.7. Case 7: MD and MAC both exponential functions

In this case it is assumed that both MB and MAC are exponential of the form[7]:

$$\text{MAC}(z) = g(z) = \beta_0 e^{\beta_1 z}, \quad \beta_0 > 0$$

$$\text{MD}(z) = \varphi(z) = \theta_0 e^{\theta_1 z}, \quad \theta_0 > \beta_0 > 0$$

The target is to evaluate the z_0 point of the optimal pollution level as:

$$\varphi(z_0) = g(z_0) \Leftrightarrow \beta_0 e^{\beta_1 z_0} = \theta_0 e^{\theta_1 z_0} \Leftrightarrow \frac{\beta_0}{\theta_0} = e^{(\theta_1 - \beta_1)z_0} \Leftrightarrow z_0 = \frac{1}{(\beta_1 - \theta_1)} \ln \frac{\theta_0}{\beta_0}$$

From the above relation it is clear that there is no intersection when $\theta_1 = \beta_1$. The corresponding Benefit Area (BA_{EE}) is evaluated as:

$$BA_{EE} = \int_0^{z_0} (\varphi(z) - g(z))dz = \frac{\beta_0}{\beta_1}(e^{\beta_1 z_0} - 1) - \frac{\theta_0}{\theta_1}(e^{\theta_1 z_0} - 1) \tag{23}$$

Finally, the case that $\varphi(z)$ is exponential and $g(z)$ is of second order is the dual of case 6a. In this way we have investigated all possible cases modelling MAC and MD. A compact view of all the cases is presented in Table 1. The results obtained above have been adopted in an empirical application discussed in the next section.

Table 1. The compact presentation of the results.

Case	MD = φ(z)	MAC = g(z)	$z_0 > 0$	BA	Restrictions
1	$\alpha + \beta z$	$\beta_0 + \beta_1 z$	$-\frac{\beta_0 - \alpha}{\beta_1 - \beta}$	(3)	$\alpha > \beta_0$ $\beta_1 > \beta$
2	$\alpha + \beta z$	$\beta_0 + \beta_1 z + \beta_2 z^2$	$-\frac{\beta_1 - \beta}{2\beta_2}$	(7), (8)	$\beta_2 > 0$ $\beta > \beta_1$ $\alpha > \beta_0$
3	$\alpha + \beta z$	$\beta_0 e^{\beta_1 z}$	Numerically (10)	(11)	$\beta_1 > 0$
4	$\alpha z^2 + \beta z + \gamma$	$\beta_0 + \beta_1 z$			$0 < \beta_0 < \gamma$
4a	$\alpha > 0$ D = 0				$\beta_1 > \beta$ $\beta < 0$ $0 < \beta_0 < \gamma$
4b	$\alpha > 0$ D > 0		$Z_0 = \frac{\beta_1 - \beta}{2\alpha}$	(15)	$\beta < 0 < \alpha\gamma < (\beta/2)^2$ $\beta_1 > \beta_0$
4c	$\alpha < 0$ D > 0				$\beta_1 < \beta$ $\alpha\gamma < 0$
5	$\alpha z^2 + \beta z + \gamma$	$\beta_0 + \beta_1 z + \beta_2 z^2$			$\alpha \neq 0$ $\beta_2 \neq 0$
5a	$\alpha > 0$ D = 0	$\beta_2 > 0$ d ≤ 0			
5b	$\alpha > 0$ D > 0	$\beta_2 > 0$ d ≤ 0	$Z_0 = -\frac{\beta - \beta_1}{2(\alpha - \beta_2)}$	(20)	$\beta_1 > \beta$ $\alpha > \beta_2$
6	$\alpha z^2 + \beta z + \gamma$	$\beta_0 e^{\beta_1 z}$		(20)	$\beta_0 > 0$ $\alpha > 0$
6a	$\alpha > 0$, D > 0		Numerically	(22)	$\gamma > \beta_0$
7	$\theta_0 e^{\theta_1 z}$	$\beta_0 e^{\beta_1 z}$	$z_0 = \frac{1}{\beta_1 - \theta_1} \ln \frac{\theta_0}{\beta_0}$	(23)	$\beta_1 \neq \theta_1$

4. An empirical application

In our empirical application we will use estimates for the available data for different European countries. For this purpose we discuss how the two curves, the abatement cost $g(z)$ and the damage cost $\varphi(z)$, can be approximated. The abatement cost function measures the cost of reducing tonnes of emissions of a pollutant, such as sulphur (S), and differs from country to country depending on the local costs of implementing best practice abatement techniques as well as on the existing power generation technology. For abating sulphur emissions various control methods exist with different cost and applicability levels such as gas oil desulphurisation, heavy fuel oil desulphurisation, hard coal washing, in furnace direct limestone injection, flue gas desulphurisation and fluidised bed combustion (Halkos 1995).

To calculate total emissions of the pollutant after (any) control from each source (TE_p) the annual emissions for a given pollutant in each sector for each European country are calculated. Total emissions are then determined as:

$$TE_p = \sum [PR_{ijt} \times (1 - \alpha_t) E_{pij} \times AR_{ijtf}] \quad (24)$$

where i stands for country, j for sector, t for technology, f for fuel and p for pollutant. Similarly, PR stands for production levels; α_t for the abatement efficiency of method t and AR for the application rate (Halkos 2013).

In the same way, given the generic engineering capital and operating control cost functions for each efficient abatement technology, total and marginal costs of different levels of pollutant's reduction at each individual source and in the national (country) level can be constructed. According to Halkos (1995, 2013), the cost of an emission abatement option is given by the total annualised cost (TAC) of this abatement option, including capital and operating cost components. Specifically:

$$TAC = \{(TCC)[r/(1-(1+r)^{-n})]\} + VOMC + FOMC \tag{25}$$

Where TCC is the total capital cost; VOMC and FOMC stand for the variable and fixed operating and maintenance cost, respectively; $r/[1-(1+r)^{-n}]$ is the capital recovery factor at real discount rate r, which converts a capital cost to an equivalent stream of equal annual future payments, considering the time value of money (represented by r). Finally, n stands for the economic life of the asset (in years).

For every European country a least cost curve is derived by finding the technology on each pollution source with the lowest marginal cost per tonne of pollutant removed in the country and the amount of pollutant removed by that method on that pollution source. In this way the first step on the country's abatement curve is constructed. Iteratively the next highest marginal cost is found and is added to the country curve with the amount of pollutant (say sulphur) removed on the X-axis. In the national cost curve each step corresponds to a control measure that leads to an emission reduction of an extra unit at the least cost. Figure 7 shows the Total Abatement Cost curve for Austria in the year 2000.

For analytical purposes, it is important to approximate the cost curves of each country by adopting a functional form. Extending the mathematical models described above to stochastic models (as the error term from the Normal distribution with mean zero and variance σ^2) we have found that least squares equations of the form

$$g(z) = AC_i = \beta_{0i} + \beta_{1i}SR_i + \varepsilon_i \tag{26.1}$$

or

$$g(z) = AC_i = \beta_{0i} + \beta_{1i}SR_i + \beta_{2i}SR_i^2 + \varepsilon_i \tag{26.2}$$

lead to satisfactory approximations for all the countries analysed in this paper.[8] In these equations SR_i represents sulphur removed in country i, AC_i abatement cost in country i and ε_i the disturbance term with the usual hypotheses.

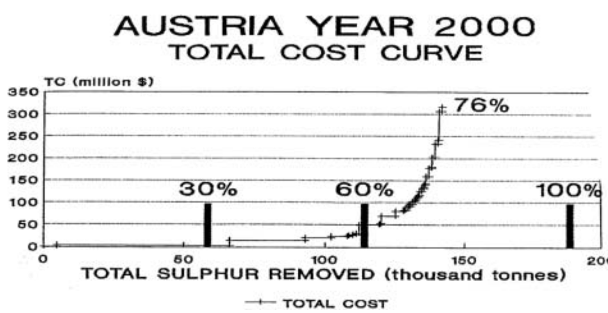

Figure 7. Total Abatement Cost curves for Austria in the year 2000.

Next, the calculation of the damage function $\varphi(z)$ is necessary. The problem of estimating damage cost functions is far more difficult compared to the estimation of abatement costs, as the effects of pollution cannot be identified with any accuracy and sometimes it takes a long time to realise the consequences. In our case and in order to extract the damage estimates we use the case of acidification which is related to transboundary pollution, requiring that the model takes account of the distribution of the externality among the various countries (victims).

Each country receives a certain number of pollutant's units whose deposition is due to the other countries' emissions as well as its own emissions. The deposition of sulphur in country i is given by:

$$D_i = B_i + d_{ii}(1-\alpha_i)E_i + \sum\nolimits_{i \neq j} d_{ij}(1-\alpha_j)E_j = B_i + \sum\nolimits_j d_{ij}(1-\alpha_j)E_j \qquad (27)$$

where E_j is the total annual sulphur emission in country j; D_i is the total Annual sulphur Deposition in country i; α_i is the abatement efficiency coefficient in country i and d_{ij} is the transfer coefficient from country j to i, indicating what proportions of emissions from any source country are ultimately deposited in any receiving country; B_i is the level of the so-called background deposition attributable to natural sources (such as volcanoes, forest fires, biological decay, etc.) in receptor-country i, or to pollution remaining too long in the atmosphere to be tracked by the model, i.e. is probably attributable not only to natural sources but also to emissions whose origin cannot be determined. This assignment is summarised in the European Monitoring and Evaluation Programme transfer coefficient matrix (EMEP).

We do not directly estimate the damage function, but instead we infer its parameters assuming that countries currently equate national marginal damage cost with national marginal abatement cost.[9] The restrictions on the derivatives of the damage cost function are important. The total cost resulting from a specific level of pollutant (such as sulphur) for country i is:

$$\mathrm{TC}_i = \text{abatement cost} + \text{damage cost} = \mathrm{AC}_i + \mathrm{DC}_i.$$

As previously mentioned, abatement costs are estimated by linear and quadratic functions of sulphur removed (as in Equations (26.1) and (26.2)) and we also assume that damage costs are linear and quadratic in deposits.[10]

Table 2 presents the estimated damage and abatement cost coefficients in the more complex case of assuming that both cost functions are quadratic.[11] Similarly, Table 3 presents the corresponding 'calibrated' Benefit Area (BA^c) indexes evaluated from the available parameter estimates in the case of linear-quadratic (LQ) and quadratic-quadratic (QQ) damage and abatement cost functions, respectively, for 20 European countries.[12] In the first of the two case studies the associated efficiency index[13] is presented.[14] The area for which the BA is evaluated can be 'country', 'province' or 'municipality' areas. This is a measure of what percentage of the adopted policy covers that policy which provides the maximum benefit area. Similarly, the evaluation of the optimal damage reduction, z_0 as has been denoted in this paper, provides evidence that the larger it is the better the adopted environmental policy.

As can be seen, countries with high optimal damage reductions are the UK and France (in both cases LQ and QQ) and Former Czechoslovakia, Spain and Turkey (in the LQ case). On the other hand, countries with low damage reductions are Greece, Hungary,

Table 2. Coefficient estimates in the case of quadratic MD and MAC functions.

Countries	c_0	c_1	c_2	b_0	b_1	b_2
Albania	0.7071	0.01888	0.0001397	−3.3818	0.015	0.0048
Austria	8.57143	0.055012	0.0001145	3.274	−0.221	0.004
Belgium	2.2424	0.03869	0.0001688	0.497	−0.124	0.003
Former Czechoslovakia	37.794	0.100323	0.000059	11.241	0.2358	0.00018
Denmark	10	0.1923	0.0060811	−2.49	0.099	0.0053
Finland	4.021	0.0781	0.00014587	2.343	−0.098	0.0046
France	33.158	0.277352	0.000197	42.374	−0.053	0.0018
Greece	3.7373	0.034133	0.0000491	−1.614	0.342	0.0006
Hungary	5.101	0.031488	0.0000417	2.506	0.216	0.0004
Italy	21.01	0.030036	0.0000191	12.5	0.36	0.0003
Luxembourg	0.421	0.3161	0.0272381	−0.7272	0.01	0.09234
Netherlands	8.353	0.19513	0.00351442	−6.18	0.41	0.0009
Norway	1.421	0.07852	0.00017008	0.94	−0.244	0.0164
Poland	6.212	0.023153	0.000071	−8.023	0.324	0.00009
Romania	9.091	0.011364	0.00006237	5.502	0.19	0.0001
Spain	11.7	0.007288	0.00497419	10.21	−0.021	0.00014
Sweden	2.4	0.06423	0.0000932	4.074	−0.252	0.004
Switzerland	2.4	0.56027	0.002803	5.7543	−1.6289	0.11203
Turkey	14.9	0.01781	0.00001223	8.0622	0.011	0.00036
UK	19.1	0.06879	0.0000467	15.54	0.0264	0.0003

Italy, Romania (in the case of LQ cost functions), Finland, Sweden, Turkey (in the case of QQ) and Norway and Switzerland in both cases. Large industrial upwind counties (like Denmark, France and the UK) seem to have very large benefit areas. Looking at the EMEP transfer coefficients matrix, it can be seen that the countries with large benefit areas are those with large numbers on the diagonal. This shows the importance of the domestic sources of pollution. The large off-diagonal transfer coefficients indicate in general the major effects one country has on another, and especially the externalities imposed by the Eastern European countries on the others.

Similarly downwind or near to the sea countries seem to have small benefit areas. In addition, the damage caused by acidification depends on where the depositions occur. In the case of occurrence over the sea it is less likely to have much harmful effect, as the sea is naturally alkaline. In the same way, if it occurs over sparsely populated areas with acid tolerant soils then the damage is low (Newbery 1990).

5. Conclusions and policy implications

The analysis of the effectiveness of environmental programmes and regulations requires the comparison of damage and control costs associated with the reduction of different pollutants. It is worth mentioning that although there is an obvious uncertainty in damage costs we cannot ignore that uncertainty is also present in the abatement cost functions due to abatement efficiencies that may differ between countries and between adopted scenarios.

The typical approach to define the optimal pollution level has been to equate the marginal damage of an extra unit of pollution with the corresponding marginal abatement cost. An efficient level of emissions maximises the net benefit that is the difference

Table 3. Calculated ‘calibrated’ Benefit Areas (BA^c)

	Linear - Quadratic						Quadratic - Quadratic	
Countries	D	z_0	g(z_0)	G(z_0)	BA	Eff	z_0	BA
Albania	0.07852	29.594	−3.38	−52.05	81.24	3.5872	0.416282	1.460084
Austria	0.160942	84.649	3.3	294.1	628.6	27.756	35.51795	34.38386
Belgium	0.047413	63.406	0.5	37.2	182.8	8.0715	28.73163	58.58427
Former Czechoslovakia	0.037791	160.988	11.24	5119.8	2264.6	100		
Denmark	0.273493	58.138	−2.5	369.72	536.7	23.698	−507.79	66016.29
Finland	0.061886	46.182	2.4	154.72	114.3	5.0453	19.76705	1.578593
France	0.042777	149.22	42.4	7726.3	309.2	13.65	103.0412	950.4364
Greece	0.107625	16.83	−1.62	22.23	45.5	2.0095		
Hungary	0.038197	13.66	2.51	54.72	17.9	0.7901		
Italy	0.119088	25.22	12.5	431.19	108.1	4.7726		
Luxembourg	0.517796	5.56	−0.73	1.4	5.8	0.2572	2.351007	2.359444
Netherlands	0.098488	54.98	−6.18	329.7	424.4	18.741	−73.6549	85.02478
Norway	0.135573	21.056	0.94	16.75	30.6	1.3508	9.93597	2.085391
Poland	0.095634	46.67	−8.03	−18.57	333.7	14.734		
Romania	0.033347	19.87	5.5	147.1	35.8	1.5803		
Spain	0.001635	245.43	10.2	2563.2	527.8	23.305	−2.92603	11.43726
Sweden	0.073218	73.35	4.1	147.1	201.7	8.9075	40.47174	55.68209
Switzerland	3.289337	17.87	5.56	55.8	76.5	3.378	10.02119	24.05788
Turkey	0.009866	147.82	8.1	1698.5	698.65	30.851	9.819189	58.17368
UK	0.006069	200.5	15.6	4452.1	759.9	33.551	83.67548	1813.895

between abatement and damage costs. Therefore the identification of this efficient level shows the level of benefits maximisation, which is the output level resulting if external costs (damages) are fully internalised.

In this paper the corresponding optimal cost and benefit points were evaluated analytically. It is shown that this is feasible in the linear and quadratic cases while in the exponential case only approximated values can be obtained. The explicit evaluation of the benefit area was also discussed and analytical forms for this particular area were calculated for different policies. In this way the optimal level was also evaluated.

We show that the optimal pollution level can be evaluated only under certain conditions, as were derived in section 3. Specifically, it is required that in all of the cases $\alpha > \beta_0$ if we assume that MAC is an increasing function and MD is a decreasing function. That is, the constant term in the damage cost function (we may think of the background deposition) is bigger than the abatement cost at level $z = 0$ (we may think of fixed costs of operating an abatement method at level $z = 0$). In cases of both linear or both quadratic functions we have $\beta > \beta_1$. The slope of the benefit function must be greater than the marginal abatement cost at level $z = 0$. For the quadratic case it is required that $\beta_2 > 0$ while for the exponential case β_0, $\beta_1 > 0$. Both the quadratic and the exponential cases obey the same form of evaluating the benefit area.

From our empirical findings it is clear that the evaluation of the 'calibrated' Benefit Area, as it was developed, provides an index to compare the different policies adopted from different countries on the basis of how large calibrated Benefit Area eventually provides. In this way a comparison of different policies can be performed. Certainly the policy with the maximum Benefit Area is the best, and the one with the minimum is the worst. Clearly the index BA^c provides a new measure for comparing the adopted policies.

An important finding (in the case of transboundary pollution) is that domestic pollution sources are important while large industrial upwind counties seem to have a very large benefit area. On the other hand, countries downwind near to the sea or over sparsely populated areas with acid tolerant soils seem to have small benefit areas. As mentioned, the empirical results derived are only indicative and very sensitive to the assumptions of calibration.

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Notes

1. Rabl and Holland (2008) illustrated an impact pathway framework for analysing external costs of environmental burdens, together with the inventory stage of life cycle assessment.
2. The cost functions may not behave well or may not satisfy the conditions of convexity or concavity. In the case of the damage cost function this may take place by threshold effects as well as by any irreversibility where pollution reaches a critical point at which the receptor (rivers, lakes, etc.) is damaged completely and cannot sustain any life. If one or both of the cost functions are not well behaved then our results will be different. At the same time, the

distinction between flow and stock pollutants is important because for stock pollutants the persistence has to be taken into consideration due to the accumulation (and decay) of pollutant(s) in time (Perman *et al*. 2011). As an example, we may consider the case of F-Gases with the very high global warming potentials (Halkos 2013).

3. This is clear as if it is assumed that $\alpha<\beta_0$ there is no intersection (no benefit area) and if we let $\alpha=\beta_0$ the benefit area coincides with the point, namely A = B = I, that a one point area is created.
4. The numerically evaluated root is under an error ε, say $|z_{0,n+1}-z_0|<10^{-6}=\varepsilon$. Therefore the values of $MAC(z_0)$ and $MD(z_0)$ are approximated values.
5. Practically that results in the value of the difference $MAC(z_0)$-$MD(z_0)$ is not zero, but close to zero, with a certain accuracy $\pm\zeta$, say $\zeta=10^{-3}$ or 10^{-6}.
6. The graphical presentation of this case is similar to Figure 6.
7. In such a case there are 'fast' increases/decreases of both marginal abatement and marginal damage costs, indicating possibly for the former the existence of limited and expensive control methods, and for the latter very high instant damages.
8. Equations were fitted across the range 5-55% of maximum feasible abatement. The estimated coefficients of both specifications were statistically significant in all cases with only exception in the estimate of β_1 in the quadratic specification of Spain.
9. Rabl, Spadaro, and van der Zwaan (2005) approximated the damage function by a linear function of the pollution emissions and they claimed that linearity is found to be appropriate approximation in the case of PM, SO_2 and NO_X emissions, while for CO_2 linearity is probably acceptable for emissions reductions in the 'foreseeable' future period.
10. Damage is a function of deposits, which depend on the transfer coefficients [d_{ij}] matrix as explained before. In the more complex case of both quadratic abatement and damage cost functions the total cost function may be expressed as:

    $TC_i=[\beta_{0i}+\beta_{1i}\,SR_i+\beta_{2i}\,SR_i^2]+[\gamma_{0i}+\gamma_{1i}\,D_i+\gamma_{2i}\,D_i^2]$ I = 1,2,. . .,n

    To 'calibrate' the damage function, we assume that national authorities act independently (as Nash partners in a non-cooperative game with the rest of the world), taking as given deposits originating in the rest of the world. Specifically, we minimise TC_i with respect to SR_i and we calibrate the damage function by taking the first order conditions (for more details see Hutton and Halkos 1995, 265).
11. Estimates of c_0 were derived by assuming countries act in a Nash behaviour.
12. The empirical results presented are indicative and very sensitive to the assumptions of calibration.
13. Following Halkos and Kitsos (2005) the efficiency (Eff) of the benefit area, in comparison with the maximum evaluated from the sample of countries under investigation, can be estimated using as measure of efficiency the expression:

$$\mathrm{Eff}=\left[\frac{\mathrm{BA}}{\mathrm{maxBA}}\right]^{*}100$$

    This efficiency is evaluated for the same class of model, referring to different data sets in each case.
14. Germany dominates the picture in Europe as it has a very high initial abatement level (≈42%) and its calibrated damage function ensures high abatement levels (Hutton and Halkos 1995). For this reason the efficiency index was constructed on the second highest benefit area (former Czechoslovakia).

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Appendix

Proof of Proposition 1

The order of the roots $0<\rho_1<\rho_2$ is equivalent to the set of relations:

$$D > 0, \quad \alpha\varphi\left(-\frac{\beta}{2\alpha}\right) < 0, \quad \alpha\varphi(0) > 0, \quad 0 < \frac{\rho_1 + \rho_2}{2}. \tag{16.1}$$

The first is valid, as we have assumed $D > 0$. For the imposed second relation from Equation (16.1) we have $\alpha\varphi\left(-\frac{\beta}{2\alpha}\right) < 0 \Leftrightarrow \alpha\frac{4a\gamma-\beta^2}{4\alpha} < 0 \Leftrightarrow \Delta > 0$, which holds. As both the roots are positive $\rho_1, \rho_2 > 0$, then the product $\rho_1\rho_2 > 0$ therefore $\dfrac{\gamma}{\alpha} > 0 \Leftrightarrow \alpha\gamma > 0$ which is valid as $0 < a\gamma < \left(\frac{\beta}{2}\right)^2$. The third relation $\alpha\varphi(0) = \alpha > 0$, in (16.1) is true already and $0 < \frac{\rho_1+\rho_2}{2} = -\frac{\beta}{2\alpha}$ equivalent to $\beta < 0$. Therefore we get $\beta < 0 < a\gamma < \left(\frac{\beta}{2}\right)^2$.

Proof of Proposition 2

The imposed assumption is equivalent to $\alpha\varphi(0) < 0 \Leftrightarrow \alpha\gamma < 0$ true as $\rho_1\rho_2 < 0$ $\alpha\varphi\left(-\frac{\beta}{2\alpha}\right) < 0 \Leftrightarrow \alpha\gamma < \left(\frac{\beta}{2}\right)^2$. Therefore the imposed restrictions are $\alpha\gamma < 0 < \left(\frac{\beta}{2}\right)^2$

Actually $\alpha\gamma < 0$. Case 4c requires that $\beta_0 < \gamma$ and $\beta_1 > 0$. To calculate z_0 we proceed as in (12) and z_0 is evaluated as in (13) with $\alpha < 0$, we have therefore $\beta_1 - \beta < 0$ i.e. $\beta_1 < \beta$. Thus for $\beta_1 < \beta$, $\alpha\gamma < 0$, the BA as in (15) is still valid.

Proof of Proposition 3

If we consider the equation $F(z) = \alpha(z-\rho_1)(z-\rho_2) - \beta_0 e^{\beta_1 z}$ then under the imposed restrictions $F(0) = \gamma - \beta_0 > 0$ as $\rho_1\rho_2 = \frac{\gamma}{\alpha}$ and $F(\rho_1) = -\beta_0 e^{\beta_1\rho_1} < 0$. Therefore, as $F(0)F(\rho_1) < 0$ there exists a number $z_0 \in (0, \rho_1) : F(z_0) = 0$ and therefore Equation (21) is true that there exists a real solution (root) z_0 for it.