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A stochastic vehicle routing model and its specifications

E. C. Chrysohoou[1] A.Ziliaskopoulos[2]
[1,2] The University of Thessaly,
Department of Mechanical Engineering
, System Optimization Laboratory
,Volos, Greece

Outline

- Emerging technology trends of the freight and logistic management
- Stochastic programming & robust optimization
- Stochastic vehicle routing problem through inventory routing problem
- Problem formulation
- Experimental design
- Remarks & conclusions

Emerging Technology trends of freight ITS

- Freight **Intelligent Transportation Systems (ITS)** change the way that transportation activities are performed.
- Its solutions require the development of operational research models capable of transforming real time data to intelligent advice for advance systems and fleet planning as well as management operation and control systems.
- Although that many new technologies are introduced that gives such advantages, methodologies – models and algorithms - are yield back to its developments.

Emerging Technology trends of freight ITS

Vehicle routing is **critical** in successful logistics execution and freight transportation.

The emerge of technologies and information systems allowing for seamless mobile and wireless connectivity between delivery vehicles and central distribution facilities is paving the way for innovative approaches to both **VRP** , **distribution** and **inventory** management.

Stochastic Optimization

- **Stochastic programming (SP)** models try to take advantage of the fact that probability distribution governing the data are known or can be estimated.
- The goal is to find **the policy** that is **feasible** for almost all possible parameter realization and **optimize the expectation** of some function of the decisions and the random variables.
- A complication in the setting of the model could be the **choice of the objective function** which might be related to a definition of a new measure of performance of the model.

Robust Optimization

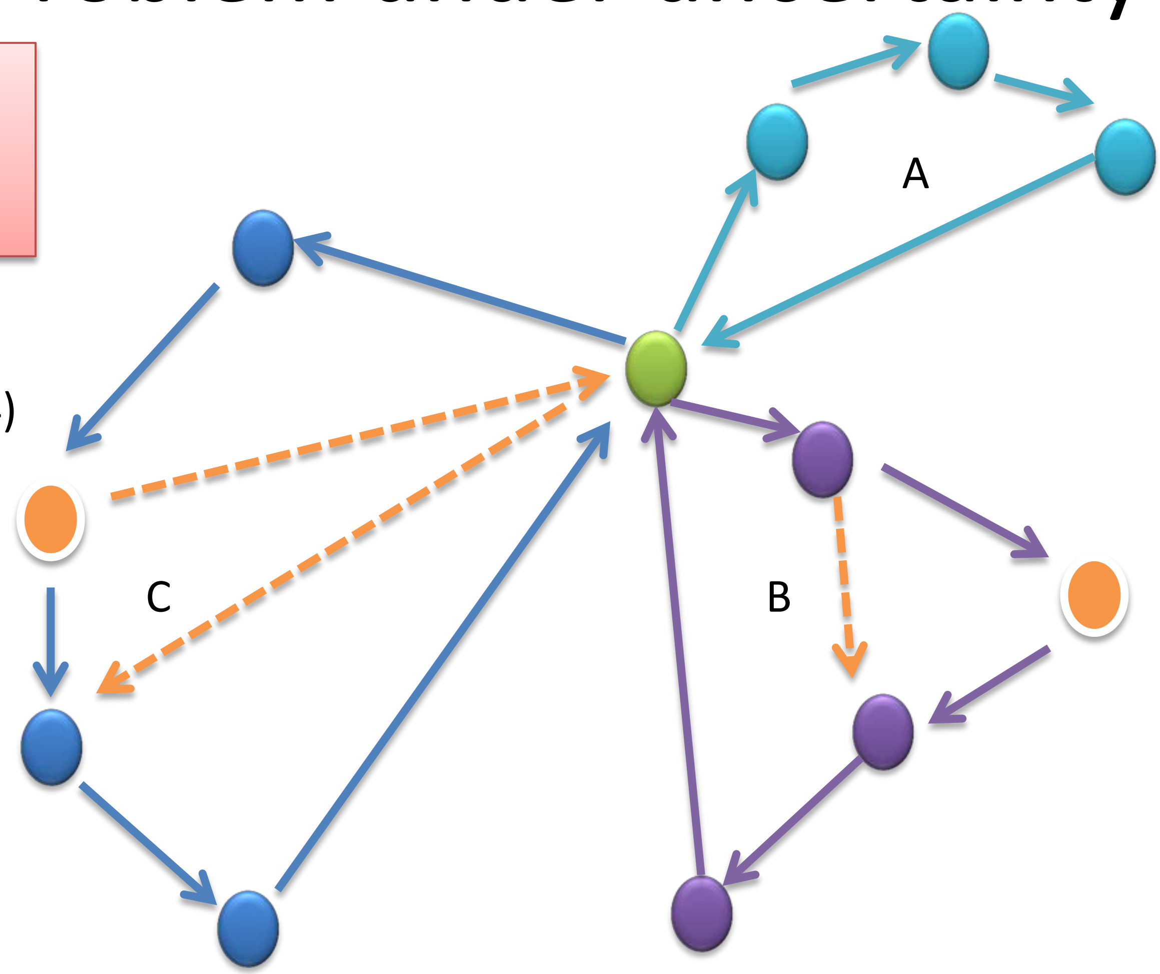
- **Robust Optimization (RO),** is a more recent approach to optimization under uncertainty, in which the uncertainty model is not stochastic, but rather deterministic and set-based.
- Instead of seeking to immunize the solution in some probabilistic sense to stochastic uncertainty, here the decision-maker constructs a solution that is optimal for any realization of the uncertainty in a given set.

Soyster (1973) , Ben Tal &Nemirovski (2000) , Betstimas &Sim (2004)

Vehicle Routing Problem under uncertainty

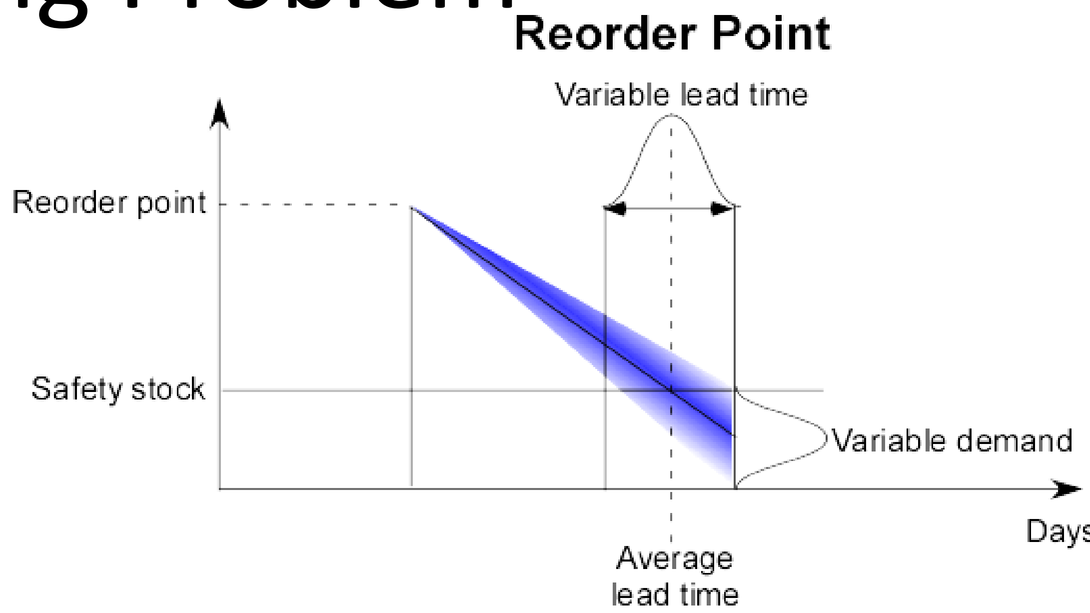
Motivation : Data Velocity
Matching the speed of decisions
with the speed of action

P.Jaillet, (1985)
Bertsimas (1988)
Seguin , Gendreau and Laporte (1994)



Inventory Routing Problem

- Inventory management
- Vehicle routing problem



- **GOAL:** Optimal trade off strategy between inventory policy and delivery policy with the aim of minimizing the distribution cost over an infinite horizon.

Stochastic Inventory Routing Problem

In the Stochastic Inventory Routing problem (SIRP)

- the supplier (vendor) controls the inventories of retailers
- The supplier decides **who, when** and **how much to deliver**
- The demand of each customer is stochastic therefore **stock – out** may be unavoidable
- Customers wish to have as possible least safety stock quantities

Problem Formulation - Notation

Definition of Sets

- O: the node that represents the warehouse of the supplier
- M= { 1,2,…, n} set of retailers
- T = {1, 2… H} set of equal time periods of horizon H
- V = {1,2,…, v} set of fleet of the supplier

Problem Formulation

Parameters

- Retailer (Customer) has
  - C_i: $capacity\ of\ the\ inventory\ \ of\ i - th\ retailer$
  - h_i: $inventory\ holding\ cost\ of\ i - th\ retailer$
  - π_i: $backorder\ penalty\ cost\ of\ i - th\ retailer$
  - d_{ti}: $demand\ of\ i - th\ retailer\ per\ time\ period\ t$
- Vehicle has
  - q_v: $capacity\ of\ the\ v -$
    $th\ vehicle\ starting\ from\ depot\ \ at\ the\ beginning\ of\ each\ period$
- Transportation costs
  - f_t: $fixed\ transportation\ cost\ per\ vehicle\ which\ depends\ on\ the\ period\ t$
  - c_{ij}: $transportation\ cost\ between\ i\ ans\ j\ (satisfies\ trianglular\ inequality)$

Variables

$$x_{i,j,t}^{v} = \begin{cases} 1 \text{ if vehicle } v \text{ travels from } i \text{ to } j \text{ at period } t \\ 0 \text{ otherwize} \end{cases}$$

$$y_{i,j,t}^{v} = \text{amount transported on the trip}$$

$$I_{i,t} = \text{inventory at customer } i \text{ at time period } t$$

$$B_{i,t} = \text{backorder at customer } i \text{ at time period } t$$

Problem Formulation

Objective function

$$\min \sum_{t=1}^{H} \left[\sum_{j=1}^{n} \sum_{v=1}^{V} f_t x_{0,j,t}^{v} + \sum_{i=0}^{n} \sum_{\substack{j=0 \\ i \neq j}}^{n} \sum_{v=1}^{V} c_{i,j} x_{ijt}^{v} + \sum_{i=1}^{n} \left(h_i I_{i,t} + \pi_i B_{i,t} \right) \right]$$

Fixed Transportation cost which depends on the time period t	Transportation cost of the travels from i to j	Inventory cost of holding and backorder penalties

Constrains related to vehicle routing

$$\sum_{\substack{j=0 \\ j \neq i}}^{n} x_{ijt}^{v} \leq 1 \; i = 0,1, \dots n, t = 1, \dots, H, v = 1, \dots, V \qquad (1)$$

$$\sum_{\substack{k=0 \\ k \neq i}}^{n} x_{ikt}^{v} - \sum_{\substack{l=0 \\ l \neq i}}^{n} x_{ilt}^{v} = 0 \; i = 0,1, \dots n, t = 1, \dots, H, v = 1, \dots, V \qquad (2)$$

$$y_{ijt}^{v} - q_{v} \, x_{ijt}^{v} \leq 0 \;\; i,j = 0,1, \dots n, t = 1, \dots, H, v = 1, \dots, V \qquad (3)$$

$$\sum_{\substack{k=0 \\ k \neq i}}^{n} y_{ikt}^{v} - \sum_{\substack{l=0 \\ l \neq i}}^{n} y_{lit}^{v} \leq 0 \;\; i = 0,1, \dots n, t = 1, \dots, H, v = 1, \dots, V \qquad (4)$$

Constrains related to inventory management

$$I_{it-1} - I_{it} - (B_{it-1} - B_{it}) + \sum_{v=1}^{V} \left(\sum_{\substack{k=0 \\ k \neq i}}^{n} y_{ikt}^{v} - \sum_{\substack{l=0 \\ l \neq i}}^{n} y_{lit}^{v} \right) = d_{it} \quad (5)$$

$$I_{it} \leq C_i \quad (6)$$

Domain constrains

$$I_{it} \geq 0 \quad (7)$$

$$B_{it} \geq 0 \quad (8)$$

Transformation to stochastic programming model

1st APPROACH

- Chance constrains formulation by assuming that the parameter of demand of each retailer is unknown and follows some known probability distribution.
- a_D: confidence level of the chance constraint

$$P\{d \;/\; I_{it-1} - I_{it} - (B_{it-1} - B_{it}) + \sum_{v=1}^{V}\left(\sum_{\substack{k=0 \\ k\neq i}}^{n} y_{ikt}^{v} - \sum_{\substack{l=0 \\ l\neq i}}^{n} y_{lit}^{v}\right) = d_{it}\} \geq 1 - a_D$$

Transformation to stochastic programming model

2nd APPROACH

- Stochastic programming with recourse
- Incorporate in the objective function the expected cost of the second stage corrective recourse action

Transformation to stochastic programming model

- **1st Recourse action:** Reduce of distributed amounts of products to remaining suppliers in order to avoid return to depot
- **2nd recourse action :** Return to depot to reload again quantities
- **3rd recourse action :** Load additional vehicle to satisfy the extra demand

$$\min \sum_{t=1}^{H} \left[\sum_{j=1}^{n} \sum_{v=1}^{V} f_t x_{0,j,t}^{v} + \sum_{i=0}^{n} \sum_{\substack{j=0 \\ i \neq j}}^{n} \sum_{v=1}^{V} c_{i,j} x_{ijt}^{v} + \sum_{i=1}^{n} \left(h_i I_{i,t} + \pi_i B_{i,t} \right) \right]$$

Experimental Design

– Structure and formulate appropriate alternative scenarios
– Perform experimental text by solving deterministic equivalent with variety of distribution
– Evaluate the relation of the decision with the values of inventory penalty and transportation costs
– Evaluate it in coherent among and within alternative corrective action

Remarks – Conclusions

- Nowadays of **unstable global economic conditions** the demand of products become highly uncertain in many business areas.
- **Sustainability** of business depend on the ability to handle market uncertainties.
- Research should focus on development of models and methods that fit the industries needs of **robust flexible plans to handle the uncertainties.**

Thank you for your attention

Contact details

Evangelia Chrysochoou

Ph.D. Candidate

Email: echryso@certh.gr

Tel. +302310498481

Greece

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