

A Fast CMS Technique for Computational Efficient System Re-analyses in Structural Dynamics

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Abstract

Sensitivity analyses, model calibration techniques, uncertainty quantification methods, reliability computations and design optimization methods require a moderate to large number of system re-analyses to be performed for different values of system parameters. For finite element models involving hundreds of thousands or even million degrees of freedom and localized nonlinear actions activated during system operation, the computational demands from repeated system re-analyses may be excessive. This study, reported in this paper, develops fast efficient component mode synthesis (CMS) algorithms to alleviate the computational burden arising in methods involving system re-analyses.

Keywords: component mode synthesis, structural dynamics, Bayesian inference.

1 Introduction

Component mode synthesis (CMS) techniques [1-2] are often used to carry out dynamic analyses in a reduced coordinate space. CMS techniques can be effective in reducing the high computational effort involved in the repeated system evaluations for different values of system parameters required in sensitivity analyses, model calibration, uncertainty quantification, reliability computations and design optimization. For this, one needs to avoid the re-computation of the eigenproperties at the component or system level. For small variations of the model parameters from a reference model, perturbation techniques [3-4] can be used to provide accurate results. For large variation of the model parameters, approximate methods have been proposed that approximate the modes at the component or system level in terms of the modes of a family of structures corresponding to support points in the parameter space [5]. Linear and quadratic interpolations of the structural mass and stiffness matrix and the matrix of eigenvectors have been proposed in [6] to achieve this.

Recently, a new efficient method [7] has also been proposed that applies under certain parameterization cases. Specifically it applies when the division of the structure into components is guided by the FE parameterization scheme so that the stiffness matrix that arise for each one of the introduced components to depend linearly on only one of the parameters to be estimated. In this case the fixed-interface and constraint modes of the components for any value of the model parameters can be obtained exactly from the fixed-interface and constraint modes corresponding to a single reference FE model, avoiding re-analyses at component level. Additional substantial reductions in computational effort are obtained by reducing the number of interface DOF using characteristic interface modes through a Ritz coordinate transformation [8]. The repeated solutions of the component and interface eigen-problems are avoided, reducing drastically the computational demands in FE formulations, without compromising the solution accuracy.

In this work, the CMS framework for effective system re-analyses proposed in [7] is reviewed and extended to cover more general cases of FE model parameterization. The formulation is demonstrated for the case of uncertainty quantification and model parameter calibration using Bayesian techniques. However, the proposed approach can be applied in other cases requiring system re-analyses. The computational efficiency of the proposed approach is illustrated by using the Bayesian method to update a finite element model of a bridge involving hundreds of thousands of degrees of freedom.

2 Component Mode Synthesis for System Re-analyses

Dynamic reduction techniques such as CMS can be implemented with Bayesian uncertainty quantification and propagation framework [9-10] in order to alleviate the computational burden associated with each model run in the re-analyses required in the optimization and stochastic simulation methods. CMS techniques 1 divide the structure into components with mass and stiffness matrices that are reduced using fixed-interface and constrained modes. Dividing the structure into components and reducing the number of physical coordinates to a much smaller number of generalized coordinates certainly alleviates part of the computational effort. However, at each iteration or TCMC sampling point one needs to re-compute the eigen-problem and the interface constrained modes for each component. This procedure is usually a very time consuming operation and computationally more expensive than solving directly the original matrices for the eigenvalues and the eigenvectors. It was recently shown [7] that for certain parameterization schemes for which the mass and stiffness matrices of a component depend linearly on only one of the free model parameters to be updated, often encountered in finite element model updating formulations, the full re-analyses of the component eigen-problems are avoided. The eigenproperties and the interface constrained modes as a function of the model parameters can be computed inexpensively from the eigenproperties and the interface constrained modes that correspond to a nominal value of the model parameters.

Specifically let Δ_j be the set of structural components that depend on the j -th parameter θ_j . Consider the case for which the stiffness matrix of a component $s \in \Delta_j$ depends linearly on θ_j and the mass matrix is independent of θ_j , i.e. $K^{(s)} = \bar{K}^{(s)}\theta_j$ and $M^{(s)} = M_0^{(s)}$. It can be readily derived that the stiffness and mass matrices of the Craig-Bampton reduced system admits the representation

$$\hat{K}^{CB} = \hat{K}_0^{CB} + \sum_{j=1}^{N_\theta} \hat{K}_{,j}^{CB}\theta_j \quad \text{and} \quad \hat{M}^{CB} = \hat{M}_0^{CB} \quad (1)$$

where the coefficient matrices \hat{K}_0^{CB} , $\hat{K}_{,j}^{CB}$ and \hat{M}_0^{CB} in the expansion (1) the component stiffness and mass matrices. It is important to note that the assembled matrices \hat{K}_0^{CB} , $\hat{K}_{,j}^{CB}$ and \hat{M}_0^{CB} of the Craig-Bampton reduced system in the expansion (1) are independent of the values of $\underline{\theta}$. In order to save computational time, these constant matrices are computed and assembled once and, therefore, there is no need this computation to be repeated during the iterations involved in optimization or TCMC sampling algorithms for model updating due to the changes in the values of the parameter vector $\underline{\theta}$. This saves substantial computational effort since it avoids (a) re-computing the fixed-interface and constrained modes for each component, and (b) assembling the reduced matrices from these components.

Further reduction in the generalized coordinates can be achieved by replacing the interface DOFs by a reduced number of constraint interface modes [8] formed by a reduced basis. Selecting the reduced basis to be constant, independent of $\underline{\theta}$, the formulation significantly simplifies. The reduced basis can be kept constant at each iteration involved in the optimization algorithm or updated every few iterations in order to improve convergence and maintain accuracy.

Following the formulation proposed in [7], the aforementioned framework can be extended to handle the case for which the component stiffness and mass matrices depend nonlinearly on a single parameter θ_j of the system parameter set $\underline{\theta}$. Specifically, consider the case for which the stiffness and mass matrices of a component $s \in \Delta_j$ depend nonlinearly on θ_j , i.e. $K^{(s)} = \bar{K}^{(s)}f^{(s)}(\theta_j)$ and $M^{(s)} = \bar{M}^{(s)}g^{(s)}(\theta_j)$, where $f^{(s)}(\theta_j)$ and $g^{(s)}(\theta_j)$ are nonlinear functions of the parameter θ_j . Then the interface modes, the modal frequencies and the interface constrained modes of a component can readily be computed by the corresponding interface modes, modal frequencies and interface constrained modes of the same component for a reference structural configuration corresponding to a particular nominal value of the parameter set $\underline{\theta}$ as well as the current value of the parameter θ_j . In the nonlinear case, a representation similar to (1) is no longer applicable and the reduced mass and stiffness matrices of the reduced structure should be re-assembled from the component mass and stiffness matrices for the new value of θ_j . This procedure also saves substantial computational effort since it avoids re-computing the fixed-interface and constrained modes for each component.

3 Application to a bridge structure

The computational efficiency and accuracy of the proposed CMS is demonstrated by using the Transitional TCMC [11] stochastic simulation algorithm to update a FE model of the Metsovo bridge using simulated modal data. A detailed FE model of the bridge is created using 3-dimensional tetrahedron quadratic Lagrange FEs. An extra coarse mesh, chosen to predict the lowest 20 modal frequencies and mode shapes of the bridge, results in a minimum 97,636 FEs and 562,101 DOF.

3.1 Effectiveness of CMS technique

For demonstration purposes, the bridge is divided into nine physical components with eight interfaces between components as shown in Figure 1. Each deck component consists of several 4-5m deck sections. The tallest pier also consists of several sections. The size of the elements in the extra coarse mesh is the maximum possible one that can be considered, with typical element length of the order of the thickness of the deck cross-section.

The cut-off frequency ω_c is introduced to be the highest modal frequency that is of interest in finite element model updating. In this study the cut-off frequency is selected to be equal to the 20th modal frequency of the nominal model. i.e. $\omega_c = 4.55$ Hz. The effectiveness of the CMS technique as a function of the number of modes retained for each component is next evaluated. For each component it is selected to retain all modes that have frequency less than $\omega_{\max} = \rho\omega_c$, where the ρ values affect computational efficiency and accuracy of the CMS technique. Representative ρ values range from 2 to 10.

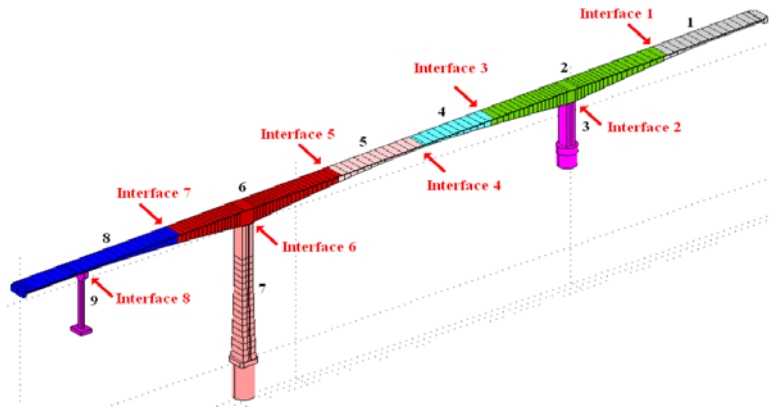


Figure 1: Components of FE model of Metsovo Bridge

A large number of generalized coordinates for the reduced system arises from the interface DOFs. A further reduction in the number of generalized coordinates for the reduced system can be achieved by retaining only a fraction of the constrained

interface modes. For each interface, it is selected to retain all modes that have frequency less than $\omega_{\max} = \nu\omega_c$, ν is user and problem dependent. The total number of internal DOFs before the model reduction is applied and the number of modes retained for various ρ values are given in Table 1 .

	Full Model	Reduced Model (Retained Modes)					
		$\rho = 8$	$\rho = 5$	$\rho = 2$	$\rho = 8$ $\nu = 200$	$\rho = 5$ $\nu = 200$	$\rho = 2$ $\nu = 200$
Internal DOF	558,801	286	100	31	286	100	31
Interface DOF	3,300	3,300	3,300	3,300	306	306	306
Total DOF	562,101	3,586	3,400	3,331	592	406	337
Highest Percentage Error [%]	0.00	0.02	0.17	1.10	0.20	0.30	1.20

Table 1: Number of DOF and percentage modal frequency error for the full (unreduced) and reduced models

Figure 2 shows the fractional error between the modal frequencies computed using the complete finite element model and the modal frequencies computed using the CMS technique as a function of the mode number for $\rho = 2, 5$ and 8 . It can be seen that the error for the lowest 20 modes fall below 10^{-5} for $\rho = 8$, 10^{-4} for $\rho = 5$ and 10^{-3} for $\rho = 2$. A very good accuracy is achieved even for the case of $\rho = 2$. Results are also shown in Figure 2 for $\nu = 200$. It can be seen that the fractional error for the lowest 20 modes of the structure fall below 10^{-3} for $\nu = 200$. Obviously the number of generalized coordinates is drastically reduced.

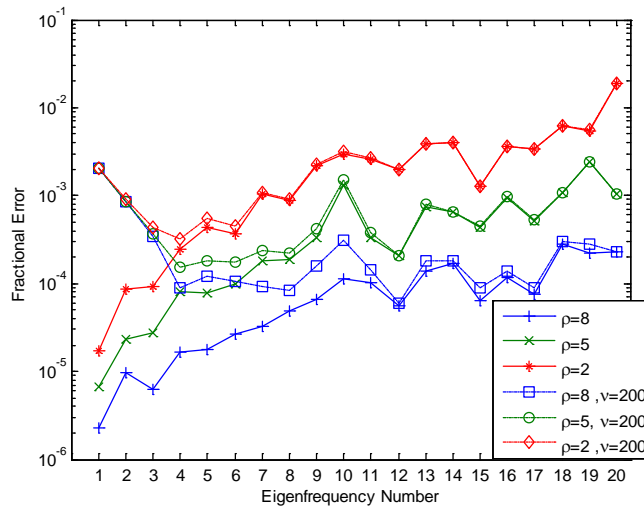


Figure 2: Fractional modal frequency error between predictions of the reduced and the full model.

3.2 Finite element model updating using CMS

The finite element model is parameterized using five parameters associated with the modulus of elasticity of one or more structural components shown in Figure 3. Specifically, the first two parameters θ_1 and θ_2 account respectively for the modulus of elasticity of the pier components 3 and 7 of the bridge. The parameter θ_3 accounts for the modulus of elasticity of the components 1 and 2 of the deck, the parameter θ_4 accounts for the components 4 and 5, while the parameter θ_5 accounts for the components 6 and 8. The component 9 is not parameterized.

Simulated, noise contaminated, measured modal frequencies and mode shapes are generated by adding a 1% and 3% Gaussian noise to the modal frequencies and modeshape components, predicted by the nominal non-reduced finite element models. The added Gaussian noise reflects the differences observed in real applications between the predictions from a model of a structure and the actual (measured) behavior of the structure. A sensor configuration involving 36 sensors is considered. The sensors are placed along the deck and the piers, measuring along the longitudinal, transverse and vertical directions. The finite element model is updated using simulated modal data for the lowest ten modes.

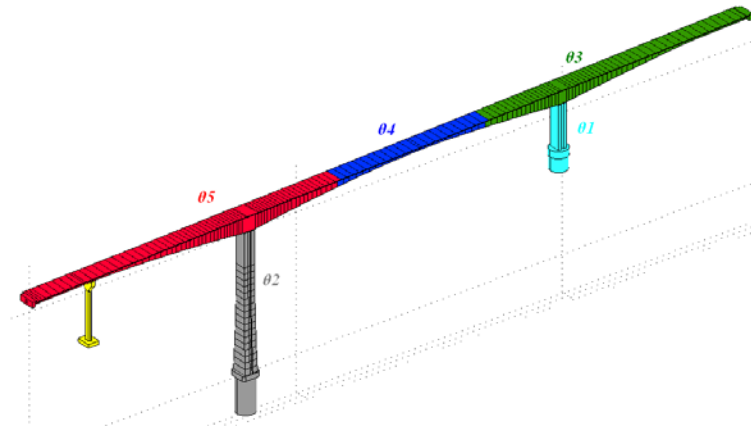


Figure 3: FE model parameterization based on 5 parameters.

The following settings of the parameters of the stochastic simulation algorithm TCMC are used to perform the model updating: $\text{tolCov}=1.0$, $\beta=0.2$ and 1000 samples per TCMC stage. The number of FE model runs for the five-parameter model class depends on the number of TCMC stages which was estimated to be 19. The resulting number of FE model re-analyses are 19,000. The parallelization features of TCMC [12] were also exploited, taking advantage of the available four-core multi-threaded computer unit to simultaneously run eight TCMC samples in parallel. For comparison purposes, the computational effort for solving the eigenvalue problem of the original unreduced FE model is approximately 139 seconds. Multiplying this by the number of 19,000 TCMC samples and

considering parallel implementation in a four-core multi-threaded computer unit, the total computational effort for the model class is expected to be of the order 7 days. In contrast, for the reduced-order models for $\rho = 8$, the computational demands for running the model class are reduced to approximately 13 hours (759 minutes), while for the reduced-order models for $\rho = 8$ and $\nu = 200$ these computational demands are drastically reduced to 14 minutes. It is thus evident that a drastic reduction in computational effort for performing the structural identification based on a set of monitoring data is achieved from approximately 7 days for the unreduced model class to 14 minutes for the reduced model classes corresponding to $\rho = 8$ and $\nu = 200$, without compromising the predictive capabilities of the proposed parameter estimation methodology. This results in a factor of over 500 reduction in computational effort.

It should be noted that surrogate methods can also be used to further reduce the computational effort by more than one order of magnitude so that the updating of the 562,101 DOF finite element model, requiring 19,000 model runs, can be performed in less than two minutes which is a remarkable reduction in computational effort.

4 Conclusions

Sensitivity analyses, model calibration techniques, uncertainty quantification and propagation methods, reliability computations and design optimization methods, require a moderate to large number of system re-analyses to be performed for different values of system parameters. A fast and accurate CMS technique is proposed, consistent with the finite element model parameterization schemes, to achieve drastic reductions in computational effort. The computational efficiency of the proposed CMS technique is demonstrated with applications in model calibration of a bridge using the TMCMC stochastic simulation algorithm. Substantial reductions in computational effort are achieved. Further computational savings can also be achieved by integrating into the formulation surrogate models to substantially reduce the number of costly finite element-based function evaluations, as well as using parallel computing algorithms to efficiently distribute the computations in available multi-core CPUs.

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References

- [1] R.R. Craig Jr., M.C.C. Bampton, “Coupling of substructures for dynamic analysis”, *AIAA Journal* 6(7) (1965) 678–685.
- [2] R.R. Craig Jr., “Structural Dynamics - An Introduction to Computer Methods”, John Wiley & Sons, New York, 1981.
- [3] L. Hinke, F. Dohnal, B. Mace, T. Waters, N. Ferguson, “Component mode synthesis as a framework for uncertainty analysis”, *Journal of Sound and Vibration* 324(1–2) (2009) 161–178.
- [4] B. Mace, P. Shorter, “A local modal/perturbational method for estimating frequency response statistics of built-up structures with uncertain properties”, *Journal of Sound and Vibration* 242(5) (2001) 793–811.
- [5] E. Balmes, “Parametric families of reduced finite element models. Theory and applications”, *Mechanical Systems and Signal Processing* 10(4) (1996) 381–394.
- [6] B. Goller, H.J. Pradlwarter, G.I. Schueller, “An interpolation scheme for the approximation of dynamical systems”, *Computer Methods in Applied Mechanics and Engineering* 200 (2011) 414–423.
- [7] Papadimitriou, C., Papadioti, D.C., “Component Mode Synthesis Techniques for Finite Element Model Updating”, *Computers and Structures*, DOI: 10.1016/j.compstruc.2012.10.018, 2012
- [8] M.P. Castanier, Y.-C. Tan, C. Pierre, “Characteristic constraint modes for component mode synthesis”, *AIAA Journal* 39(6) (2001) 1182–1187.
- [9] C. Papadimitriou, J.L. Beck, L.S. Katafygiotis, “Updating robust reliability using structural test data”, *Probabilistic Engineering Mechanics* (2001), Vol. 16, No. 2, pp. 103–113.
- [10] J.L. Beck, S.K. Au, “Bayesian updating of structural models and reliability using Markov chain Monte Carlo simulation”, *Journal of Engineering Mechanics, ASCE* (2002), Vol. 128, No. 4, pp. 380–391.
- [11] J. Ching, Y.C. Chen, “Transitional Markov Chain Monte Carlo method for Bayesian updating, model class selection, and model averaging”, *Journal of Engineering Mechanics, ASCE* (2007), Vol. 133, pp. 816–832.
- [12] Angelikopoulos, P., Papadimitriou, C., Koumoutsakos, P., “Bayesian Uncertainty Quantification and Propagation in Molecular Dynamics Simulations: A High Performance Computing Framework”, *The Journal of Chemical Physics* 137(14), 2012 DOI: 10.1063/1.4757266.