An exact method for the inventory routing problem

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Abstract

Vendor inventory management is a concept which is adapted highly nowadays where the decision maker in the process is the supplier. The combination of the inventory management with the vehicle routing problem constitutes one of the latest trends of logistics and supply chain management and constitutes the backbone of vendor managed inventory systems. As new emerging technologies are introduced in the context of freight transportation systems, research requires the development of new models and algorithms that can incorporate their advantages. In this context, this paper aims to discuss all significant elements of inventory routing problem. New valid inequalities are proposed to stronger the formulation of the transported quantities and enhance the Maximum Level (ML) policy. This approach was motivated by the fact that, nowadays where infrastructures were manufactured for much higher consumption rates of goods, retailers are opposed to the Order – Up – to level (OU) policy and look for more economic and competitive inventory plans. A branch and cut algorithm was developed to solve the problem exactly. In order to evaluate the performance of the algorithm the benchmark instances set for the single vehicle case created by Arhetti et al. (2007) was used. Computational results have shown that this approach improves the optimal solution on an average at least 20%.

Keywords: Vendor Managed Inventory, Inventory Routing, Transshipment, Branch and Cut

1. Introduction

Vendor Managed Inventory (VMI) systems seem to be one of the most tractable business model nowadays in global logistics and supply chain operations. This is increasingly the case for electronics and automotive parts manufactured in China and assembled in the European Union countries. Most of these parts are assembled in five (5) major plants in Central Europe, operating with Just – In – Time production procedures, using the VMI principles. The general concept behind this model is that replenishments and distribution making process is centralized at the supplier level. It is characterized as a win – win situation for both supplier and manufacturers, or in general retailers due to the fact that it provides the ability to the supplier to combine and coordinate the demand and shipments of a network of retailers (or more generally stock holding entities, such as manufacturers, wholesalers, retailers or 3rd party logistics providers). On the other hand these retailers secure the shortage of their inventories without allocating resources to control and manage them. Backbone of the VMI system is the solution of inventory routing problem (IRP) which is one of the most interesting extensions of routing problems. IRP combines the decision process of inventory management and distribution – transportation of goods. The decision maker in such a model has to make three decisions: the amount to be transported, the frequency of shipments as well as the distribution plan. However, the IRP in practice becomes meaningful when customers' demand is considered to be stochastic instead of assuming a fixed usage rate. The basic difference behind the SIRP and the deterministic IRP is the level of realism and the difficulty of solving instances given the data in a probabilistic sense. In a two stage stochastic program a long term anticipatory decision must be made prior to the full information of the random parameter of the problem and short terms decisions are available as recourse actions once the uncertainty has been revealed. The overall aim is to make "here and now" a decision which minimizes the total expected cost associated with both the long term and the short term decisions (Carøe and Tind. ,1998). IRP was introduced 30 years ago by the seminal paper of Bell et al., (1983) which studied the case with stochastic demand accounting only for transportation costs. They proposed a linear programming model to solve the deterministic version of the problem. To the best of our knowledge there are two very recent literature reviews on the subject. We refer to the work of Andersson et al. (2010) which was focused mostly on industrial aspects and Coelho et al. (2014) which provides the most up to date overview of the problems and methodologies of the VMI problem. Bertazzi, Palettas and Speranza (2002) introduced a practical VMI policy, called deterministic order – up – to – level (OU) policy for the IRP. Based on the proposed policy Arhetti et al. (2007) developed the fist exact algorithm using a branch and cut scheme for the single vehicle. Based on their work very recently Coelho and Laporte (2013) and Adulyasak et al. (2014) have solved multivehicle version of IRP in a branch and cut fashion under OU and maximum level (ML) policies. Solyali and Sural (2011) also based on the work of Arhetti et al. (2007) proposed a strong formulation for the inventory replenishment part of the IRP. In this paper new valid inequalities are introduced to enhance the computational process of the optimal transported quantities under the ML policy. This approach was motivated by the fact that in the context of a deterministic model all parameters are known at the beginning of the process; thus a vendor can take advantage of the fact that the he knows the total demand of each stock keeping venue in advance and can transport quantities in an early stage in order to fulfill the future known demand. However, the amounts that he is able to transport are bounded by the amounts that are made available to him at each stage. This seems to be an important issue for major multinationals that ship parts from China to Europe to be assembled in a number of locations in Central Europe, but also keep inventory either in 3rd party facilities or at the production sites. Transshipment is in fact a recourse action they use in practice in case that there is shortage at a particular venue. These important realizations gave us the motivation to introduce new inequalities in order to enhance the ML policy. The key deference of ML policy in contrast with the OU policy is that the supplier is free to decide about any quantity to be transported to the inventories of his retailers (in fact stock keeping venues) bounded only by their capacity or maximum level defined by them. On the other hand OU policy restricts the amount to be such that fills the inventory to its capacity. However nowadays where infrastructures were manufactured for much higher consumption rates of goods retailers are opposed to the OU policy and look for more economic and competitive inventory plans. A convenient approach to address these particularities is Coelho and Laporte's (2013) proposed new tactical policy, called optimized target level that yields lower cost and inventory levels than the OU policy. Reviewing their approach in comparison to the strong formulation of Solyali and Sural (2011) this paper was motivated to introduce new bound in order to determine optimal quantities to be transported. Therefore, we introduce a modification for the mixed integer programming model of the IRP. To the best of our knowledge this assumption was not proposed before. The remainder of this paper is organized as followed. In §2, we give the formal description of the deterministic IPR model and the branch and cut. Computational results are provided in §3. Significant remarks as well as conclusions are given in§4.

2. Inventory routing problem deterministic model

We consider an inventory routing problem where a supplier denoted by node 1 is distributed to N-1 retailers over a finite discrete time T, using a single vehicle of capacity C. Traditionally the problem is defined on an undirected graph G=(V,E) where {1} is the vertex representing the supplier and vertices V' = {2,3,…,N-1} represent the set of stock keeping venues (will be called retailers from thereafter as it is commonly called in the literature). $A = \{(i,j): i \neq j, i, j \epsilon V\}$ is the set of arcs . Inventory holding cost occurs for both supplier and set of retailers and is denoted as $h_i : i\epsilon V$ per period and each vertex has and inventory capacity $C_i : i\epsilon V$. The length of the discrete planning horizon is H where $t\epsilon T = \{1, \dots, H\}$. At the beginning of the planning horizon the decision maker knows that (1) each period the quantities r^t is made available to the supplier in order to fulfill the request of his retailers; (2)the initial inventory levels of both supplier and t retailers are known { $I_1^0, I_i^0, i\epsilon V'$} (3) of each retailer at each period is denoted with $d_i^t, i\epsilon V'$. A single vehicle can perform the route one at each period with capacity C, and a routing cost $c_{i,j}$is associated with arc$(i,j)\epsilon A$. Throughout the paper we assume that since the supplier has the information of the demand of his retailers in advance he can transport the quantities q_i^t to meet the demand of period t and subsequent periods as well. However the available quantities r^t shall be added to the total available quantities at period t can be used for deliveries to retailer in the same period t and subsequent periods. The objective function is defined in a way to minimize the total transportation and inventory cost of the whole planning horizon while meeting the demand of each retailer.

$$\text{minimize} \sum_{t\epsilon T} \sum_{i\epsilon V} h_i I_i^t + \sum_{t\epsilon T} \sum_{i\epsilon V} \sum_{\substack{i\epsilon V \\ i<j}} c_{ij} x_{i,j}^t \quad (1)$$

Subject to the following constrains:

$$I_1^t \geq 0 \; \forall t \in T \quad (2)$$

$$I_1^t = I_1^{t-1} + r^t - \sum_{i\epsilon V'} q_i^t \; \forall t \in T \quad (3)$$

$$I_i^t \geq 0 \; \forall t \in T, \forall i \in V' \quad (4)$$

$$I_i^t = I_i^{t-1} + q_i^t - d_i^t \; \forall t \in T, \forall i \in V' \quad (5)$$

$$I_i^t \leq C_i \; \forall t \in T, \forall i \in V' \quad (6)$$

$$\sum_{i\epsilon V'} q_i^t \leq C \; \forall t \in T \quad (7)$$

$$\sum_{t\epsilon T} q_i^t = \sum_{t\epsilon T} d_i^t - I_i^0, \forall i \in V' \quad (8)$$

$$q_i^t \leq y_i^t \sum_{j=t}^{H} d_i^j, \forall i \in V', \forall t \in T \quad (9)$$

$$\sum_{j=1}^{t} q_i^j \leq I_1^0 + \sum_{j=1}^{t} r^j, \forall t \in T, \forall i \in V' \quad (10)$$

$$\sum_{t\epsilon T} \sum_{j\epsilon V'} x_{1j}^t \leq H \quad (11)$$

$$\sum_{j\epsilon V'} x_{1j}^t \leq y_1^t \; \forall t \in T \quad (12)$$

$$\sum_{j\epsilon V'} x_{ij}^t = \sum_{j\epsilon V'} x_{ji}^t \; \forall t \in T, \forall i \in V' \quad (13)$$

$$\sum_{j\epsilon V'} x_{ij}^t + \sum_{j\epsilon V'} x_{ji}^t = 2y_i^t \; \forall t \in T, \forall i \in V' \quad (14)$$

$$x_{i,j}^t \leq y_i^t \forall t \in T, \forall i,j \in V' \quad (15)$$

$$x_{i,j}^t \leq y_j^t \forall t \in T, \forall i,j \in V' \quad (16)$$

$$C(1 - x_{i,j}^t) + u_i^t \geq u_j^t + q_j^t$$
$$\forall i,j \in V' : i \neq j, t \in T \quad (17)$$

$$u_i^t \leq q_i^t \quad \forall i,j \in V', t \in T \quad (18)$$

$$u_i^t \leq y_i^t * C \quad \forall i \in V', t \in T \quad (19)$$

$$y_i^t \leq y_1^t \forall t \in T, \forall i \in V' \quad (20)$$

$$q_i^t, u_i^t \geq 0 \forall i \in V', t \in T \quad (21)$$

$$x_{i,j}^t \in \{0,1\} \forall i,j \in V' : i \neq j, t \in T \quad (22)$$

$$y_i^t \in \{0,1\} \forall i \in V, t \in T \quad (23)$$

Constraints (2) and (3) are related to the inventory level at the supplier's site. The first one expresses the fact that inventory level at the supplier level cannot be negative in any period, thus avoiding a stock out situation. The second one defines the inventory level at the supplier at the end of period t by the inventory level at the end of period t-1, minus the total quantities to be transported at period t, plus the quantities r^t that are made available at time t. Constraint (4) secures the stock out avoidance of each retailer as well. Constraint (5) defines the inventory level at each retailer at the end of period t by the inventory level at the end of period t-1, plus the quantities that is made available at period t – the demand at period t as well. Constraint (6) secures that the inventory level of each retailer cannot exceed its capacity. Constraint (7) – (10) defined the quantities delivered. These set of constrains are opposed to the OU policy instead they aim to secure the ML policy. More precisely constraint (7) secures that for each period the quantities to distribute cannot exceed the capacity of the vehicle. Constraint (8) declares that the total quantities to be transported to each retailer are equal to the total demand over the whole planning horizon minus the starting inventory level. Constraint (9) expresses the fact that the quantities to be transported to each retailer at period t can be less or equal to the demand requested at period t and subsequent periods when the retailer is served at period t. Constraint (10) ensures that the transported quantities at period t cannot exceed the suppliers staring inventory level plus the product made available since period t. Constraints (11) – (20) serves the routing counterpart of the problem. More specifically, constraint (11) secures that the total number of routes cannot exceed the number of periods of the planning horizon, however it is not necessary to perform a route for each period. Constraint (12) ensures that if a route is performed at time t it will start from the supplier and will visit only one retailer. Constraints (13) and (14) secure the flow of the route among intermediate retailers. Constraints (15) and (16) define the relationship of the two indexed of the three indexed variables of the routing constrains and stated that when a retailer is served at time t he will be an origin or a destination of a valid path. Constraints (17) – (19) is the well known sub tour elimination constrains based on the Miller-Tucker-Zemlin (MTZ) constraint formulation also suggested by Anken et. al. *(2012);* this is achieved by introducing extra variables u_i^t that express the quantities that are in the vehicle until retailer i. Constraint (20) secures that if a route is performed at period t , then there will be intermediate points in the route. Constraints (21) – (23) enforce integrality and non - negativity conditions. The IRP is NP – hard since it contains the VRP as a special case. If the problem size is relatively small the formulation can be solved by the framework of a branch and cut algorithm as follows: Initially at a generic node of the search tree the relaxed linear program defined by the (1) – (16) and (20) to (23) is solved. Next a search of violated sub tour elimination constraints (17) – (19) is made and sequentially those constraints are generated and introduced to the current problem which is then re - optimized. The process is repeated until a feasible or dominated solution is reached, or until there are no more cuts to added and then branching on fractional variables is performed.

3. Computational results

The algorithm described above was coded in C++ using IBM Concert Technology and CPEX 12.4 with 2 threads. All computations were executed in an Intel Atom 1.83 GHz and 2 GB RAM personal laptop with maximum time of 2 hours. To evaluate the performance of the algorithm, we have used the benchmark instances set for the single vehicle case created by Arhetti et al. (2007). Those instances was used to evaluate the performance of the proposed valid inequalities for the ML policy in coherent to the OU policy. The small instances up to 20 customers were used for both high and low level of inventory holding cost. The small number of experiments is indicative in order to present proposed approach potential solutions that yield almost 20% less IRP cost. The computational results are shown in table 1 -2. Table 1 provides optimal solution of each of the 5 instances with 5, 10, 15 and 20 retailers. Table 1 contains the results of instances with time horizon H = 3 and high inventory cost ($h_i \in [0.1 ,0.5]$ and $h_1 = 0.3$) and results with low inventory cost ($h_i \in [0.01 ,0.05]$ and $h_1 = 0.03$). Column 1 shows the corresponding name of the data set, columns 2 – 3 contain the CPU time (in sec) and the optimal value of the objective function as it was found by Arhetti et al (2007) . Columns 4 – 5 contain the CPU time (in sec) and the optimal value of the objective function of our model, and columns 6 – 7 contain the difference of the optimal solutions and the percentage of it as well. Analogues remain columns contain the results on low inventory cost.

	High Inventory cost , Horizon = 3						Low Inventory cost , Horizon = 3					
	Arhetti et.al.		Chrysochoou& Ziliaskopoulos		z*		Arhetti et.al.		Chrysochoou & Ziliaskopoulos		z*	
Instances	CPU	z*	CPU	z*	Diff	%Diff	CPU	z*	CPU	z*	Diff	%Diff
abs1n5.dat	0	2149,8	1	1868	281,83	13%	0	1281,7	1	1210,1	71,58	6%
abs2n5.dat	0	1959,1	1	1583,7	375,39	19%	0	1176,6	1	967,76	208,9	18%
abs3n5.dat	0	3265,4	1	2533,3	732,14	22%	0	2020,7	1	1633,4	387,3	19%
abs4n5.dat	0	2034,4	1	1677,8	356,65	18%	0	1449,4	1	1245,89	203,5	14%
abs5n5.dat	0	2362,2	1	1819,4	542,74	23%	0	1165,4	1	959,4	206	18%
abs1n10.dat	0	4970,6	13	3678,9	1291,7	26%	0	2167,4	13	2126,44	40,93	2%
abs2n10.dat	0	4803,2	11	3842,4	960,78	20%	0	2510,1	11	2142,79	367,3	15%
abs3n10.dat	0	4289,8	3	3425,8	864,03	20%	0	2099,7	3	1802,02	297,7	14%
abs4n10.dat	0	4347,1	5	3324	1023	24%	0	2188	5	1702,31	485,7	22%
abs5n10.dat	0	5041,6	6	3835,2	1206,5	24%	0	2178,2	6	1740,55	437,6	20%
abs1n15.dat	0	5713,8	6	4585	1128,9	20%	0	2236,5	6	1980,18	256,4	12%
abs2n15.dat	1	5821	520	4593,4	1227,6	21%	1	2506,2	630	2100,03	406,2	16%
abs3n15.dat	4	6711,3	37	5222	1489,3	22%	1	2841,1	37	2344,44	496,6	18%
abs4n15.dat	1	5227,6	57	4083,3	1144,2	22%	1	2430,1	110	1980,16	449,9	19%
abs5n15.dat	3	5210,9	49	3952,2	1258,7	24%	2	2453,5	46	1914,1	539,4	22%
abs1n20.dat	10	7353,8	42	5585,4	1768,4	24%	12	2793,3	43	2124,09	669,2	24%
abs2n20.dat	8	7385	605	5821,8	1563,2	21%	6	2799,9	605	2341,76	458,1	16%
abs3n20.dat	5	7904	133	6006,8	1897,2	24%	8	3101,6	143	2431,14	670,5	22%
abs4n20.dat	4	7050,9	138	5570,2	1480,7	21%	4	3239,3	132	2580,13	659,2	20%
abs5n20.dat	11	8405,8	92	6976,8	1429	17%	7	3331	92	2731,41	599,6	18%

Table 1 Computational Results on instances with time horizon H = 3 and high and low inventory cost

In the set of instances with high inventory cost the average percentage is 21. 3% and yields within the interval (13.1 – 26) %. However in the second case with low inventory cost the average percentage of improvement on the optimal solution found is 16.7% and yields within the interval (1.9 – 24.1) %. This is due to the fact that the transportation cost is higher. Thus our approach can perform significant saving in the cases where the inventory cost is high and competitive to the transportation cost.

4. Conclusions

In this paper the IRP problem was analyzed which constitutes the backbone of the well known VMI systems. New valid inequalities were introduced in order to enhance the performance of the ML policy in contrast to the OU policy which is used in most recent research papers. This approach was motivated by the fact that nowadays retailers are opposed to the OU to level policy and seek for more economic and competitive inventory plans. In the context of a deterministic model all parameters are known at the beginning of the process; thus a vendor can take advantage of the fact that the he knows the total demand of each stock keeping venue in advance and can transport quantities in an early stage in order to fulfill the future known demand. However, the amounts that he is able to transport are bounded by the amounts that are made available to him at each stage. A branch and cut algorithm was developed to solve the problem exactly. In order to o evaluate the performance of the algorithm the benchmark instances set for the single vehicle case created by Arhetti et al. (2007) was used. Computational results have shown that this approach improves the optimal solution on an average at least 20%.

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