An exact method for the stochastic inventory routing problem

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Presentation Outline

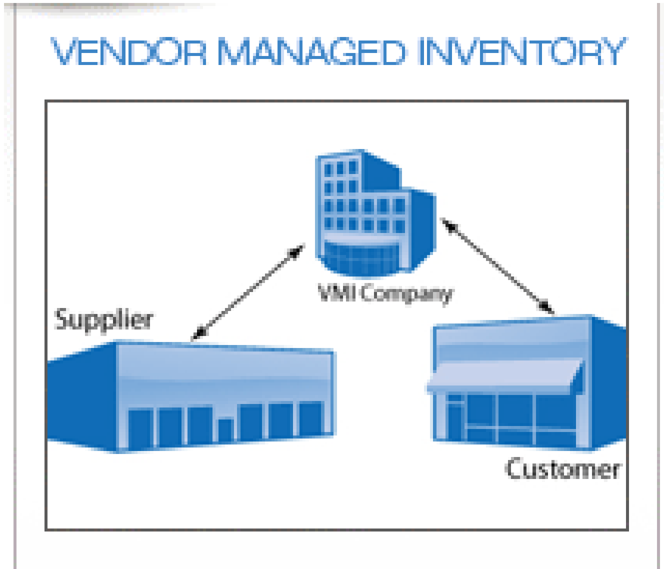
- **Introduction**
  - Vendor Managed Inventory Systems
  - Inventory Routing Problems
- **State of the art**
  - Latest findings : Main Contributors – Researcher
- **Inventory Routing Problem**
  - Model Description (Deterministic/Dynamic and Stochastic)
  - Exact Solution Method – Branch and Cut Scheme
  - Experimental Design - Initial Results
  - Next Steps – Conclusion

Introduction

- **Vendor Managed Inventory (VMI)** System seems to be one of the most promising business models of logistic and supply chain operations.
- Inventory Routing Problem (IRP) constitutes the backbone of the VMI systems.
- Concept: The replenishment & the distribution making process is centralized at supplier level.
- Application of this policy leads to an overall reduction of logistic cost.
- Often described as a win – win situation.

Win – Win Situation

1. **Supplier** save on distribution and production cost since they can combine and coordinate demands and shipments of different customers
2. **Customers** gain by not allocating resources to controlling and managing inventories.



Inventory Routing problem

IRP is considered to be one of the most significant and challenging extensions of Vehicle Routing Problem (VRP) in which decisions related to **inventory and transportation management** are jointly considered.

Decision to be taken are:

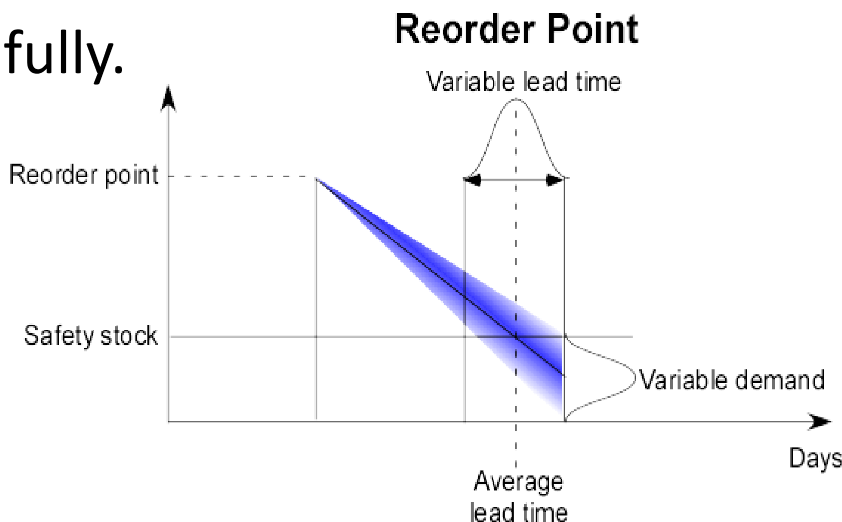
1. **When** to deliver to each customer
2. **How much** to deliver to each customer each time it is served
3. **How to route** the vehicles so as to minimize the total cost.

Total cost always include **transportation cost** and my or may not include **inventory holding cost** at **supplier** and **customers** as well as **backorder penalty cost** in case of stock – out occurrence.

Inventory Routing problem

GOAL: Optimal **trade off strategy** between inventory policy and delivery policy with the aim of minimizing the distribution cost over an infinite horizon.

- If inventory holding cost at supplier and customer is the same the component of inventory holding cost can be ignored.
- Higher inventory costs produce optimal solution where more frequently visits to customers are proposed and
- Vehicle capacities no longer be used fully.



State – of- the art

The problem was first introduced by **Bell et al.(1983)** and **Federgruen & Zipkin (1984) .**

To the best of our knowledge there are two seminal papers regarding literature review on the IRP Andersson et al.(2010) related to business models and classification of problems and Coelho et al. (2013) related to methods and algorithms.

Bertazzi ,Paletta and Speranza(2002) introduced a practical VMI policy the deterministic Order – Up – to level policy. Every customer defines a maximum level of inventory and every time a customer is visited the quantity delivered is such that the maximum level of inventory is reached.

Arhetti et al. (2007) developed the first exact method based on the OU-Policy.

Coelho & Laporte (2014) introduce the transshipment cost within IRP and developed an exact method as well as an ALNS metaheuristic for large scale instances.

State – of- the art

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Inventory Routing Problem

Formulation - Notation

Definition of Sets

- O: the node that represents the warehouse of the supplier
- M= { 1,2,…, n} set of customers/ M’ = M+{0}
- T = {1, 2… H} set of equal time periods of horizon H / T’ = T+{H+1}

Parameters

$r_{0,t}$: Product quantity made available at supplier

$r_{s,t}$: Product quantity made available at customer

$I_{0,t}$: Inventory level at supplier at time t

$I_{s,t}$: Inventory level at customer at time t

U_s: Maximum Inventory level at customer s at time t

h_0: Inventory holding cost of supplier

h_s: Inventory holding cost of customer s

$c_{i,j}$: Transportation cost from warehouse i to j

Variables

$x_{s,t} = \begin{cases} U_s - I_{s,t} \\ 0 \end{cases}$: Amount transported to s customer at t time

y_{ij}^t *Binary (if i->j at time t)*

$z_{0,t}$: *Binary (if supplier is visited at time t)*

$z_{s,t}$: *Binary (if customer s is visited at time t)*

y_{i0}^t={0,1,2}

Inventory Routing Problem

Objective function

$$\min \sum_{i \in M'} h_0 I_{0,t} + \sum_{s \in M} \sum_{t \in T;} h_s I_{s,t} + \sum_{i \in M'} \sum_{\substack{j \in M' \\ j < i}} \sum_{t \in T} c_{ij} y_{ij}^{t}$$

1. Total Inventory holding cost at supplier
2. Total inventory cost at customer
3. Total transportation cost

Inventory Routing Problem

Inventory Definition at supplier

$$I_{0,t} = I_{0,t-1} + r_{0,t-1} - \sum_{s\epsilon M} x_{st-1} \quad (1)$$

$$\forall t \in T$$

Stock out constrain at supplier

$$I_{0,t} \geq \sum_{s\epsilon M} x_{st} \quad (2)$$

$$\forall t \in T$$

Inventory Definition at customer

$$I_{s,t} = I_{s,t-1} + x_{s,t-1} - r_{s,t-1} \quad (3)$$

$$\forall t \in T, s\epsilon M$$

Stock out constrain at customer

$$I_{s,t} \geq 0 \quad (4)$$

$$\forall t \in T, s\epsilon M$$

Inventory Routing Problem

Routing Constrains

$$\sum_{s \epsilon M} x_{st} \leq c z_{0t} \quad (5)$$

$$t \in T$$

$$\sum_{\substack{j \in M' \\ j<i}} y_{ij}^t + \sum_{\substack{j \in M' \\ j>i}} y_{ji}^t = 2 z_{it} \quad (6)$$

$$i \in M, t \in T$$

$$\sum_{i \epsilon L} \sum_{\substack{j \epsilon L \\ j<i}} y_{ij}^t \leq \sum_{i \epsilon L} z_{i,t} - z_{kt} \quad (7)$$

(8)

$$L \subseteq M, t \in T, k \in L$$

Inventory Routing Problem

Capacity Constrain

$$\sum_{s\epsilon M} x_{st} \leq c \qquad (8)$$

Additional Constrains – **Order Up to level Policy** (Arhetti et al.,2007)

$$x_{st} \geq U_s z_{st} - I_{st} \qquad (9)$$

$$x_{st} \geq U_s - I_{st} \qquad (10)$$

$$x_{st} \geq U_s z_{st} \qquad (11)$$

$$\forall t \in T, s\epsilon M$$

Branch and Cut Scheme

Following the branch and Cut algorithm proposed by Arhetti , the algorithm consisted by the steps:

1. Solve the LP relaxation of the model without the SEC .
2. Indentify violation constrains by the method of Padberg & Rinaldi
3. Include the violated constrains to LP and re optimize
4. The process is repeated until a feasible or dominated solution is reached or there are no more cuts to be added
5. Cutting Plane generation proceed repeated in each node of the branching tree
6. Then branching on fractional variables (z and then y) is performed

Stochastic Inventory Routing Problem

- Stochastic programming with recourse
- Incorporate in the objective function the expected cost of the second stage corrective recourse action
- **Second Stage Policy** is the decision of transshipment as well as backorder penalized costs.

$$\min \sum_{i\in M'} h_0 I_{0,t} + \sum_{s\in M}\sum_{t\in T;} h_s I_{s,t} + \sum_{i\in M'}\sum_{\substack{j\in M' \\ j<i}}\sum_{t\in T} c_{ij} + E\left(\sum_{t\in T}\sum_{i\in M} p_i\, l_i^t + bc_{ij}\sum_{t\in T}\sum_{i,j\in M'} w_{ij}^t\right)$$

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Stochastic IRP

Stochastic problem can be solved either by using the **proactive policy** approach or the **reactive policy** approach.

Proactive Policy : Forecasting the demand and use the information in the planning process

Reactive Policy : "Wait and See"

In both policies lateral transshipments between customers is performed as emergency measure against stock out occurrence.

Trade off – between :Transshipment cost and penalization of shortage cost (backorder penalty cost)

Stochastic IRP

Pseudo code for stochastic IRP

For t =0 , H-1

  For i=1,…n

  Proactive approach: Use the forecasts to determine a plan

  Reactive approach: Use the latest demand to determine an initial plan

  After the real demand is revealed

  EVALUTE the demand :

    If Shortage occurred :

    Solve a transshipment MIL: direct delivery from supplier or from neighborhoods customers r

    End if

  End for

End for

Experimental Design – Initial Results

Benchmark instances of Coelho 2012 have been used for the stochastic and dynamic IRP .

Initial results for small instances (<50 customers, H=3) shown that transshipment is a powerful tool to mitigate lost demand.

Transshipment from other customers seems to produce less transportation cost and occur mostly in optimal solution of second stage recourse actions.

We have also to evaluate the case of seasonal data set as well as correlated data sets.

Periods	Instnace	z*	time(sec)
p=3	Small-5-low	1436	15.6
	Small-10-low	1930	40.7
	Small-15-low	2406	72.8
p=6	Small-5-low	3318	65.7
	Small-10-low	4688	153.8
	Small-15-low	5743	287.6

Next Steps – Conclusion

Continue the experiment with larger set of customers

Evaluate

- the results of seasonal and correlated data sets
- the reactive against proactive policies
- the variation of holding cost among customers

Nowadays of **unstable global economic conditions** the demand of products become highly uncertain in many business areas.

Sustainability of business depend on the ability to handle market uncertainties.

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