ELASTO-PLASTIC AND ELASTO-DAMAGE MODELS BASED ON STRAIN GRADIENT ELASTICITY WITH EVOLVING INTERNAL LENGTH

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STRAIN GRADIENT MODEL USED:

We employ a simplification of form II Midlin's strain gradient elasticity of just one internal length parameter.

Cauchy stress tensor: $\tau_{ij} = \lambda \delta_{pq} \varepsilon_{ii} + 2\mu \varepsilon_{pq}$

Double (non-classical) stress tensor: $m_{rpq} = g^2 \lambda \delta_{pq} \frac{\partial \varepsilon_{ii}}{\partial x_r} + 2g^2 \mu \frac{\partial \varepsilon_{pq}}{\partial x_r}$

g: internal length, λ and μ :Lame's constants and ε_{ij} : the strain tensor

WHY THIS MODEL?

It is the simplest possible strain gradient model that can capture size effect of composite materials such as concrete of low porosity.

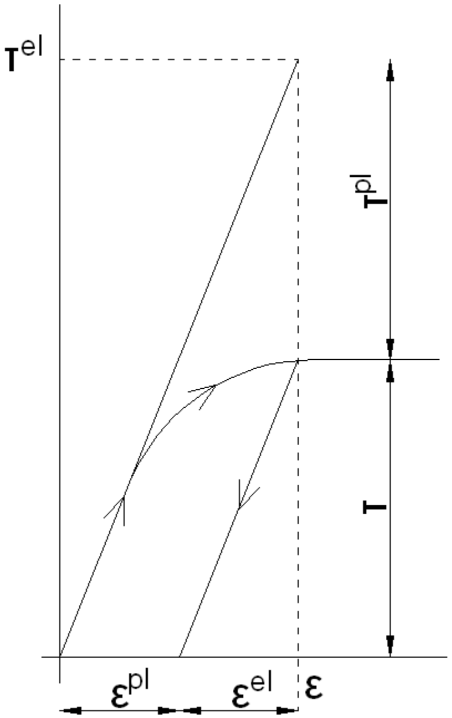
Should the internal length evolve with damage or plasticity?

Internal length used by gradient theories is associated with the microstructure of the material. → Material constant

Plasticity and damage alter the inner structure of the material → Movement of dislocations / Microcracking

Since in the inelastic region the microstructure evolves, then the internal length should evolve as well.

PLASTICITY:



Free energy potential:

$$\Psi\left(\varepsilon_{ij},\frac{\partial\varepsilon_{ij}}{\partial x_r};\tau_{ij}^{p},q,g\right)=\begin{cases}\Psi^{0}\left(\varepsilon_{ij},\dfrac{\partial\varepsilon_{ij}}{\partial x_r};g\right)\\-\varepsilon_{ij}\tau_{ij}^{p}+\Xi\left(q,\tau_{ij}^{p}\right)\end{cases}$$

Ψ^0 is the elastic strain energy:

$$\Psi^{0}=\begin{cases}\dfrac{\lambda}{2}\varepsilon_{ii}\varepsilon_{jj}+\mu\varepsilon_{pq}\varepsilon_{pq}\\+g^{2}\left(\dfrac{\lambda}{2}\dfrac{\partial\varepsilon_{pp}}{\partial x_r}\dfrac{\partial\varepsilon_{qq}}{\partial x_r}+\mu\dfrac{\partial\varepsilon_{pq}}{\partial x_r}\dfrac{\partial\varepsilon_{pq}}{\partial x_r}\right)\end{cases}$$

q: internal plastic variable and Ξ : plastic potential function

In the case of plasticity, we have two internal variables: the internal length and the total plastic strain

Clausius-Duhem dissipation inequality (2nd law of thermodynamics):

$$-\dot{\Psi}+\tau_{ij}\,\dot{\varepsilon}_{ij}+m_{rpq}\left(\frac{\partial\,\dot{\varepsilon}_{pq}}{\partial x_r}\right)\geq 0$$

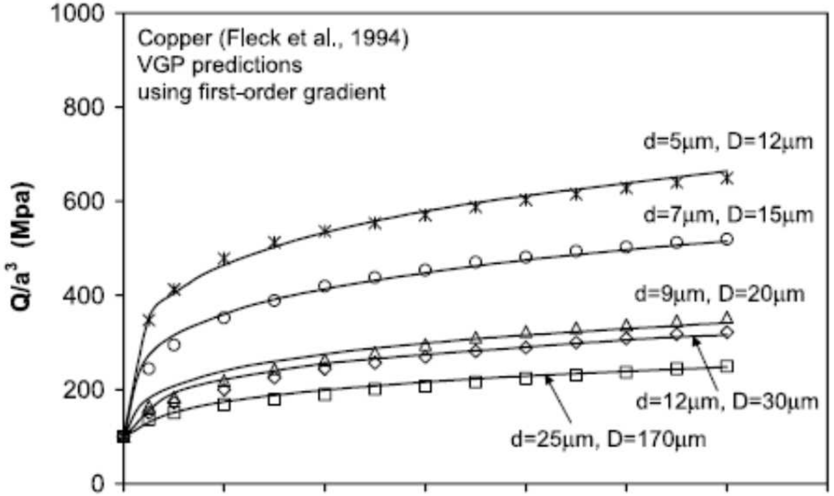
Dissipation rate (*thermodynamic flux rate of change of the int. variable*):

$$-\left(\frac{\partial\Psi^0}{\partial g}\dot{g}+\frac{\partial\Xi}{\partial q}\dot{q}+\frac{\partial\Xi}{\partial\tau_{ij}^p}\dot{\tau}_{ij}^p\right)\geq 0\Rightarrow\begin{cases}-\dfrac{\partial\Psi^0}{\partial g}\dot{g}\geq 0\Rightarrow\boxed{\dot{g}\leq 0}\\[2ex]-\left(\dfrac{\partial\Xi}{\partial q}\dot{q}+\dfrac{\partial\Xi}{\partial\tau_{ij}^p}\dot{\tau}_{ij}^p\right)\geq 0\end{cases}$$

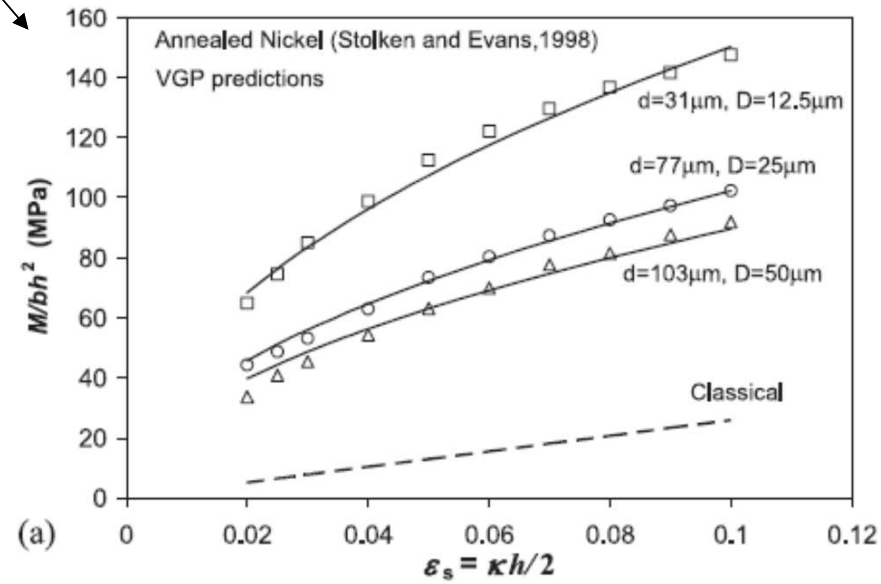
* *Stamoulis and Giannakopoulos, 2010*

Experimental proof (e.g. *Voyiadjis and Abu Al-Rub, 2005*)

Micro-torsion experiments

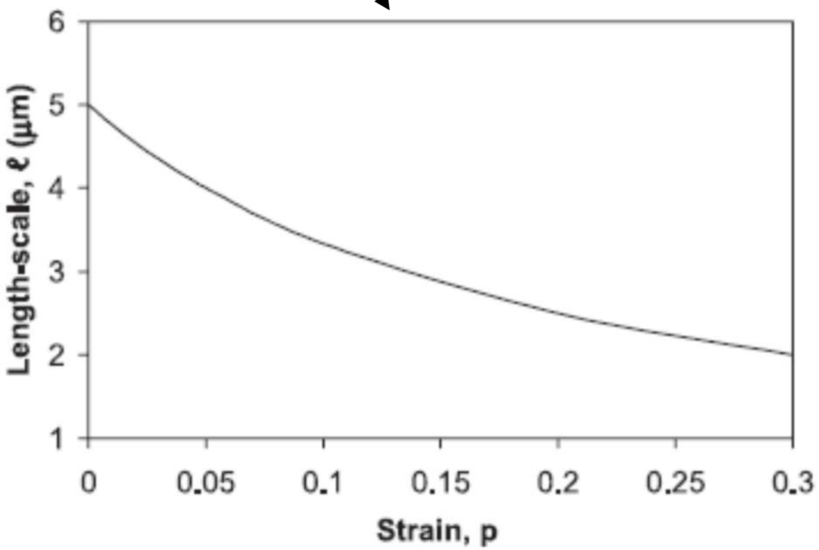


Micro-bending experiments



(a)

Internal length law assumed



Why consider a strain gradient damage model?

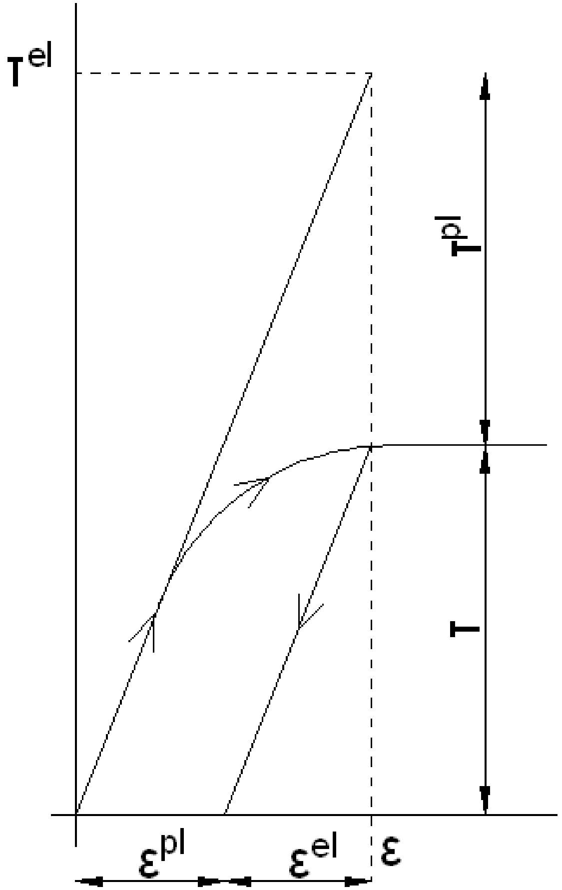
1. Materials that exhibit strain softening in uniaxial tests, i.e. after the maximum load is reached there is no yield plateau but the load declines at increasing displacements, are size sensitive.

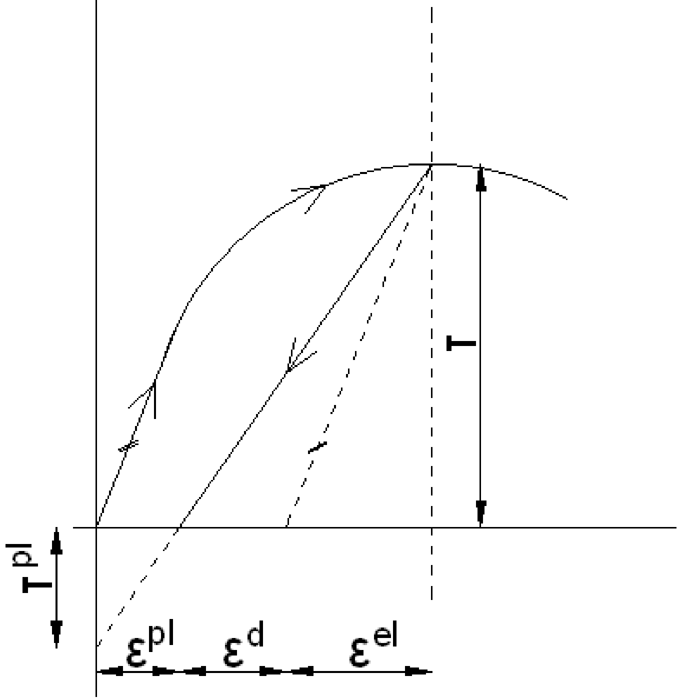
2. The inelastic response of such materials that manifests itself through microcracking ought to be non-local.

(FPZ: Fracture Process Zone)

Plasticity

Damage





Stiffness is unaffected

Loss of stiffness

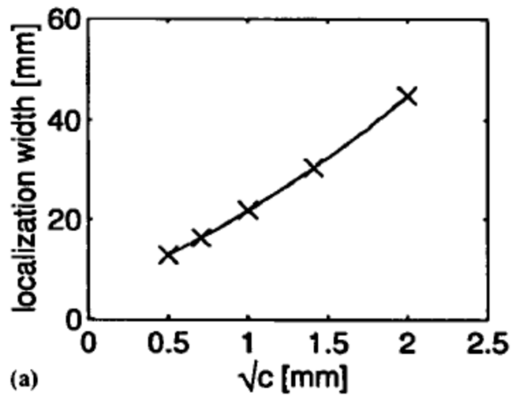
Plastic strain:

Closure effect

Should the internal length increase or decrease with damage?

Peerlings et al, 1996:

Constant int length ; FPZ increases with increasing the int. length (FEM estimates)

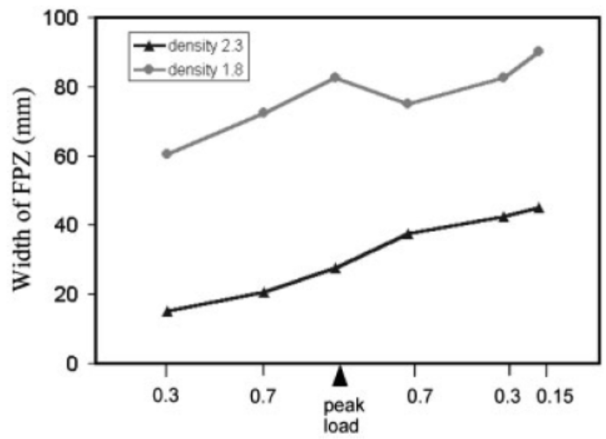


Geers et al, 1998:

An increasing int. length with an end value after a certain level of strain is necessary in order to obtain predictions with finite elements of a damage zone with finite width.

Pijaudier-Cabot et al., 2004 :

Acoustic emission experiments; micromechanical arguments



Extend *Ortiz, 1983*, rate independent damage model to include gradient effects.

Gibb's energy for isothermal process:

$$G = \frac{1}{2}\boldsymbol{\tau} : \mathbf{C} : \boldsymbol{\tau} + \frac{1}{2} g^2 \nabla\boldsymbol{\tau} \therefore \mathbf{C} : \nabla\boldsymbol{\tau} - A^c$$

A^c: free energy required to form the microcracks (distribution)

Assume that the elastic compliances have an additive structure:

$$\mathbf{C} = \mathbf{C}^0 + \mathbf{C}^c$$

$\mathbf{C}^o$: elasticity tensor of the un-cracked material

$\mathbf{C}^c$: added flexibility due to active microcracks

Damage rule (assumed): (all microcracks are active)

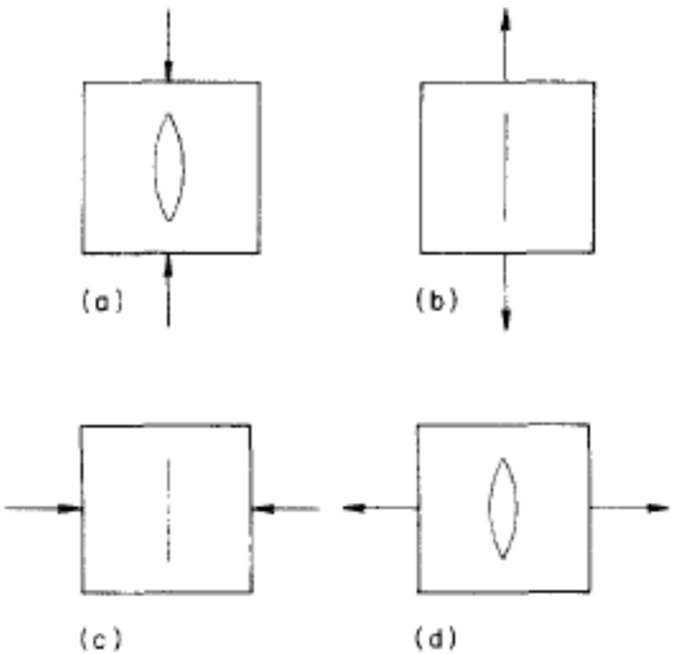
$$\dot{\overline{\mathbf{C}}}^{\mathbf{c}} = \dot{\mu}\,\mathbf{R}(\tau) = \dot{\mu}\left(\mathbf{R}_{\mathbf{I}}(\tau) + \mathbf{R}_{\mathbf{II}}(\tau)\right)$$

<u>Experimental necessity</u>

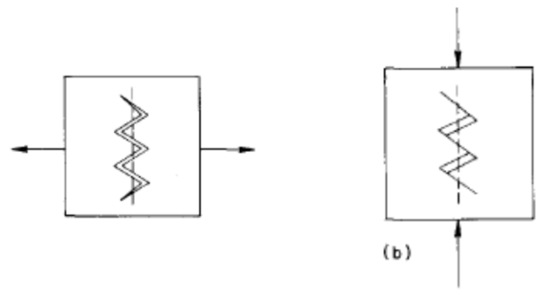
μ : scalar parameter which is a measure of <u>cumulative damage</u>

$\mathbf{R}(\tau)$: a response function of the material which determines the <u>direction</u> in which damage takes place and the open/close mode of cracks

When does a microcrack become active?



Microcracks follow a tortuous path



Cross effect: Cracks can open when acted upon by compressive stresses

Cracks acted upon compressive stresses can remain closed:

↓

Stiffening effect (spindle shape hysteric loops in cyclic bending)

(d): mode I opening or tensile mode
(a): mode II opening or compressive mode due to cross effect

We assume <u>two internal variables</u>, the <u>damage parameter</u> μ and the <u>internal length</u> g, plus a <u>constraint</u> that the internal length should be a function of damage: $g(\mu)$

Dissipation inequality demands:

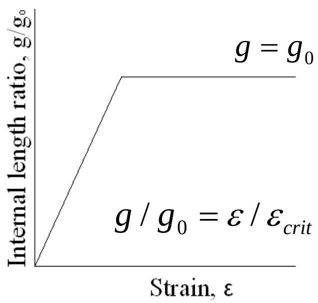
$$d = \frac{1}{2}\boldsymbol{\tau} : \dot{\mathbf{C}}^c : \boldsymbol{\tau} + \frac{1}{2} g^2 \nabla\boldsymbol{\tau} \therefore \dot{\mathbf{C}}^c : \nabla\boldsymbol{\tau} + \frac{1}{2}\left(\dot{g}^2\right)\nabla\boldsymbol{\tau} \therefore \mathbf{C}^c : \nabla\boldsymbol{\tau} - \dot{A}^c \geq 0$$

Rate of change of the internal length: $\dot{g} = \dot{\mu}\dfrac{dg}{d\mu}$

$$d = \left(\frac{1}{2}\boldsymbol{\tau}^+ : \mathbf{R}_{\mathbf{I}} : \boldsymbol{\tau}^+ + \frac{1}{2}\boldsymbol{\tau}^- : \mathbf{R}_{\mathbf{II}} : \boldsymbol{\tau}^- + \frac{1}{2} g^2 \nabla\boldsymbol{\tau} \therefore \mathbf{R}_{\mathbf{I}} : \nabla\boldsymbol{\tau} + g\frac{dg}{d\mu}\nabla\boldsymbol{\tau} \therefore \frac{\mathbf{C}^c}{E^*} : \nabla\boldsymbol{\tau}\right)\dot{\mu} - \dot{A}^c \geq 0$$

The <u>irreversible</u> character of damage necessitates: $\dot{\mu} \geq 0$ and since **R** and $\mathbf{C}^c$ are positive definite, it follows that:

$dg/d\mu \geq 0 \rightarrow$ The internal length g should <u>increase</u> with damage μ



$\varepsilon_{crit} = 0.15$

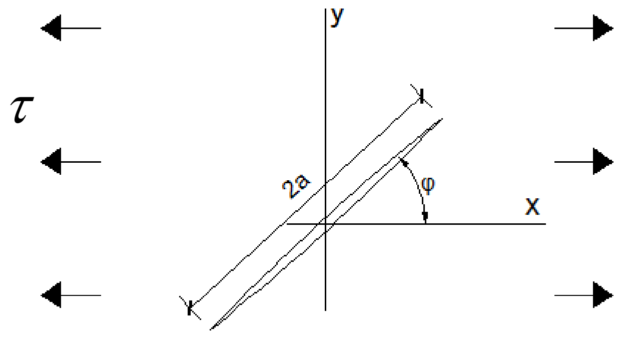
$g_0 = \sqrt{5}mm$

$E = 4500MPa$

$\nu = 0.28$

Suggested (empirically) by *Geers et al., 1998.*

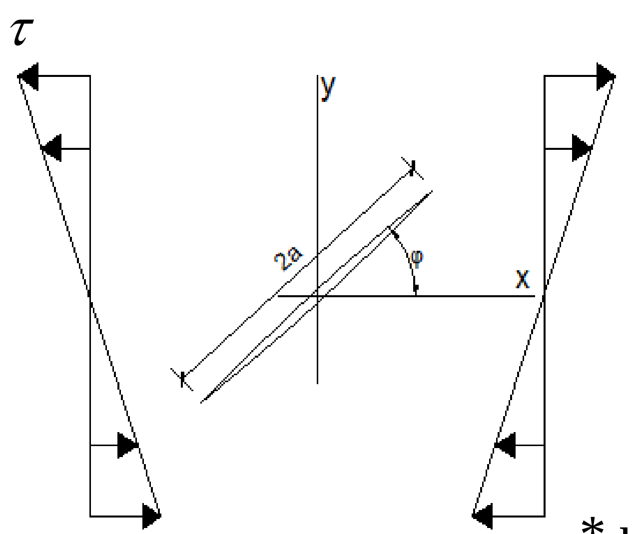
Estimation of A^c (2D case) from micromechanics



Energy release rate: $$G = \frac{K_I^{\,2} + K_{II}^{\,2}}{E^*}$$

Change of G due to increase of damage: $$\frac{dG}{da} = \frac{\pi\tau^2}{E^*}\sin^2(\varphi)$$

Average*: $$\left\langle \frac{dG}{da} \right\rangle = \frac{\pi\tau^2}{2E^*}$$



Change of G due to increase of damage: $$\frac{dG}{da} = \frac{8}{9E^*}\left(\frac{d\tau}{dy}\right)^2 a^2 \sin^4(\varphi)$$

Average*: $$\left\langle \frac{dG}{da} \right\rangle = \frac{1}{3E^*}\left(\frac{d\tau}{dy}a\right)^2$$

* uniformly distributed angle φ

$$E^* = \begin{cases} E & \text{for plane stress} \\ E/(1-\nu^2) & \text{for plane strain} \end{cases}$$

The inelastic free energy is a function of damage: $A^c = A^c(\mu)$

$$\dot{A}^c = \dot{\mu}(dA^c / d\mu)$$

$$\mu = \pi a^2 / A_0 \text{ (3D)}$$

$$\frac{dA^c}{d\mu} = \frac{\pi}{2} t(\mu)^2 + \frac{1}{3}\left(\frac{\partial t(\mu)}{\partial x} a\right)^2$$

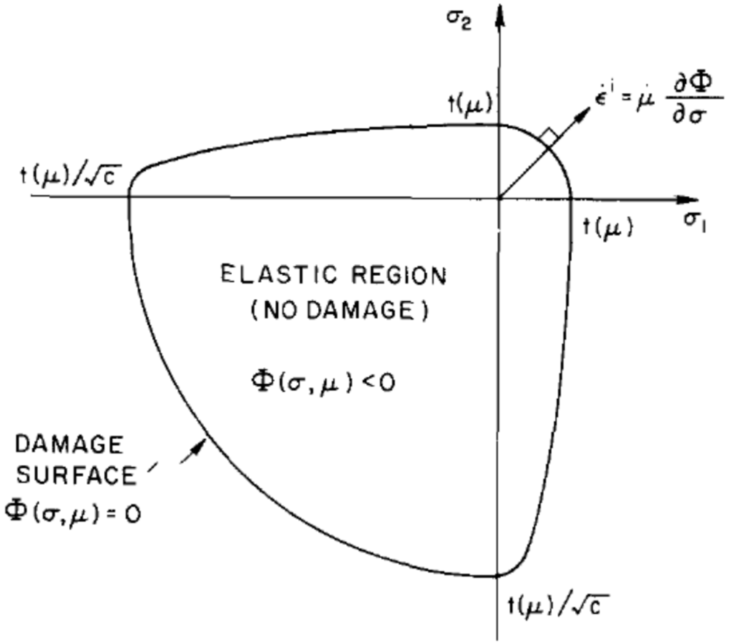
$$\mu = a / H \text{ (2D)}$$

$$g = g(\mu) = \xi(a)$$

Define a damage function: $F(\boldsymbol{\tau}) = \frac{1}{2}\boldsymbol{\tau}^{+} : \mathbf{R}_{\mathbf{I}} : \boldsymbol{\tau}^{+} + \frac{1}{2}\boldsymbol{\tau}^{-} : \mathbf{R}_{\mathbf{II}} : \boldsymbol{\tau}^{-} = F_I + F_{II}$

Example: $F(\boldsymbol{\tau}) = \frac{1}{2}\boldsymbol{\tau}^{+} : \boldsymbol{\tau}^{+} + \frac{1}{2}\left(\frac{f_t}{f_c}\right)^{\mathbf{2}} \boldsymbol{\tau}^{-} : \boldsymbol{\tau}^{-} = F_I + F_{II}$

Assume associated damage rule: $\mathbf{R}_{\mathbf{I}}(\boldsymbol{\tau}) = \frac{\partial^2 F_I}{\partial \boldsymbol{\tau}^{+} \partial \boldsymbol{\tau}^{+}}$ $\mathbf{R}_{\mathbf{II}}(\boldsymbol{\tau}) = \frac{\partial^2 F_{II}}{\partial \boldsymbol{\tau}^{-} \partial \boldsymbol{\tau}^{-}}$

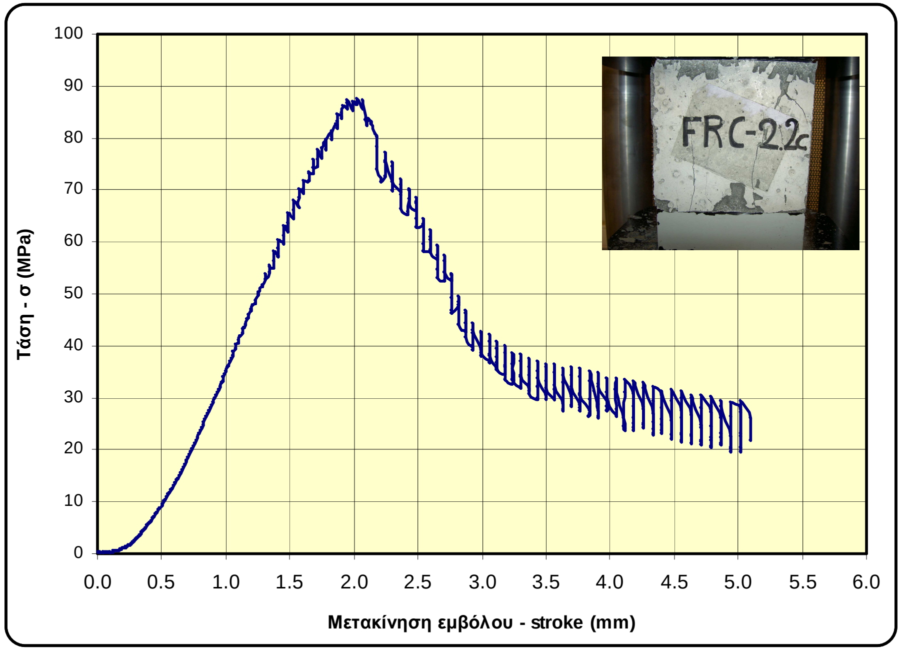


Damage surface:

$$\Phi(\boldsymbol{\tau}) = F(\boldsymbol{\tau}) - \frac{\pi}{2} t(\mu)^2 = 0$$

Instead of trying to define the tensorial quantity **R**, it is easier to define the scalar function F.

How to determine the evolution law of the internal length?



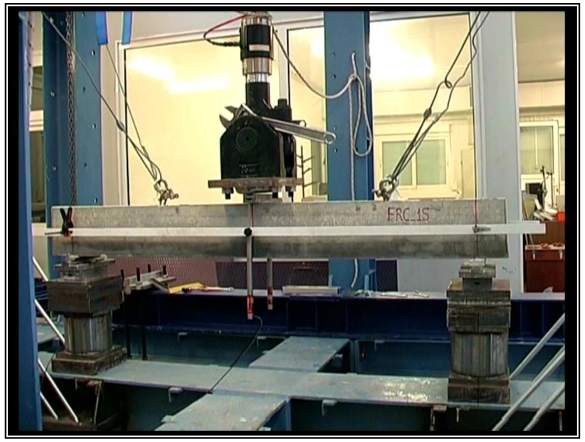
1. Compression experiment

(absence of gradient effects)

Determine the relation between the strain and the damage parameter

Stress-strain relationof Smith and Young, 1955: $\tau = f_c(\varepsilon/\varepsilon_c)\exp\left(1-\varepsilon/\varepsilon_c\right)$

$$\tau = \frac{\varepsilon}{1/E_0+\mu} \longrightarrow t(\mu) = \tau\left(\varepsilon(\mu)\right)$$

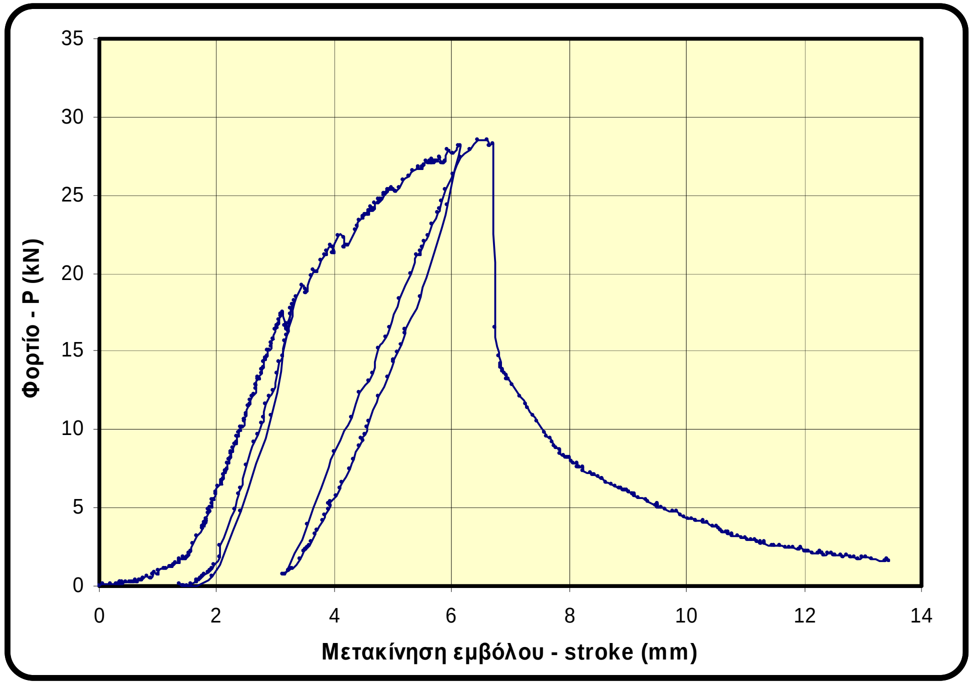


2. Bending experiment

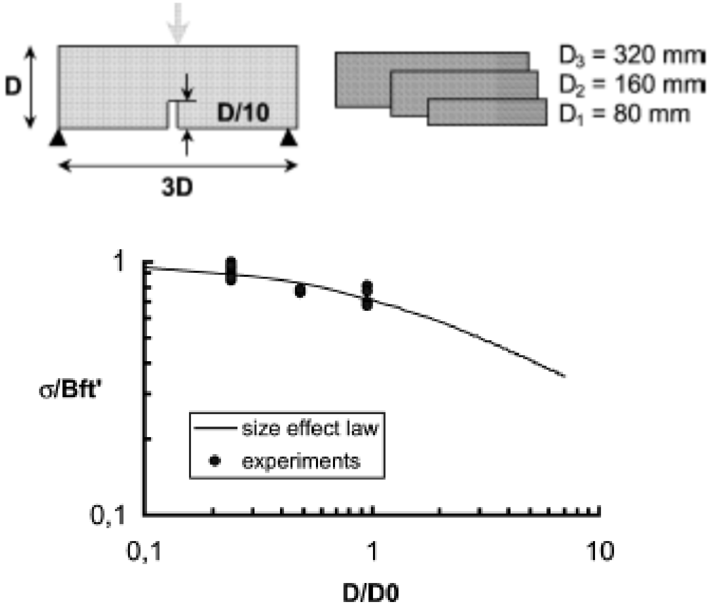
(the gradient effect is coupled with the effect of damage)

Increase of int. length → Increase of stiffness

Increase of damage → Decrease of stiffness

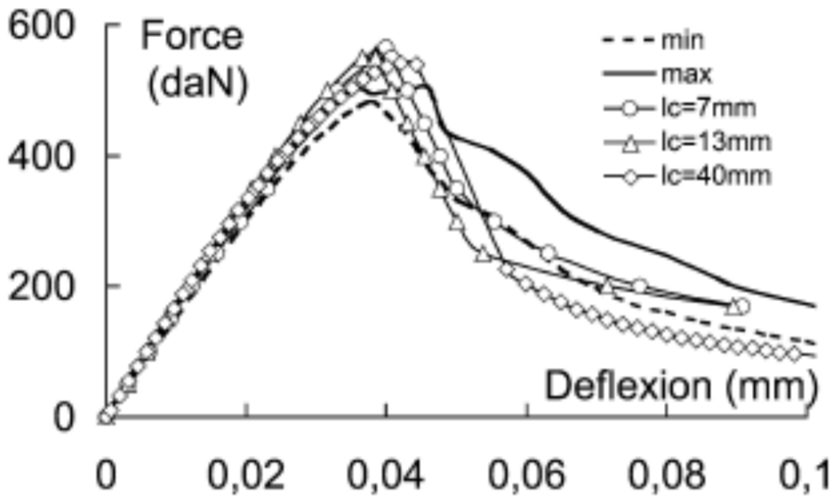


The total Stiffness is a function of both phenomena which act simultaneously but can be separated in the analysis



Size effect clearly detected

Le Bellego et al., 2003



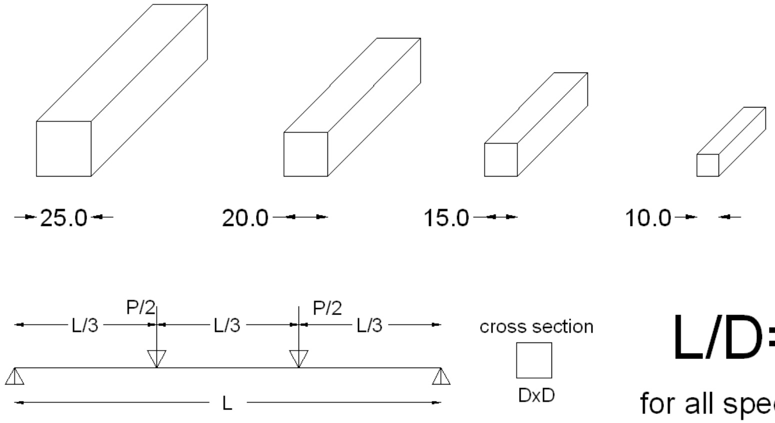
Gradient damage model:

Constant internal length & Monotonic bending

Calibration lacked objectivity

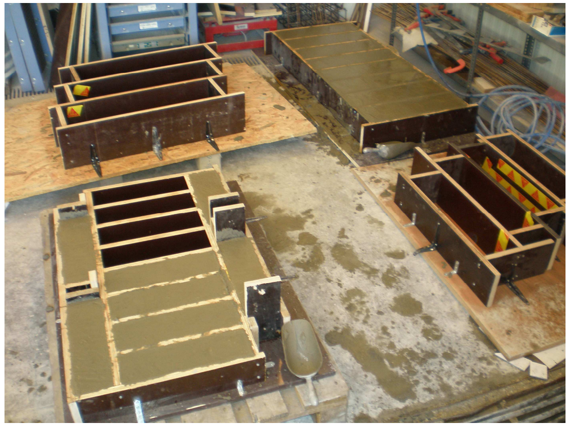
A monotonic experiment will not yield any information about the internal length evolution law. Unloading and reloading at different levels of damage is necessary in order to fully describe a strain gradient damage model for concrete

Future Work



L/D=3

for all specimens



Use experiments on geometrically similar specimens of Fiber Reinforced Concrete (FRC) to calibrate and verify the strain gradient damage model

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