PhD presentation on

“Size Effects in Semi-brittle Materials and Gradient Theories with Application to Concrete”

by

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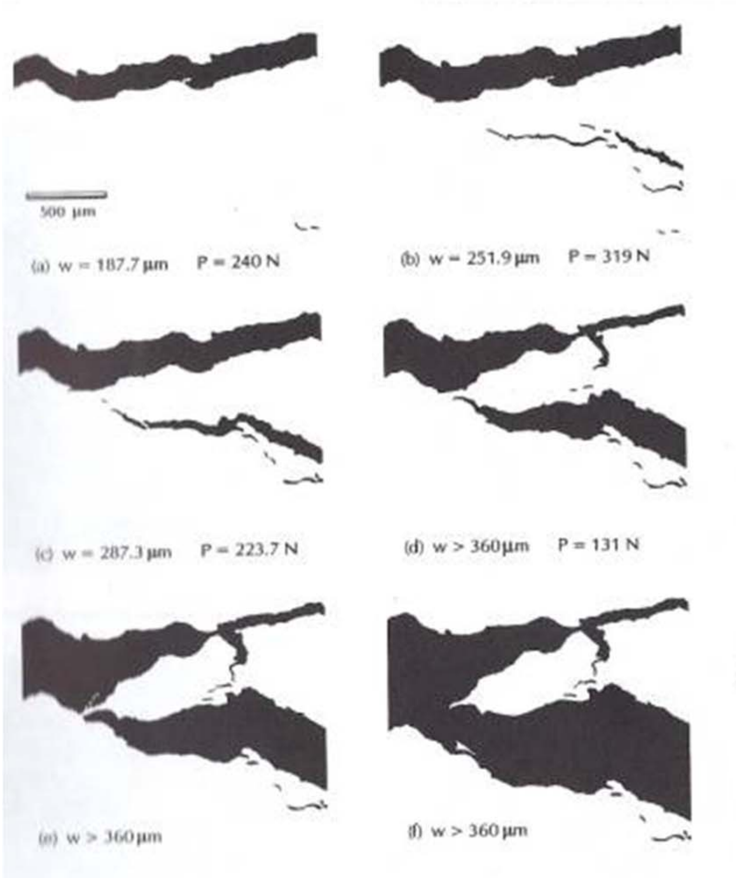
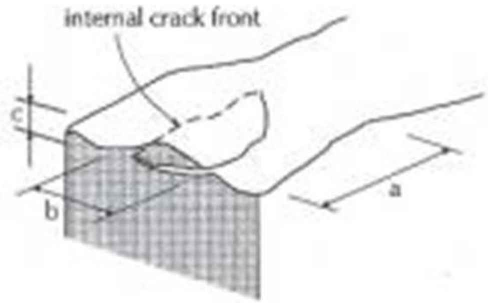
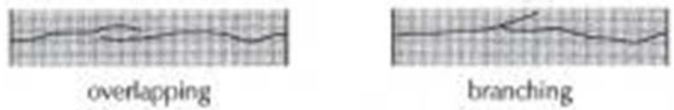
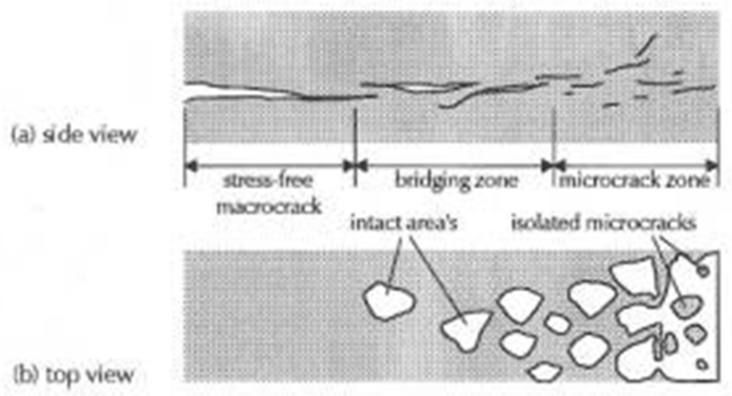
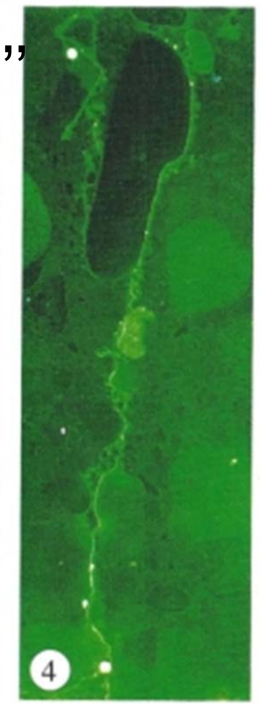
Microcracking: ''branching'' and ''crack face bridging''

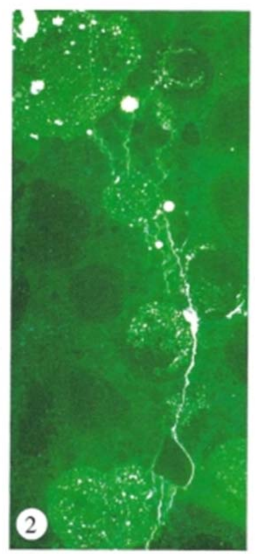












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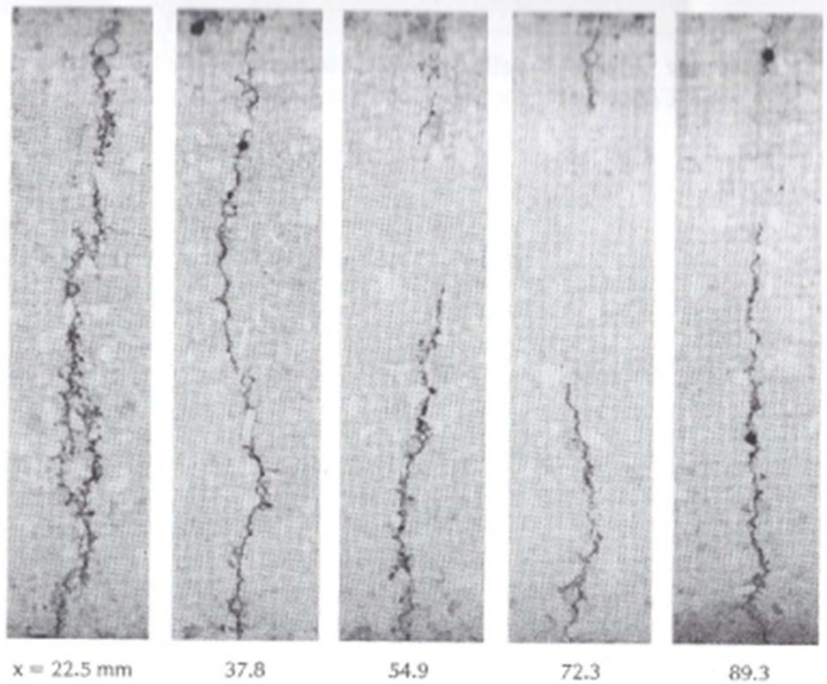
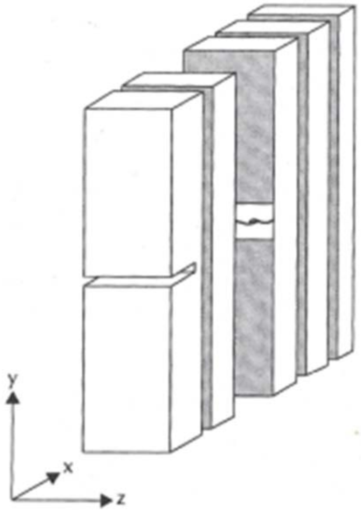
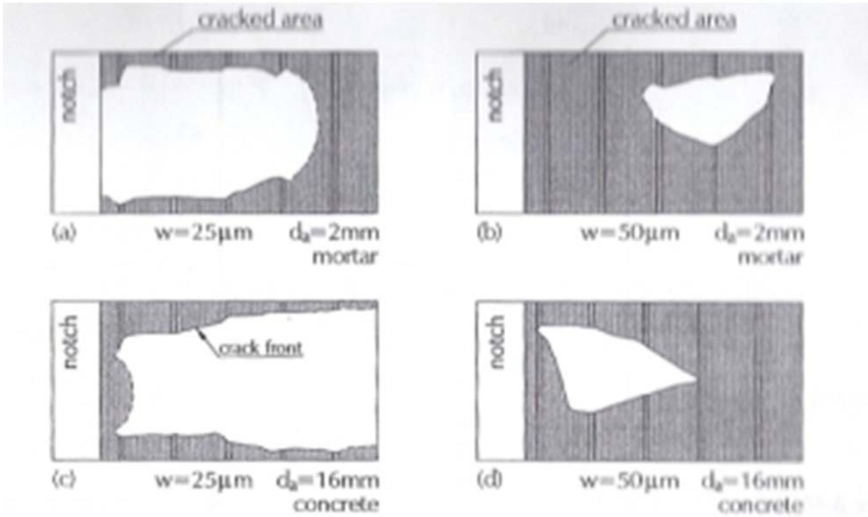
Complexity of the observed behavior of microcracking

<u>Mode I fracture</u>







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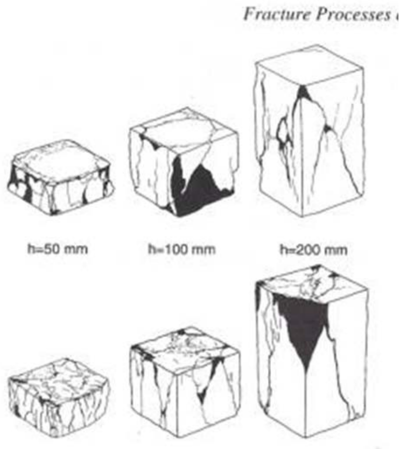
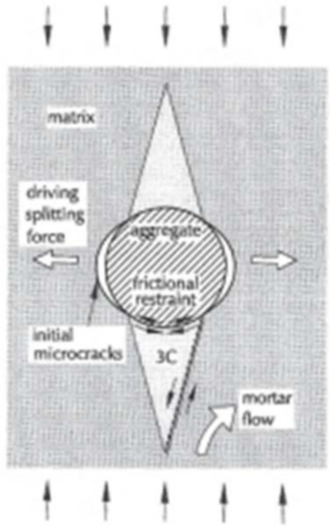
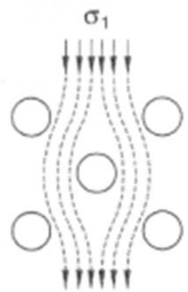
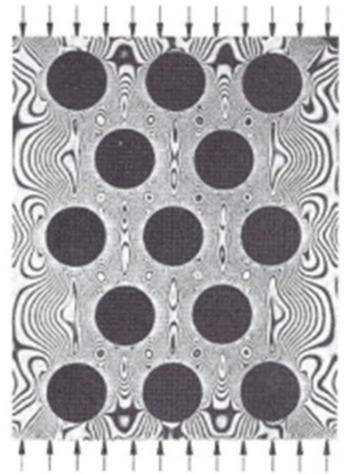
Mode I cracking in compression



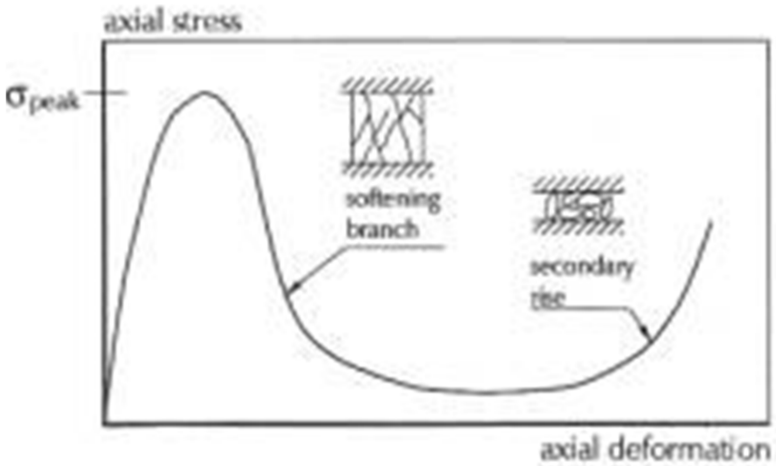


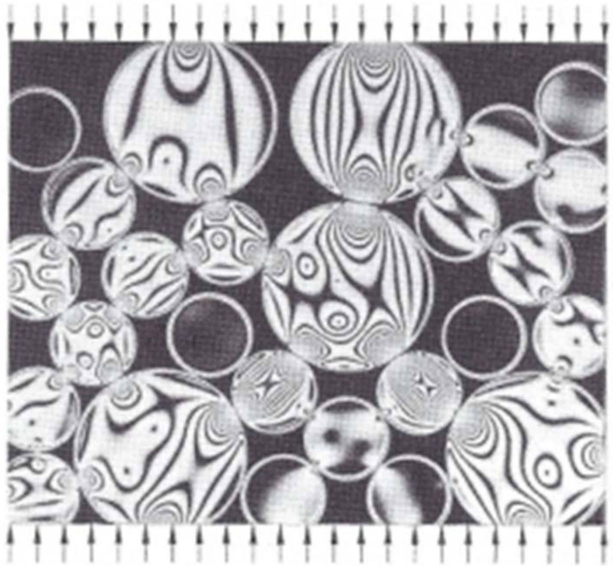


(b)



(c)





(c)

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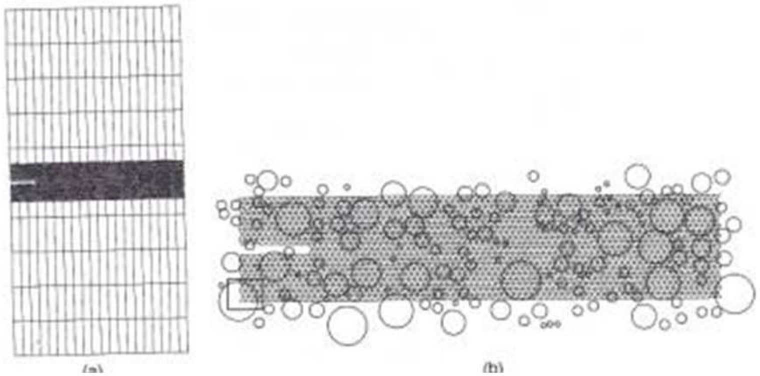
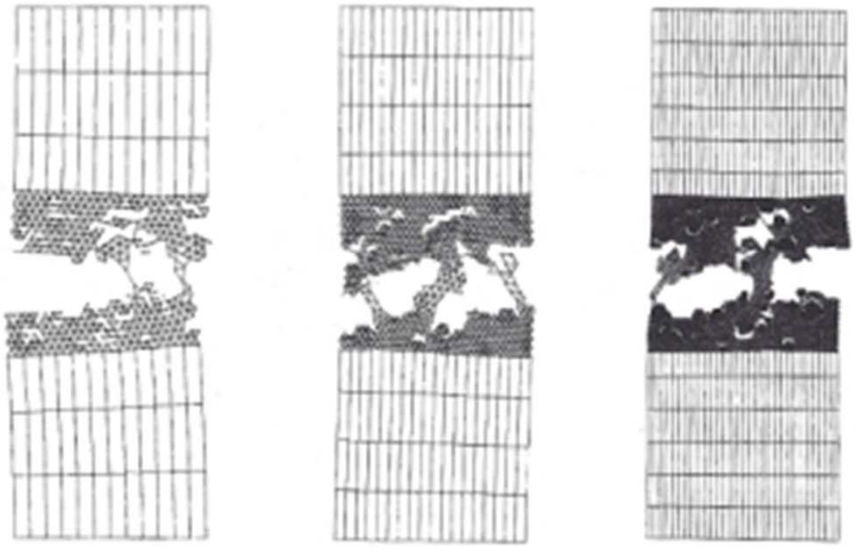
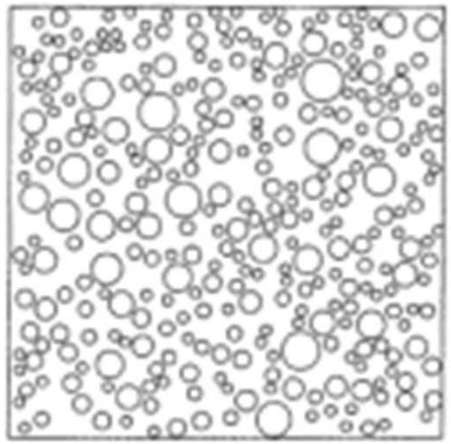
Lattice models: Modeling the full details of the microstructure

most important: particle structure







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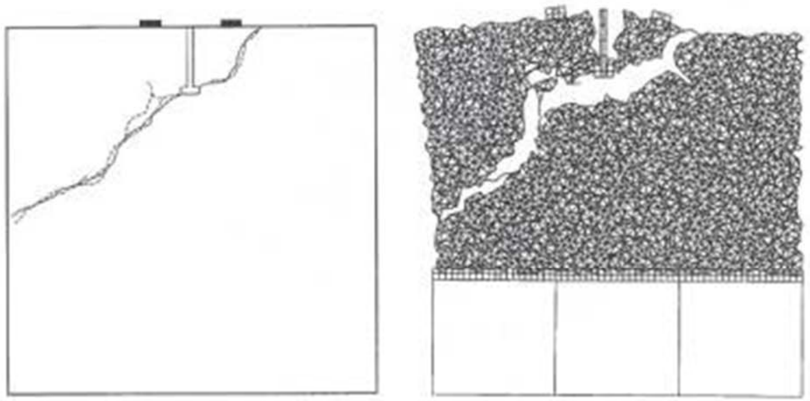
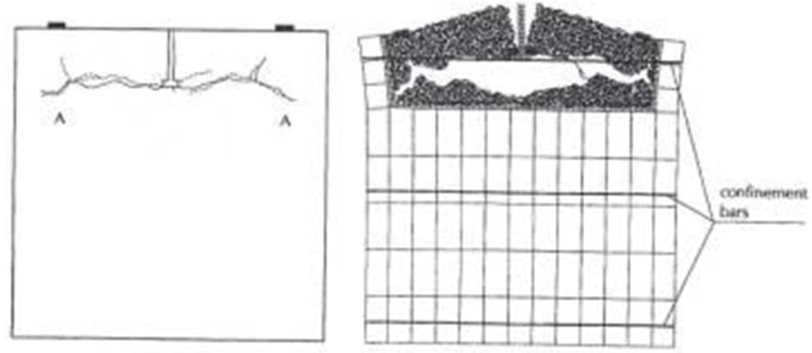
Ability to predict the complex behavior of microcracking



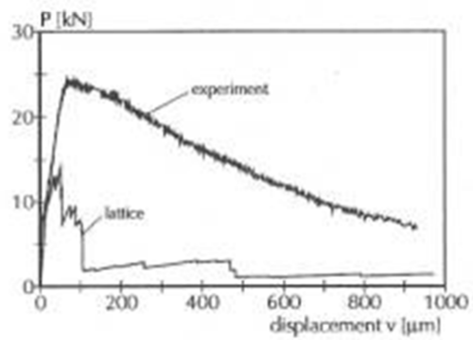
(a) (b)

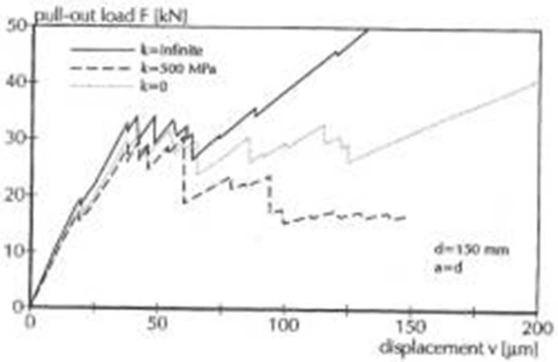
Figure 8.13 Crack growth observed in experiments with $a = d/2$ (a), and from lattice analyses (b).[509]



(a) (b)

Figure 8.16 Experiment (a) and numerical lattice simulation of a laterally confined specimen with a support span $a = 2d$ ($d = 150$ mm).[509]





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Dipolar elasticity

Strain gradient elasticity: $\sigma_{xx} = (1 - g^2\nabla^2)E\varepsilon_{xx}$ (1D, v=0)

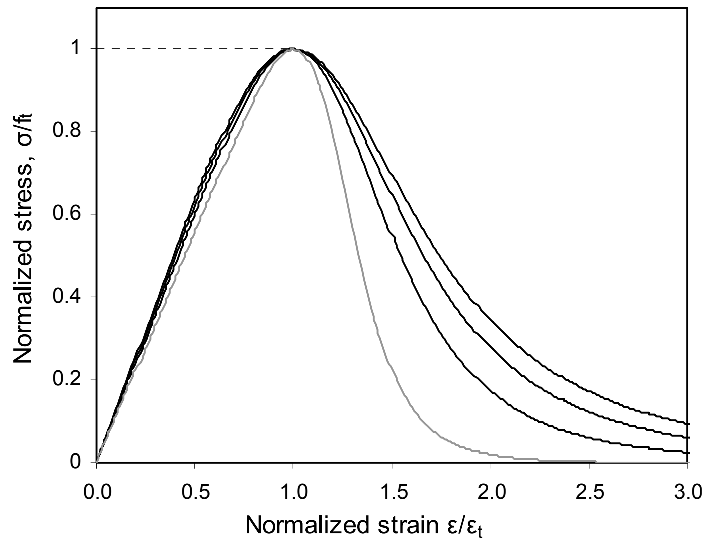
Hooke's law extended to the 2nd spatial derivative of the Cauchy strain

Non-local models:

A length scale parameter is needed for modeling materials such as concrete exhibiting size effect

Stress-strain softening law:

$$\frac{\sigma}{f_i} = \frac{\beta_i(\varepsilon/\varepsilon_i)}{\beta_i - 1 + (\varepsilon/\varepsilon_i)^{\beta_i}}$$

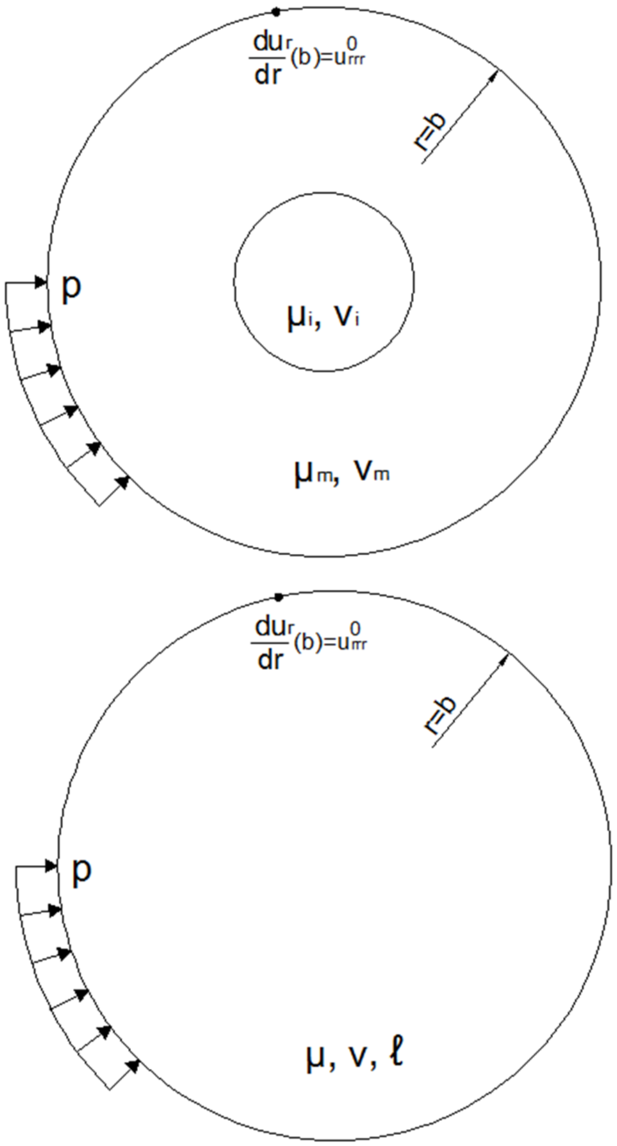


Size effect in elasticity:

Stiffer static response than that predicted by classical elasticity if g>0

What is a physical interpretation of the internal length?

The simplest case of composite material (2D case of circular inclusions)



Input parameters

Matrix and inclusion elasticity properties

Composition value, c=a/b

Effective material properties of composite material

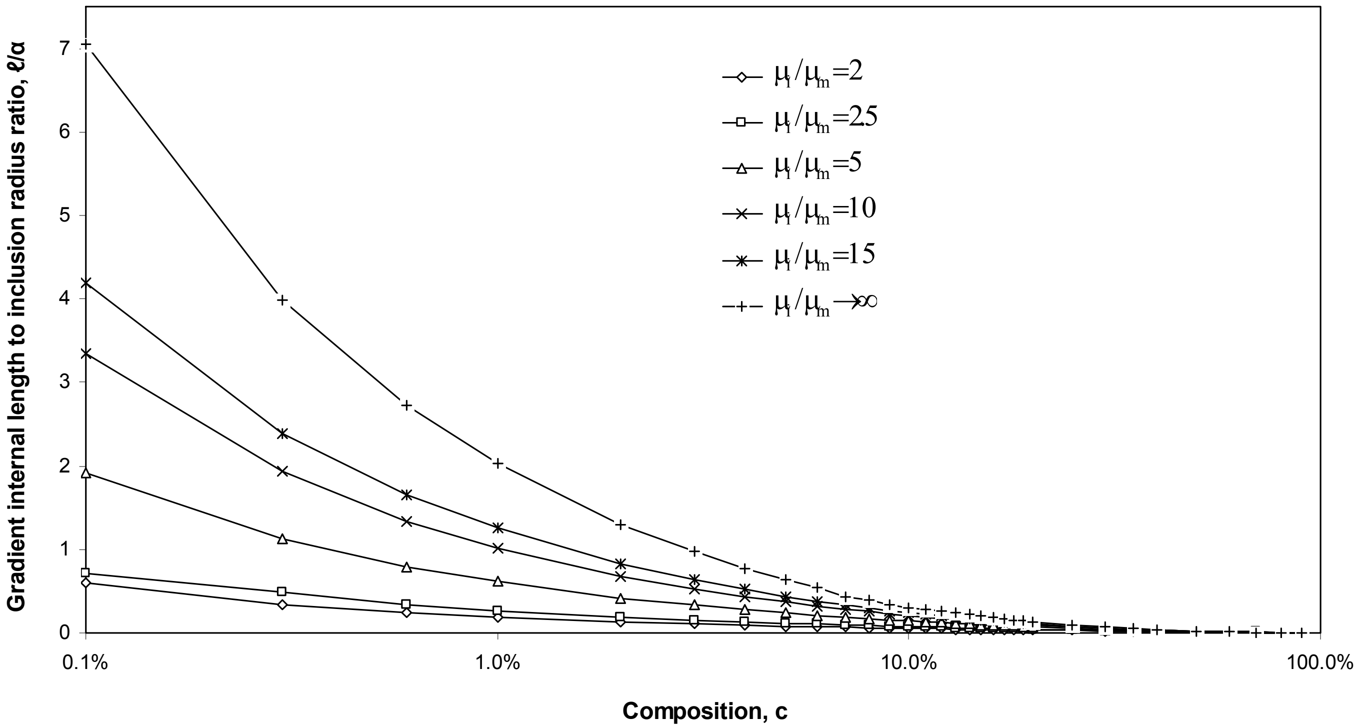
$$\mu = f_1(\mu_m, \nu_m, \mu_i, \nu_i, c)$$

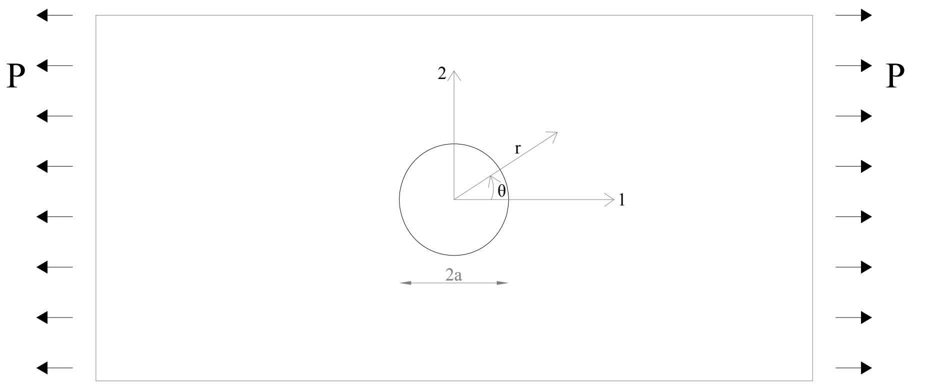
$$\nu = f_2(\mu_m, \nu_m, \mu_i, \nu_i, c)$$

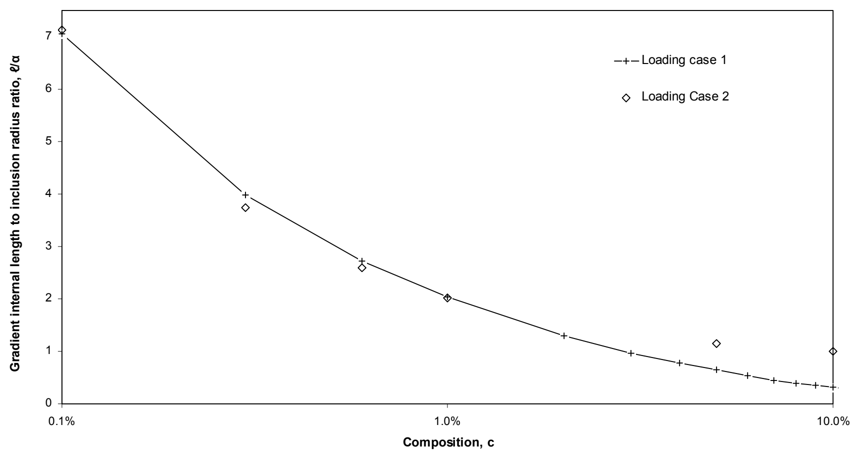
Estimation of the internal length

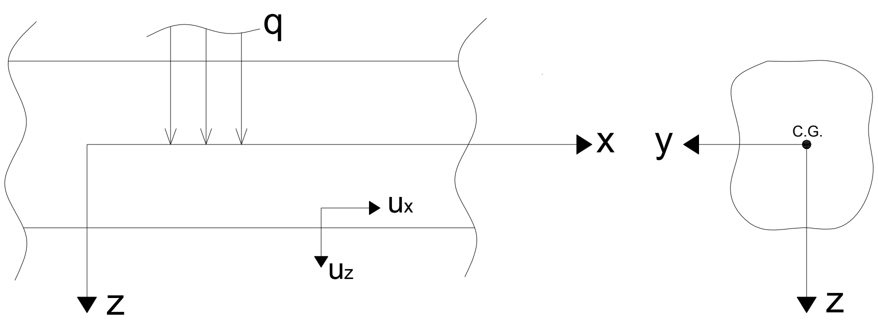
$$U_{cl} = U_{gr}$$

HOMOGENIZATION ESTIMATES OF THE INTERNAL LENGTH









Timoshenko kinematics

$$u_x = z\,\psi(x)$$

$$u_y = 0$$

$$u_z = w(x)$$

Cauchy stresses

$$\bar{\sigma}_{xx} = E\varepsilon_{xx}$$

$$\bar{\sigma}_{xz} = kG\gamma_{xz}$$

Total strain energy:

$$U_{tot} = U_{cl} + U_{gr}$$

$$U_{cl} = \frac{1}{2}\iiint_V \left(\bar{\sigma}_{xx}\varepsilon_{xx} + 2\bar{\sigma}_{xz}\,\varepsilon_{xz}\right) dxdydz$$

$$U_{gr} = \frac{1}{2}g^2\iiint_V \left(\frac{\partial\bar{\sigma}_{xx}}{\partial x}\frac{\partial\varepsilon_{xx}}{\partial x} + 2\frac{\partial\bar{\sigma}_{xz}}{\partial x}\frac{\partial\varepsilon_{xz}}{\partial x} + \underbrace{\frac{\partial\bar{\sigma}_{xx}}{\partial z}\frac{\partial\varepsilon_{xx}}{\partial z}}_{\text{Term that should not be omitted}}\right) dxdydz$$

Term that should not be omitted

Non-zero strains:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = z\frac{d\psi}{dx}$$

$$\gamma_{xz} = 2\varepsilon_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = \frac{dw}{dx} + \psi$$

$$\delta U_{tot} = \int_0^L \left\{ \begin{aligned} &\delta\psi\left(-EI\left(1 - g^2 \frac{d^2}{dx^2}\right)\frac{d^2\psi}{dx^2} + kAG\left(1 - g^2 \frac{d^2}{dx^2}\right)\left(\frac{dw}{dx} + \psi\right) - EAg^2 \frac{d^2\psi}{dx^2}\right) \\ &+ \delta w\left(-kAG\left(1 - g^2 \frac{d^2}{dx^2}\right)\left(\frac{d^2 w}{dx^2} + \frac{d\psi}{dx}\right)\right) \end{aligned} \right\} dx$$

$$+ \left[\delta\psi\left(EI\left(1 - g^2 \frac{d^2}{dx^2}\right)\frac{d\psi}{dx} + kAGg^2\left(\frac{d^2 w}{dx^2} + \frac{d\psi}{dx}\right) + EAg^2 \frac{d\psi}{dx}\right)\right]_0^L$$

$$+ \left[\delta w\left(kAG\left(1 - g^2 \frac{d^2}{dx^2}\right)\left(\frac{dw}{dx} + \psi\right)\right)\right]_0^L$$

$$+ \left[\delta\psi'\left(g^2 EI \frac{d^2\psi}{dx^2}\right)\right]_0^L$$

$$+ \left[\delta w'\left(kAGg^2\left(\frac{d^2 w}{dx^2} + \frac{d\psi}{dx}\right)\right)\right]_0^L$$

4 independent BC

w, w′, **ψ**, ψ′

(bold are the classical elasticity variables)

4 energetically conjugate quantities

Q, Y, **M**, m

(Y, m: double shear or bi-force and double moment or bi-moment)

$$\delta W = \int_0^L q\delta w dx + [Q\delta w]_0^L + [M\delta\psi]_0^L + [Y\delta w']_0^L + [m\delta\psi']_0^L$$

Equilibrium equations

$$\left(1+\frac{A}{I}g^2-g^2\frac{d^2}{dx^2}\right)\frac{d\overline{M}}{dx}=\left(1-g^2\frac{d^2}{dx^2}\right)\overline{Q} \qquad \left(1-g^2\frac{d^2}{dx^2}\right)\frac{d\overline{Q}}{dx}=-q$$

Boundary conditions

$$\left[\left\{Y-\left(g^2\frac{d\overline{Q}}{dx}\right)\right\}\delta w'\right]_0^L=0 \qquad \left[\left\{Q-\left(\left(1-g^2\frac{d^2}{dx^2}\right)\overline{Q}\right)\right\}\delta w\right]_0^L=0$$

$$\left[\left\{M-\left(\left(1-g^2\frac{d^2}{dx^2}\right)\overline{M}+\frac{A}{I}g^2\overline{M}+g^2\frac{d\overline{Q}}{dx}\right)\right\}\delta\psi\right]_0^L=0 \qquad \left[\left\{m-\left(g^2\frac{d\overline{M}}{dx}\right)\right\}\delta\psi'\right]_0^L=0$$

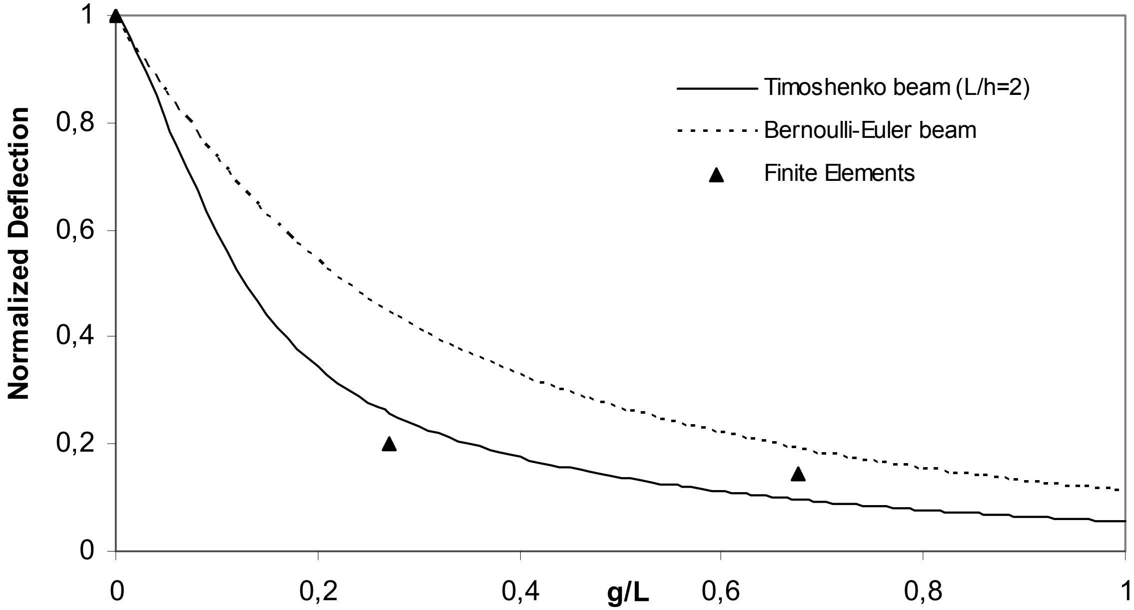
Closed-form solution

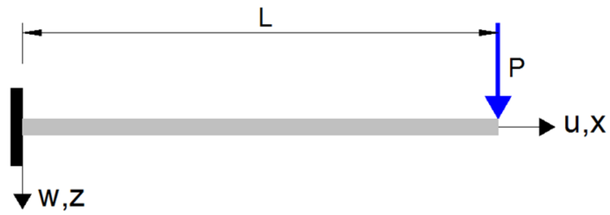
$$w(x)=\left\{\begin{array}{l}(c_3-d_3)x-\frac{d_4}{2}x^2+c_4+d_2\ell^3e^{-x/\ell}-d_1\ell^3e^{x/\ell}+c_1g^2e^{x/g}+c_2g^2e^{-x/g}\\ +\frac{q}{24EI}\left(\frac{\ell}{g}\right)^2x^4-\frac{q}{2kAG}x^2-\frac{c_3kAG}{6EI}\left(\frac{\ell}{g}\right)^2x^3\end{array}\right\}$$

8 constants to be determined from 8 B.C.'s (4 at each end)

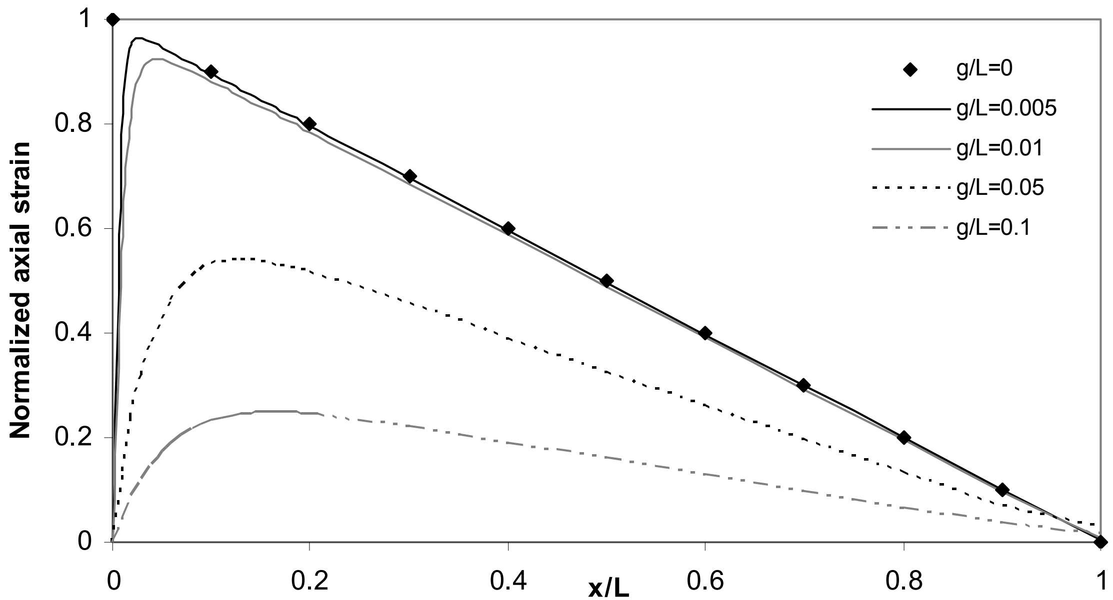
$$\psi(x)=-\frac{q}{6EI}\left(\frac{\ell}{g}\right)^2x^3+\frac{kAG}{2EI}\left(\frac{\ell}{g}\right)^2c_3x^2+d_1\ell^2e^{x/\ell}+d_2\ell^2e^{-x/\ell}+d_3+d_4x$$

STRUCTURAL ANALYSIS USING A DIPOLAR ELASTIC TIMOSHENKO BEAM





<u>Comparison with FEM results</u>



+ Effect of g/h

Clamped end: Boundary layer effect as in BE beam

Free end: zero axial stains

NON-LOCAL DAMAGE MODEL

Gibb's energy: $$G = \frac{1}{2}\boldsymbol{\tau} : \mathbf{C} : \boldsymbol{\tau} + \frac{1}{2}\boldsymbol{\lambda} \therefore \mathbf{B} : \boldsymbol{\lambda} - A^{c}$$ $$\boldsymbol{\lambda} = g^{2}\nabla\boldsymbol{\tau}$$ $$\mathbf{B} = (1/g^{2})\mathbf{C}$$

Microcracking adds flexibility: $$\mathbf{C} = \mathbf{C}^{0} + \mathbf{C}^{c}$$

Opening and closing of microcracks:

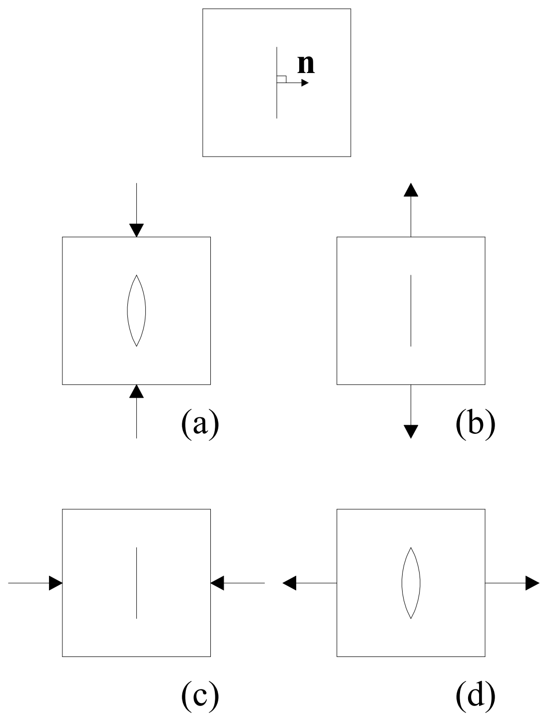
$$\mathbf{C}^{c} = \mathbf{P}^{+} : \overline{\mathbf{C}}^{c} : \mathbf{P}^{+}$$ (positive projections)

$$\boldsymbol{\varepsilon} \approx \mathbf{C}^{0} : \boldsymbol{\tau} + \mathbf{P}^{+}(\overline{\mathbf{C}}^{c} : \boldsymbol{\tau}^{+})$$

$$\nabla\boldsymbol{\varepsilon} \approx \mathbf{C}^{0} : \nabla\boldsymbol{\tau} + \mathbf{P}^{+}(\overline{\mathbf{C}}^{c} : (\nabla\boldsymbol{\tau})^{+})$$

Minimization problem solution for a given state of stress

$$\overline{\mathbf{C}}^{c} = \overline{\mathbf{C}}^{c}_{I_T} + \overline{\mathbf{C}}^{c}_{I_C}$$ Active microcracks: $\boldsymbol{\varepsilon}^{c}_{I_T} \geq 0$ $\boldsymbol{\varepsilon}^{c}_{I_C} \leq 0$



Damage rules: $\dot{\overline{\mathbf{C}}}^{c}_{I_T} = \dot{\mu}\mathbf{R}_{I_T}(\boldsymbol{\tau})$ and $\dot{\overline{\mathbf{C}}}^{c}_{I_C} = \dot{\mu}\mathbf{R}_{I_C}(\boldsymbol{\tau})$

Irreversible character of damage: $\dot{\mu} \geq 0$

Internal length is assumed to be a function of damage: $g = g(\mu)$

Energy density dissipation:

$$d = \left(\begin{array}{l} \frac{1}{2}\boldsymbol{\tau}^{+} : \mathbf{R}_{I_T} : \boldsymbol{\tau}^{+} + \frac{1}{2}\boldsymbol{\tau}^{-} : \mathbf{R}_{I_C} : \boldsymbol{\tau}^{-} + \frac{1}{2} g^{2} (\nabla\boldsymbol{\tau})^{+} \therefore \mathbf{R}_{I_T} : (\nabla\boldsymbol{\tau})^{+} \\ + g\frac{dg}{d\mu}(\nabla\boldsymbol{\tau})^{+} \therefore \overline{\mathbf{C}}^{c} : (\nabla\boldsymbol{\tau})^{+} \end{array} \right) \dot{\mu} - \dot{A}^{c} \geq 0$$

$\mathbf{R}_{I_T}, \mathbf{R}_{I_C}, \mathbf{C}^{c}_{I_T}, \mathbf{C}^{c}_{I_C}$ positive definite and $\dot{\mu} \geq 0 \Rightarrow dg/d\mu \geq 0$

Associated damage surface:

$$\Phi = \frac{1}{2}\boldsymbol{\tau}^{+} : \boldsymbol{\tau}^{+} + \frac{1}{2}c\boldsymbol{\tau}^{-} : \boldsymbol{\tau}^{-} - \frac{\pi}{2}t^{2}(\mu)$$

4-point bending: an assumed stress-strain law is sufficient input information for damage characterization

Definition of damage

$$\tau_i = E_{0i}\varepsilon_i, \text{ for } \varepsilon_i \le \varepsilon_{0i} \qquad \tau_i = (1 - D_i)E_{0i}\varepsilon_i = \frac{E_{0i}\varepsilon_i}{1 + E_{0i}\mu_i}, \text{ for } \varepsilon_i > \varepsilon_{0i}$$

Assumed stress-strain law:

$$\boxed{\tau_i = f_i \frac{\beta_i (\varepsilon / \varepsilon_i)}{\beta_i - 1 + (\varepsilon / \varepsilon_i)^{\beta_i}}}$$

Damage functions:

$$D_i = 0 \text{ for } \varepsilon < \varepsilon_{0i} \qquad D_i = 1 - \frac{\beta_i - 1 + (\varepsilon_{0i} / \varepsilon_i)^{\beta_i}}{\beta_i - 1 + (\varepsilon / \varepsilon_i)^{\beta_i}} \quad \text{for } \varepsilon \ge \varepsilon_{0i}$$

Young's modulus and isotropy:

$$E_{0i} = \frac{\beta_i f_i}{\left(\beta_i - 1 + (\varepsilon_{0i} / \varepsilon_i)^{\beta_i}\right)\varepsilon_i} \qquad \frac{\varepsilon_t}{\varepsilon_c} = \frac{\beta_t f_t \left(\beta_c - 1 + (\varepsilon_{0c} / \varepsilon_c)^{\beta_c}\right)}{\beta_c f_c \left(\beta_t - 1 + (\varepsilon_{0t} / \varepsilon_t)^{\beta_t}\right)}$$

APPLICATION TO 4-POINT BENDING

Linear strain distribution: $\varepsilon_{xx} = \varepsilon_m + kz$

$$N = b\int_{-h/2}^{h/2} \sigma_{xx}\,dz = 0$$

Input: a given value for curvature, **k**

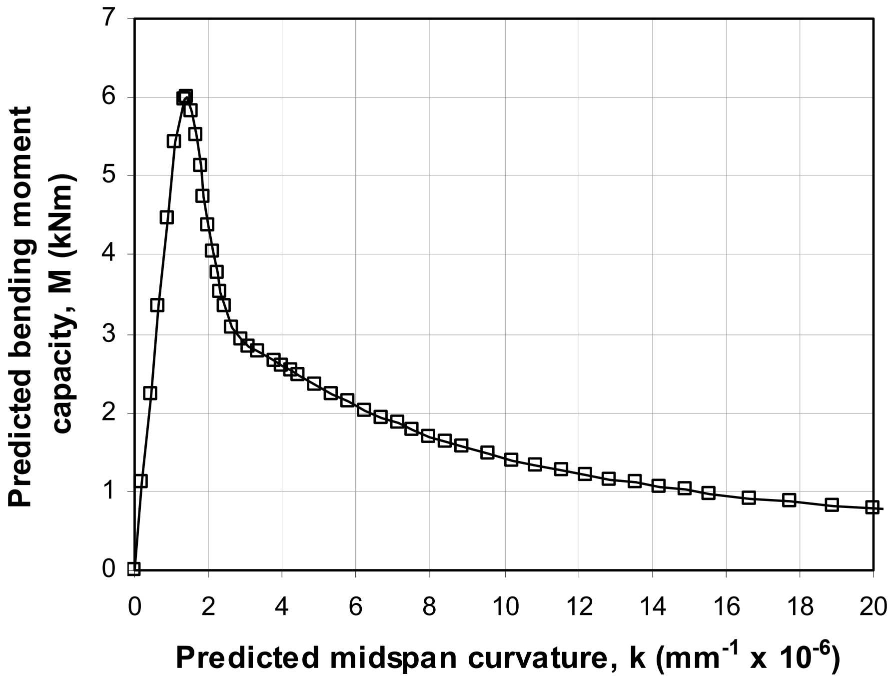
Find: $\boldsymbol{\varepsilon_m}$ that satisfies equilibrium (stress-strain law)

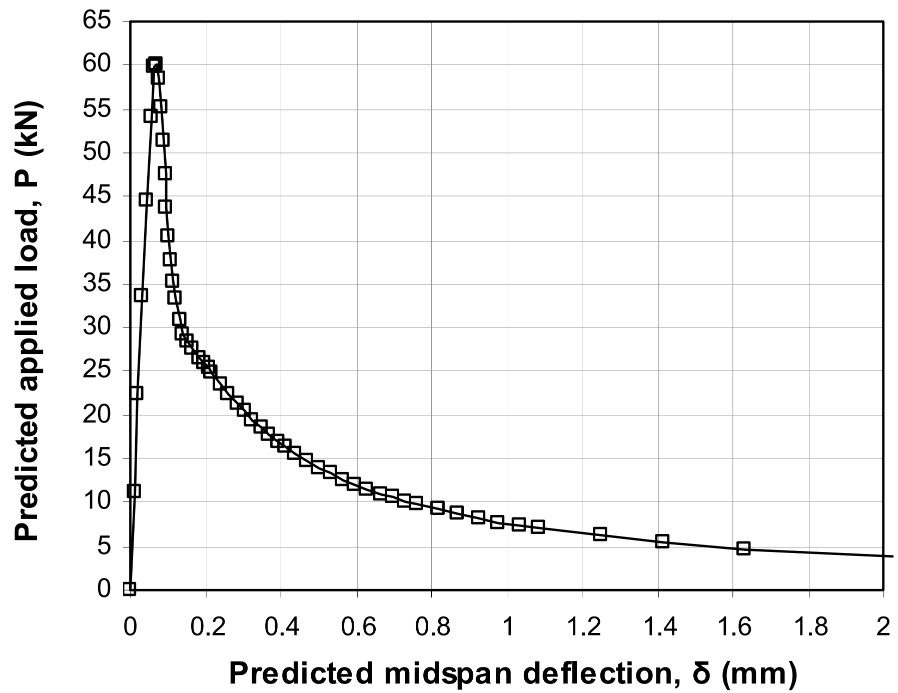
$$M = b\int_{-h/2}^{h/2} \sigma_{xx}\,z\,dz$$

Output: **M**, **k** point on the response diagram (size independent)

Input: **k-δ** kinematic relation (boundary value problem)

Final output: Force vs. midspan deflection diagram (size dependent)





Non-local predictions:

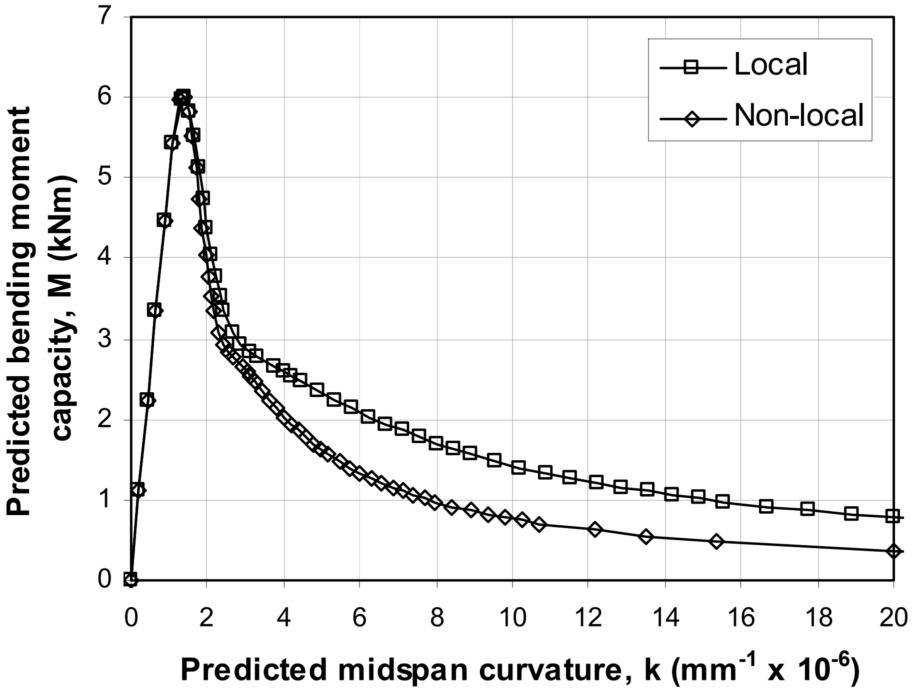
The local **M** vs. **k** is transformed to **M** vs. $\mathbf{k_{grad}}$ (scaling of curvature-elasticity)

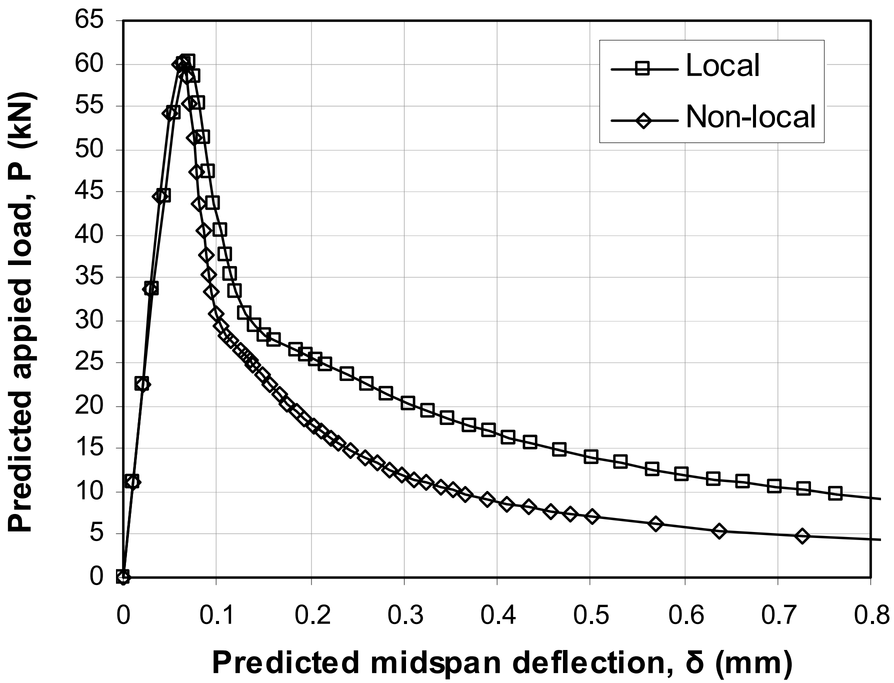
The gradient solution of the boundary value problem is used to obtain the kinematic relation, $\boldsymbol{\delta_{grad}/k_{grad} = f(g/L)}$

This relation is not constant but evolves with damage since g=g(D)

Internal length evolution law:

$$g = g_0 e^{nD}$$





Objectivity of the model:

Acoustic tensor positive definite

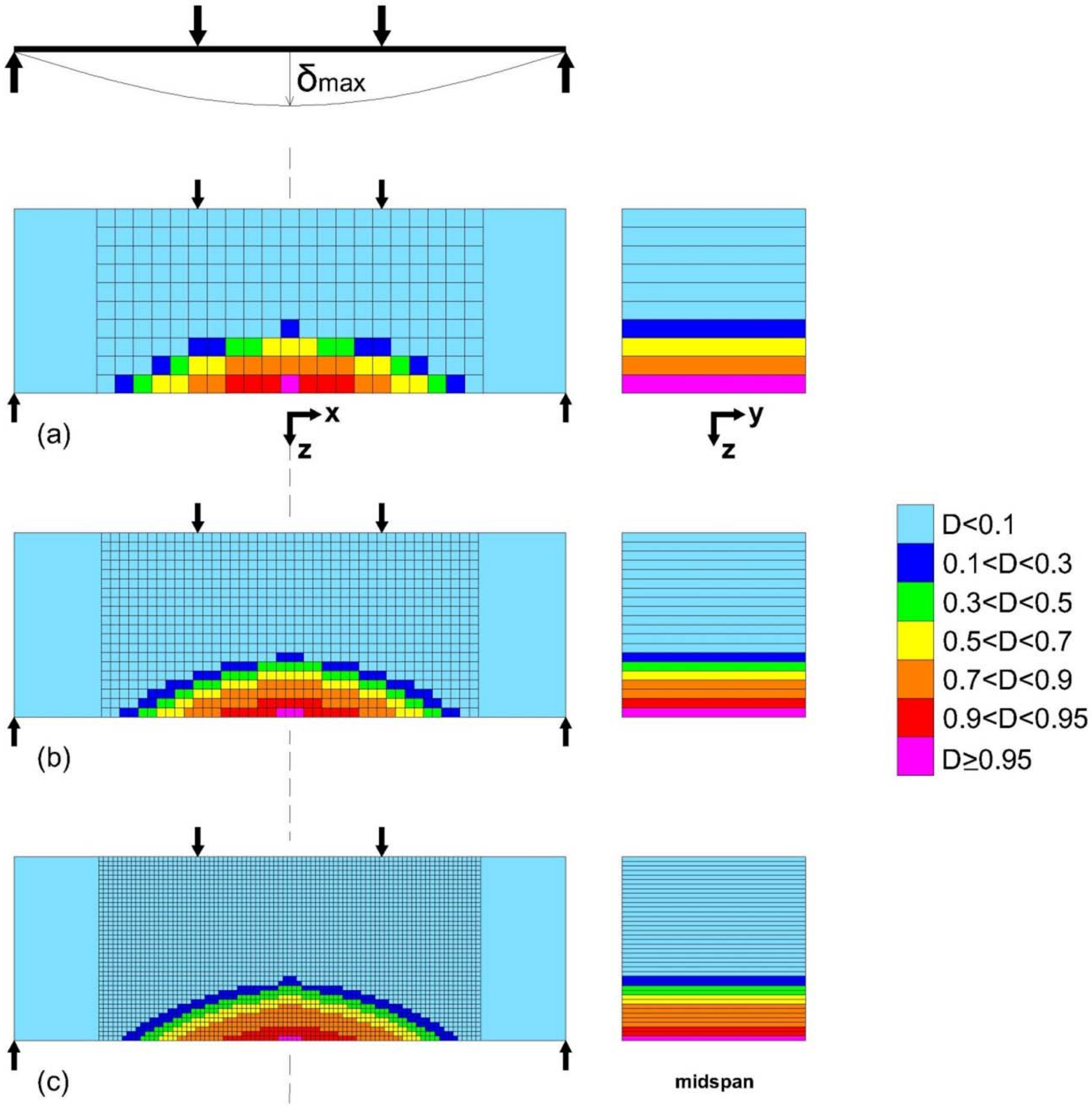
Good agreement with AE findings:

•Initiation of softening (D>0) and therefore microcracking

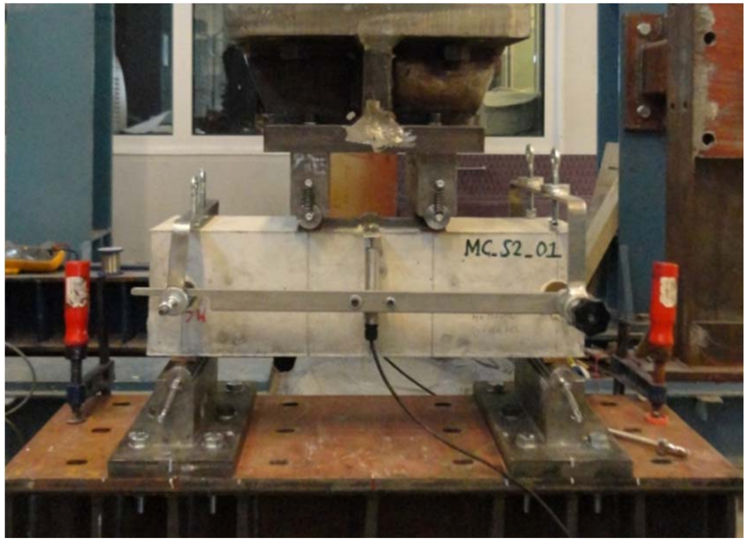
•Intense localization of AE energy release rate – macro-crack formation (D>0.95)

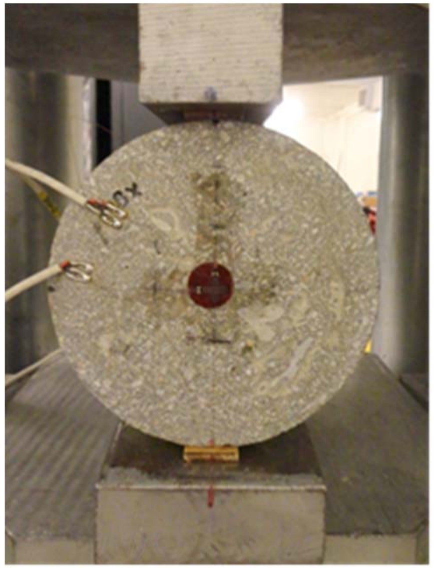
Objective damage characterization:

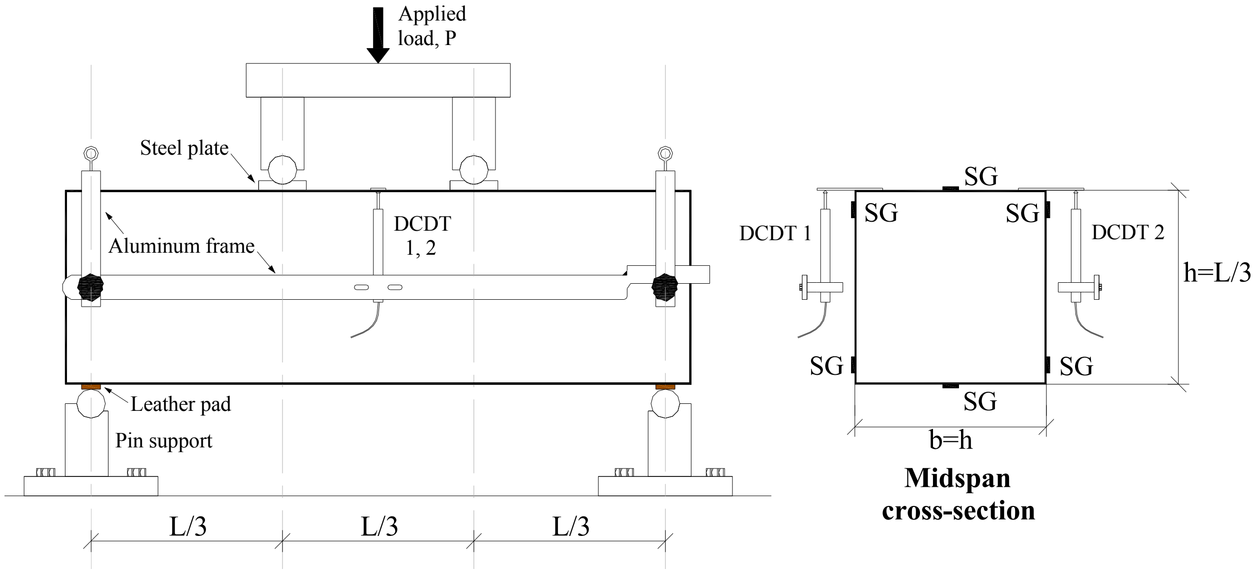
(for both g=0 and g>0)



EXPERIMENTAL PROGRAM







3D geometrical similar specimens

MATERIALS

Mix	Quantities (Kg/m^3)				Dry density (Kg/m^3)	Slump (cm)	Air-content [d] (%)
	Cement [a]	Aggregates [b]	Water (w/c ratio)	Additives [c]			
CM	450	1350	293 (0.65)	3.6	2100	- [e]	2.5
LC	208	1980	162 (0.78)	1.6	2335	25	3.0
NC	276	2080	176 (0.64)	1.5	2365	10	2.5
MC1	448	1720	204 (0.45)	4.0	2410	22.4	2.0
MC2	447	1640	207 (0.46)	6.0	2440	15.6	2.0

Sieve opening (mm)	% passing				
	LC	**NC**	**MC1**	**MC2**	**CM**
32	100	100	100	100	-
16	85.8	84.1	80.6	78.7	-
8	70.7	67.8	60.0	57.7	-
4	62.7	59.7	49.6	49.1	-
2	45.4	43.3	35.7	35.5	-
1	29.6	28.2	23.3	23.2	100
0.5	-	-	-	-	30
0.25	12.9	12.3	10.2	10.1	-
0.075	8.0	7.6	6.3	6.3	-

Four concrete mixes d_{max}=32 mm

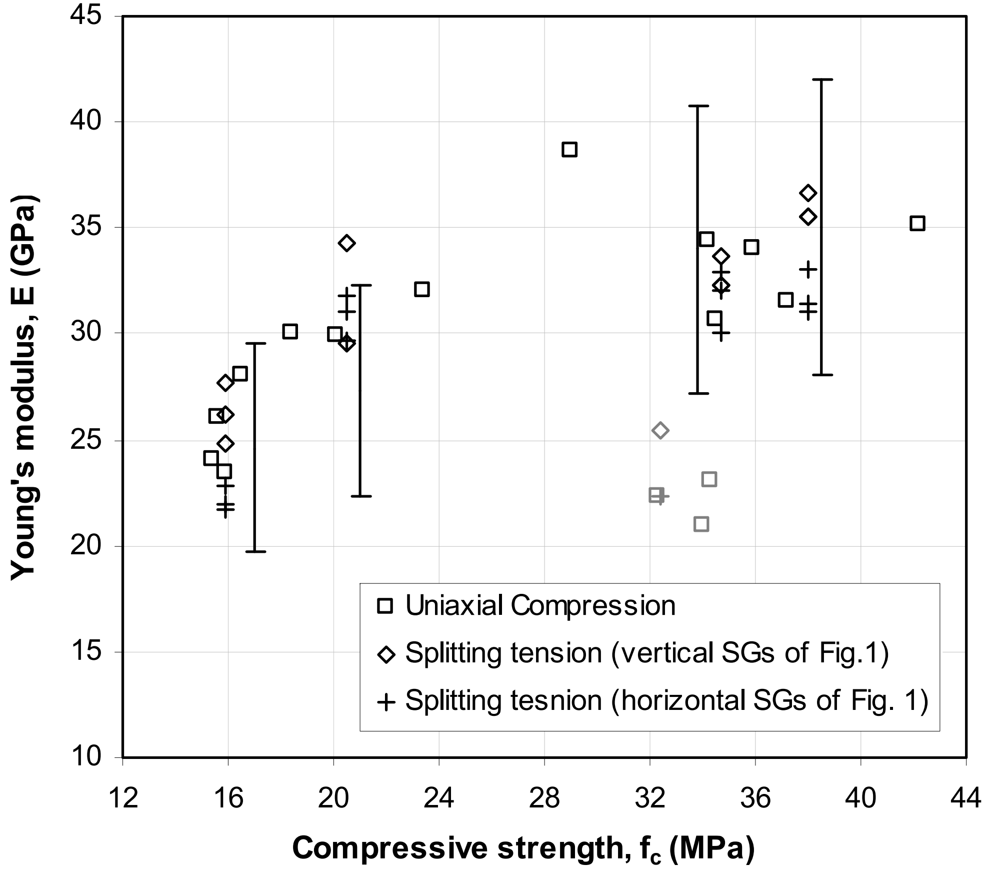
Low-strength (LC)

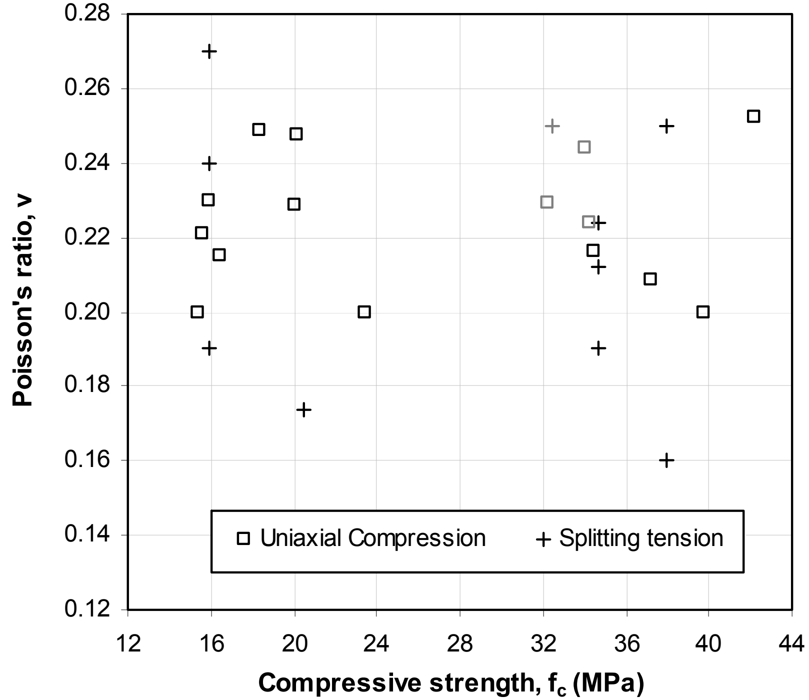
Normal-strength (NC)

Medium-strength (MC1 and MC2)

Cement mortar d_{max}=1 mm

CLASSICAL ELASTICITY PROPERTIES

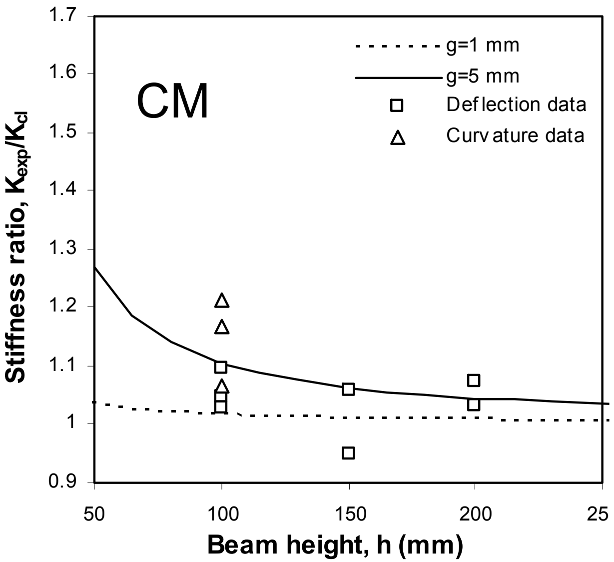


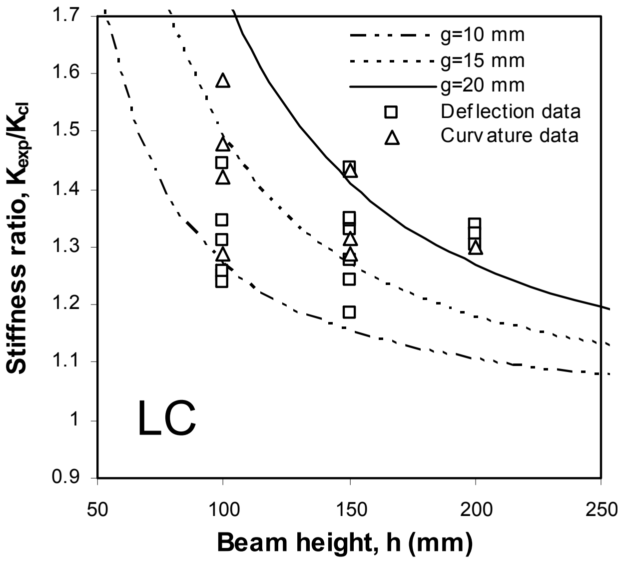


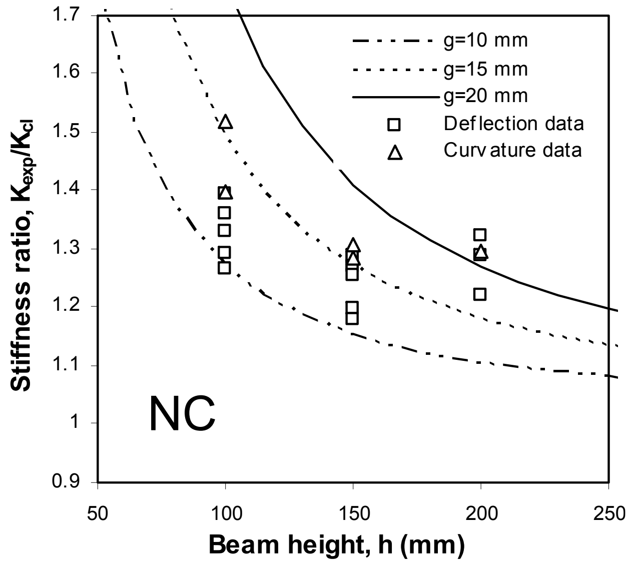
Classical elasticity properties used in the analysis

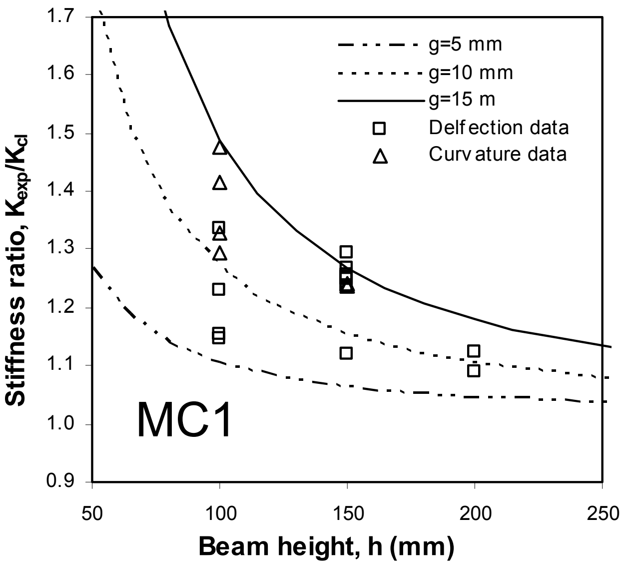
Mix	LC	NC	MC1	MC2	CM
E, GPa	25.0	30.70	32.7	34.0	22.3
ν	0.2				

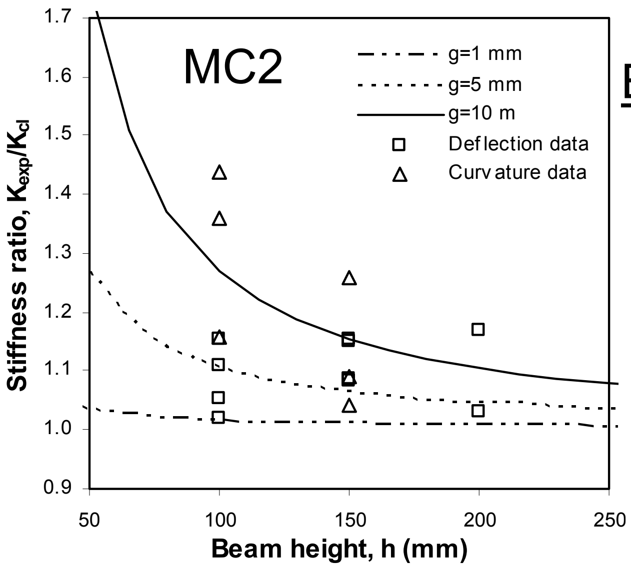
INTERNAL LENGTH AND SIZE EFFECT IN ELASTICITY











Elasticity: **$P=K\delta$ & $P=S k$**

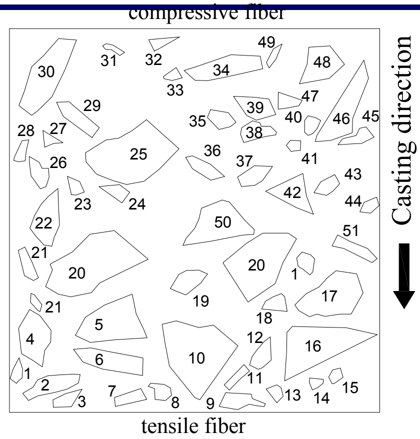
Size effect in elasticity

if: $K_{exp}/K_{cl} > 1$

$S_{exp}/S_{cl}>1$

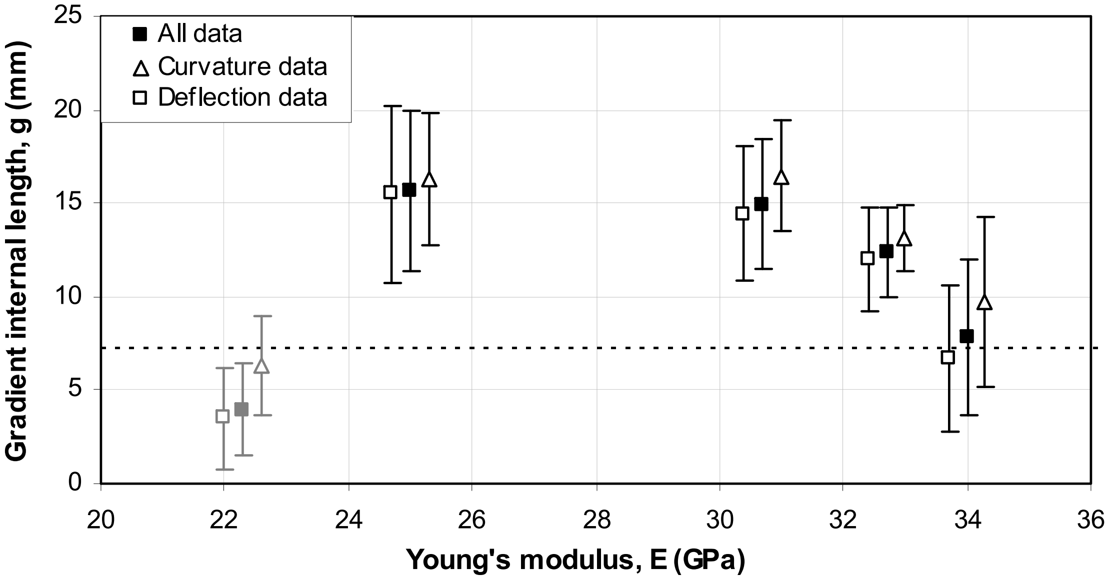
INTERNAL LENGTH AND MICROSTRUCTURE

Concrete contains different aggregate sizes in different volume fractions

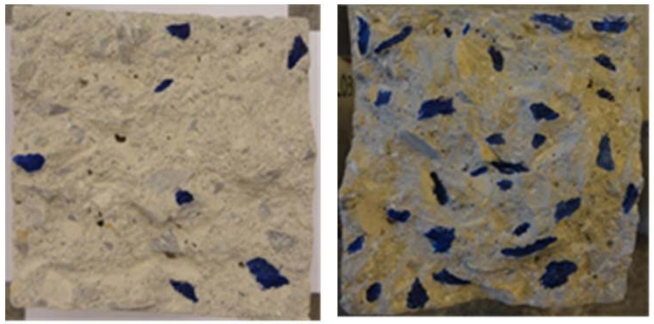


Average inclusion size ranges between

10 to 20 mm

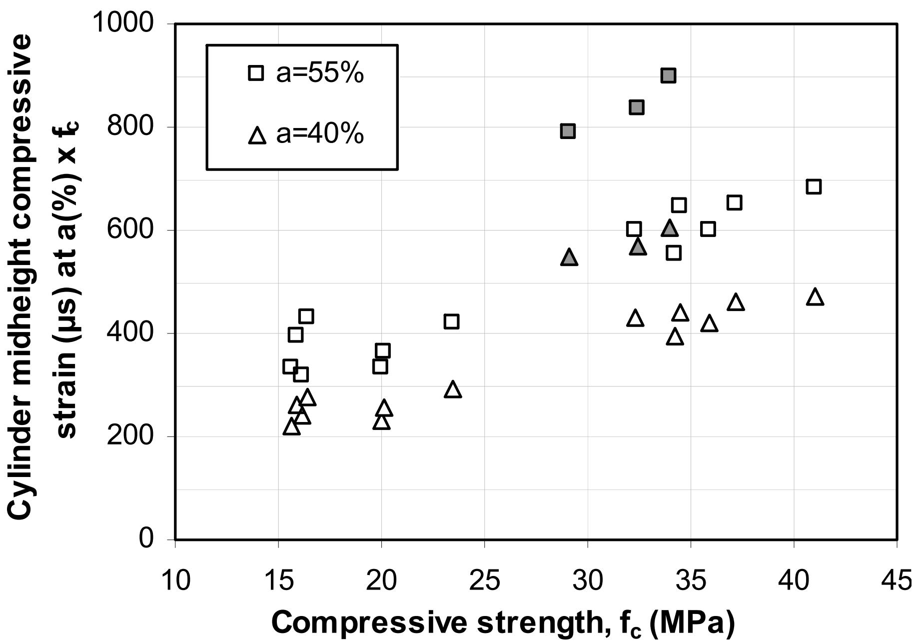


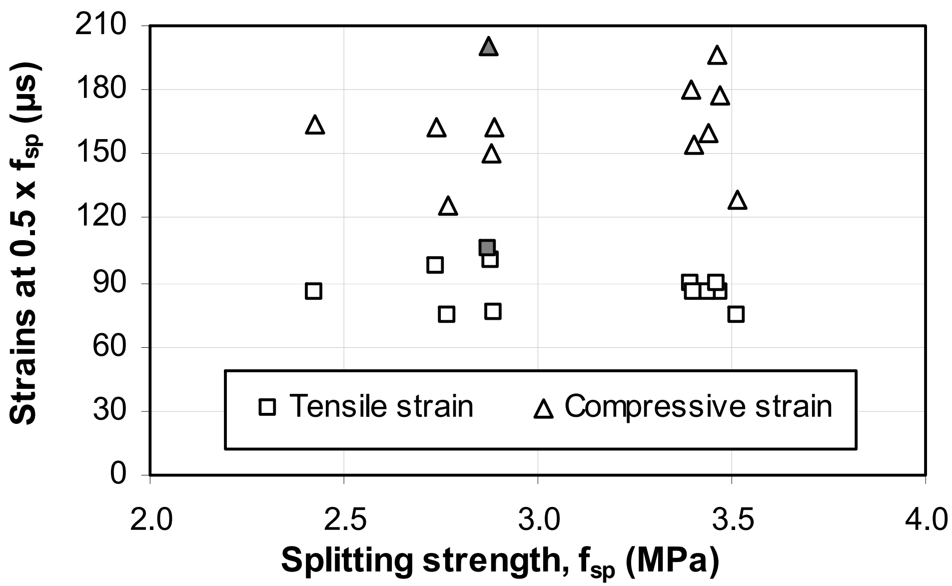
Fracture surfaces and fractured aggregates



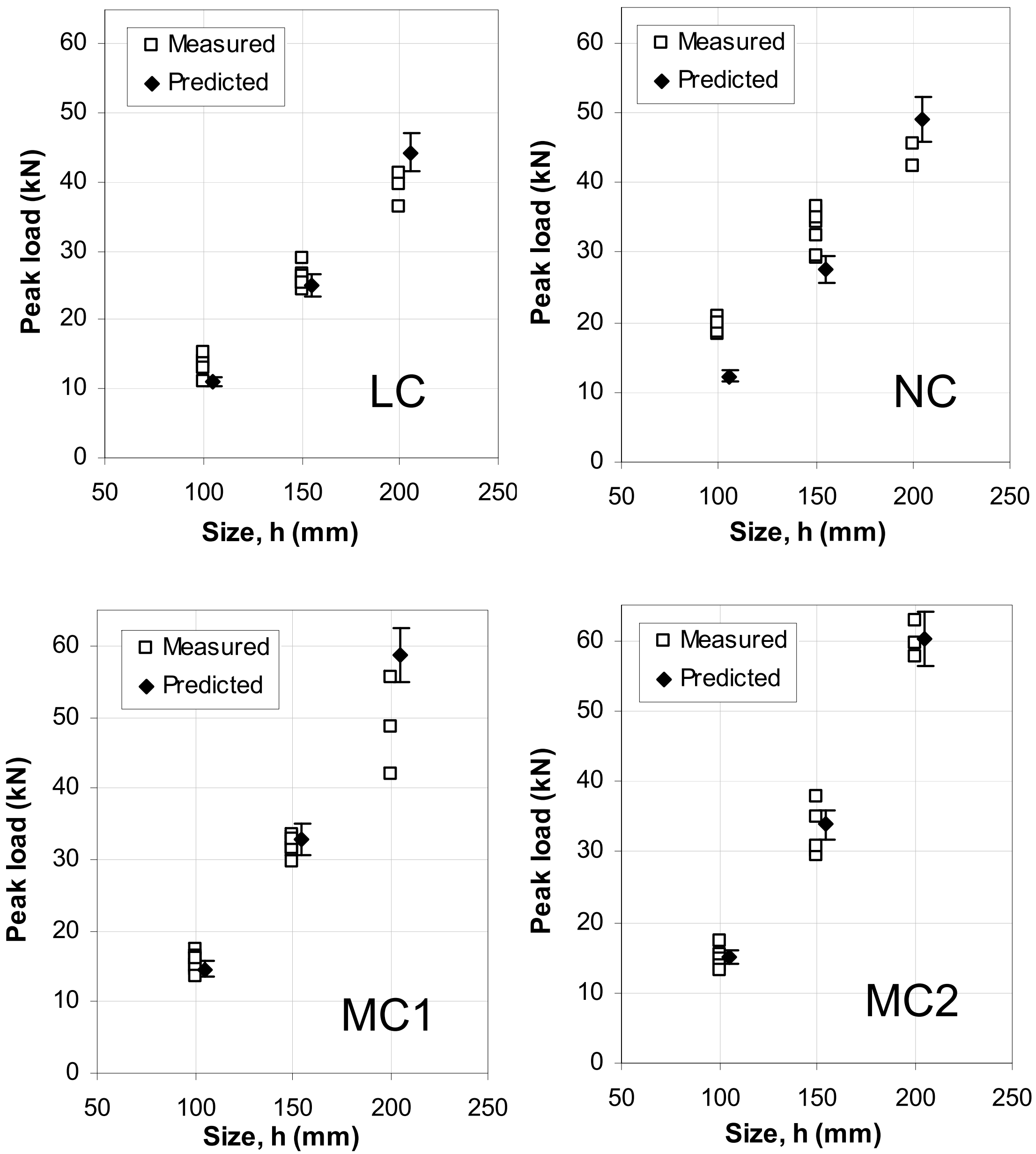
INELASTICITY: PEAK LOAD

Mix		Uniaxial stress-strain law parameters										Young's modulus
		Compression					Tension					
		f_c (a)	ε_c	ε_{0c} (b)	b_c (b)		f_t (c)/ f_{sp} (a)	b_t (c)	ε_{0t} (d)	ε_t (d)		E (a)
		(MPa)	($x10^{-6}$)	($x10^{-6}$)	-		-	-	($x10^{-6}$)	($x10^{-6}$)		(GPa)
LC		15.9	1500	250	1.643		0.80	4.5	70	110		25.0
NC		20.5	1500	270	1.707		0.85	5.0	65	100		30.7
MC1		34.7	1500	430	3.360		0.88	6.0	80	120		32.7
MC2		38.0	1500	450	3.890		0.90	6.5	80	110		34.0
CM (e)		32.4	1800	600	5.183		0.95	10	130	130		22.4





INELASTICITY: PEAK LOAD

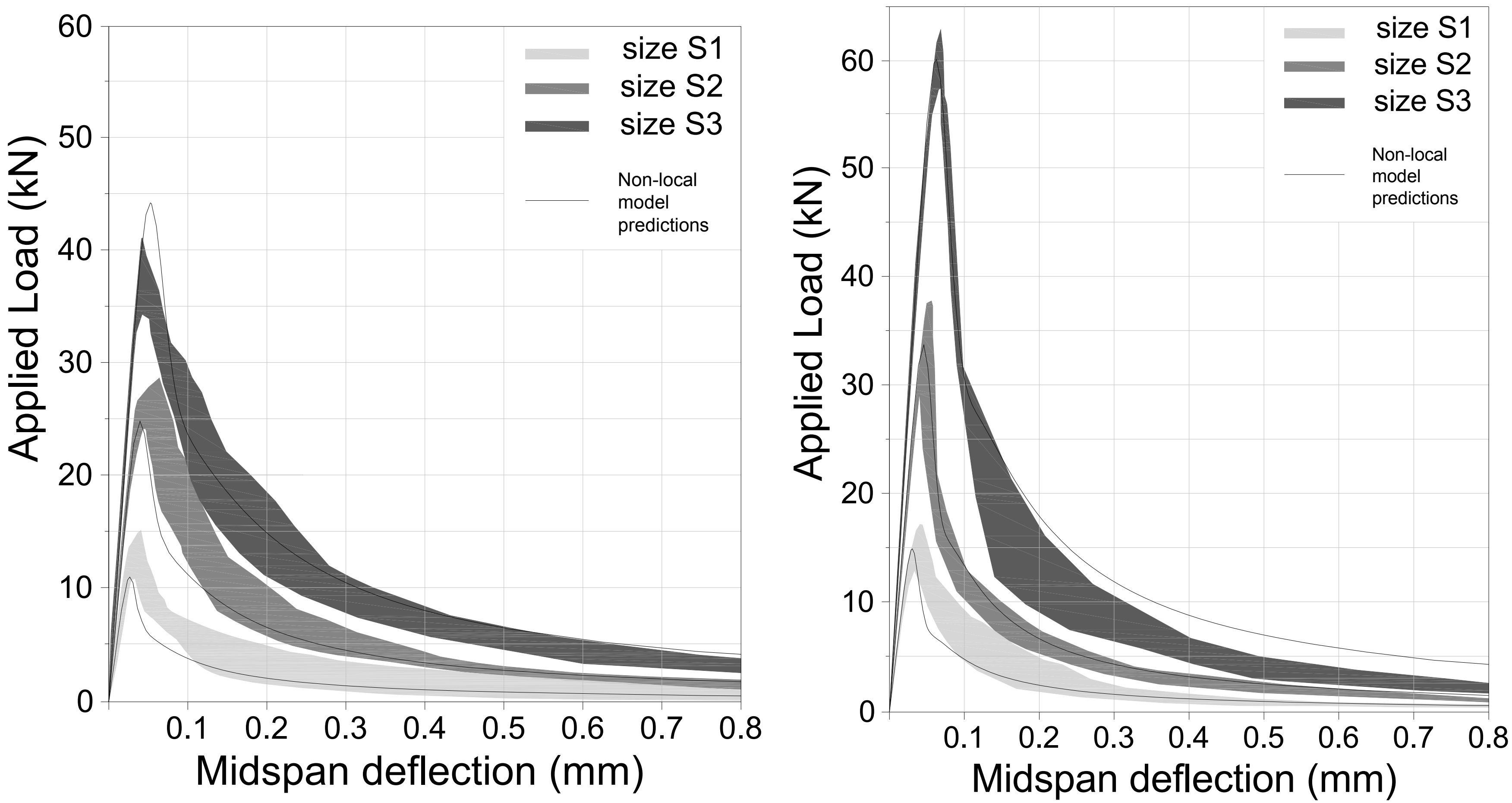


Model predictions correspond to a size independent flexural strength

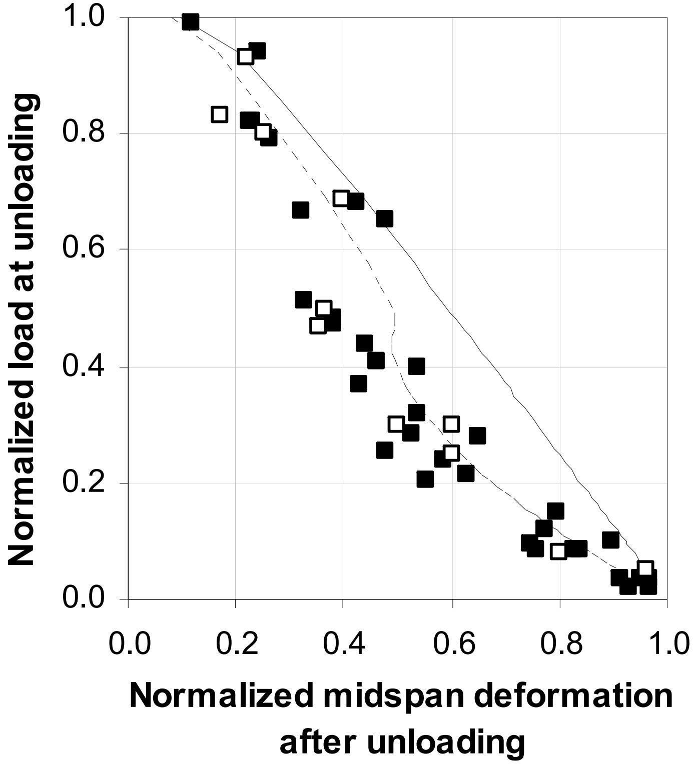
Scatter of predicted values corresponds to a +/- 5% deviation of the assumed material’s tensile strength

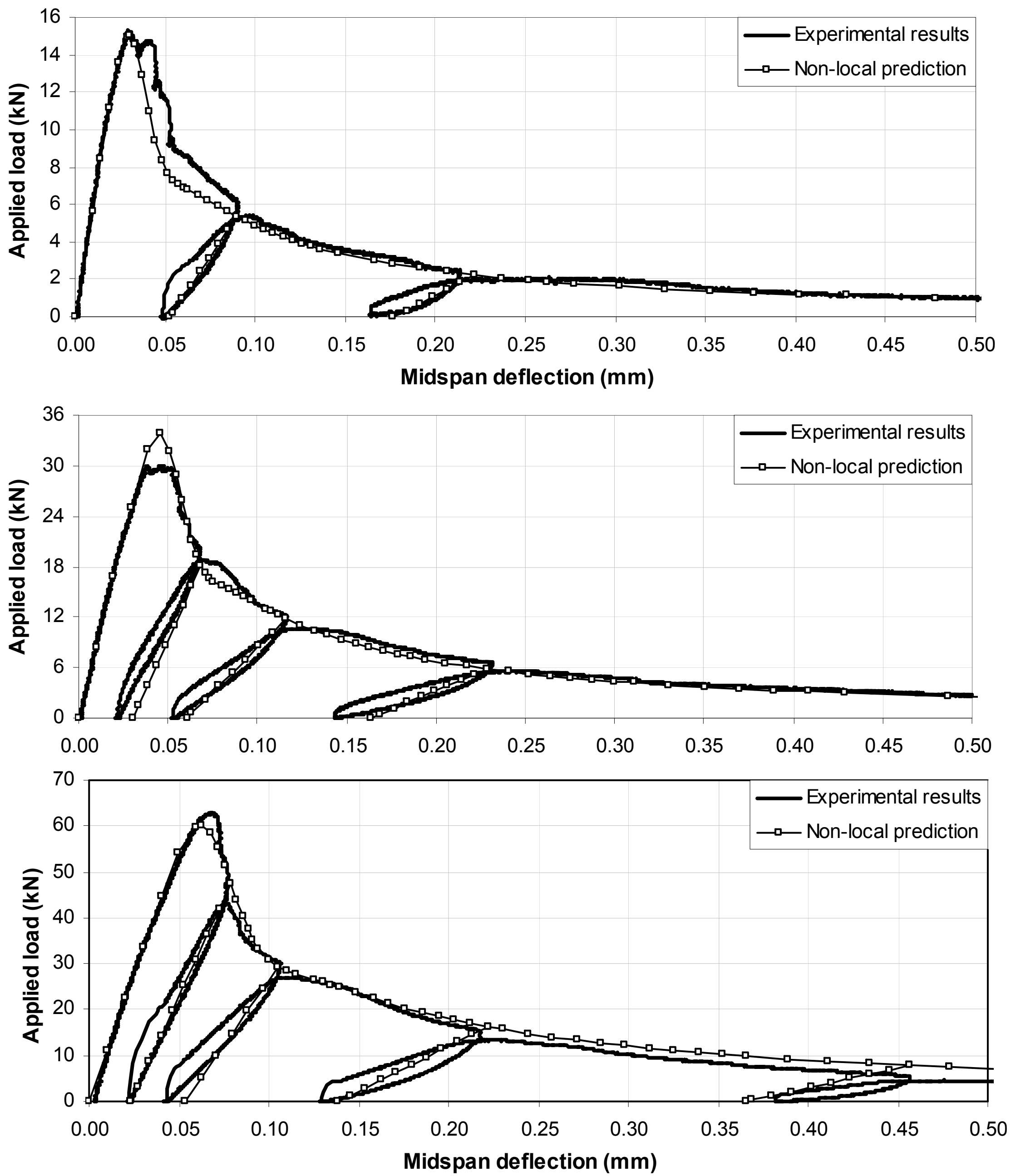
LC mix: $g = 17e^{0.9D}$ **(mm)**

MC2 mix: $g = 8e^{2D}$ **(mm)**



INELASTICITY: UNLOADING PATH



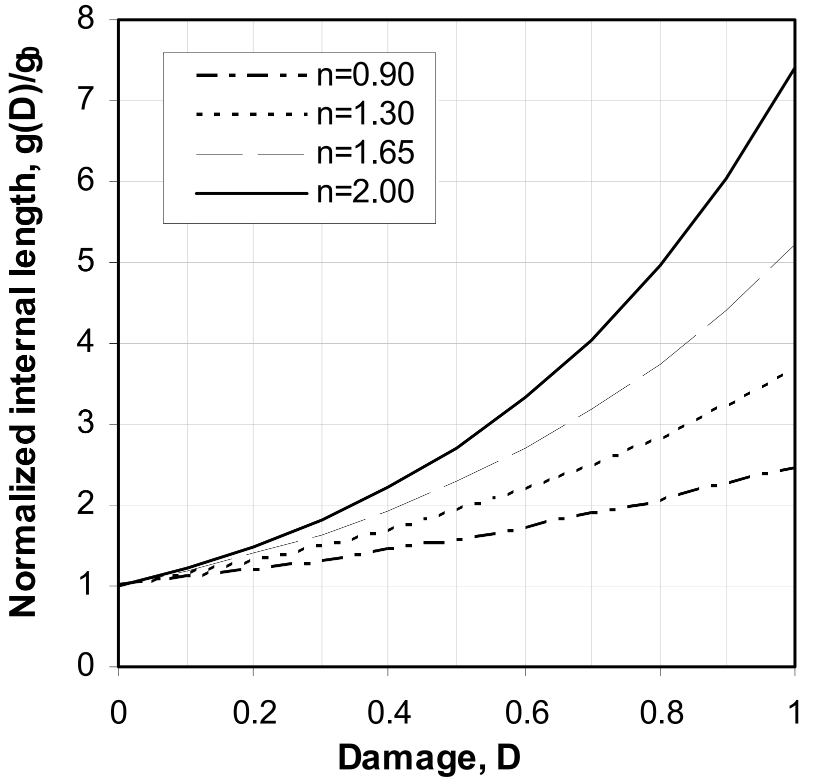


$$\delta_{pl} = \overline{\delta} - \overline{P}/(1-\overline{D})K_0$$

MC2 mix

EVOLUTION LAW OF GRADIENT INTERNAL LENGTH

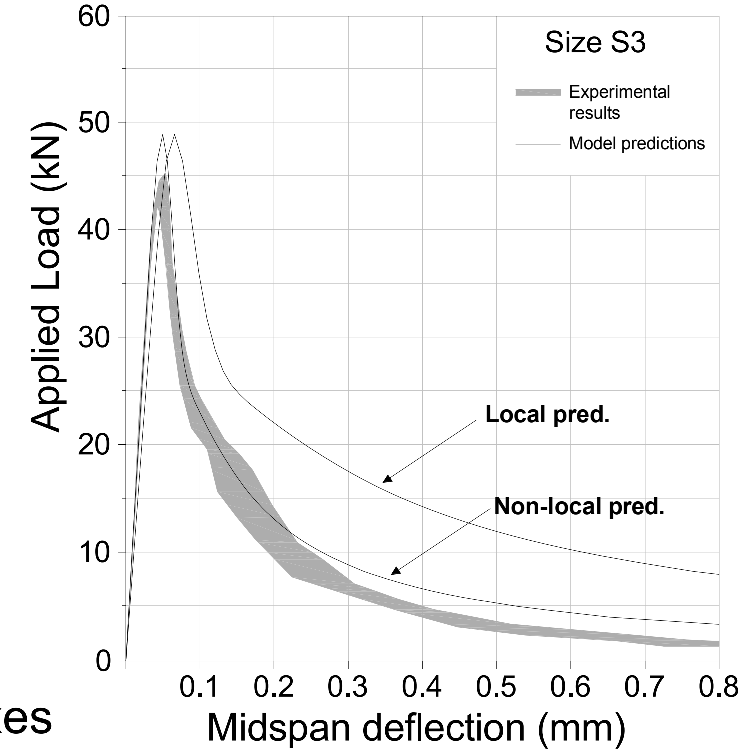
Non-local parameters	Mix			
	LC	NC	MC1	MC2
g_0 (mm)	17	15	12.5	8.0
n	0.90	1.30	1.65	2.00



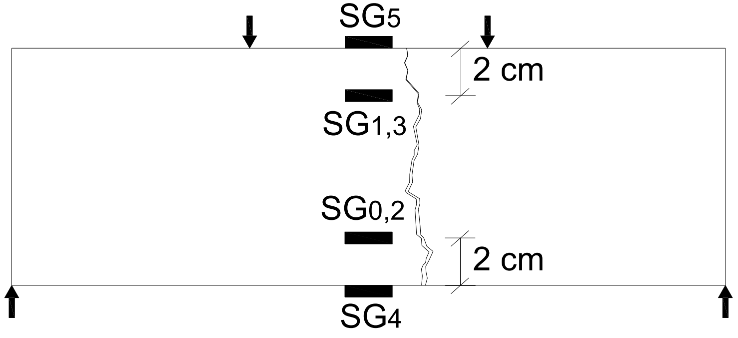
n increases as brittleness increases in the mixes

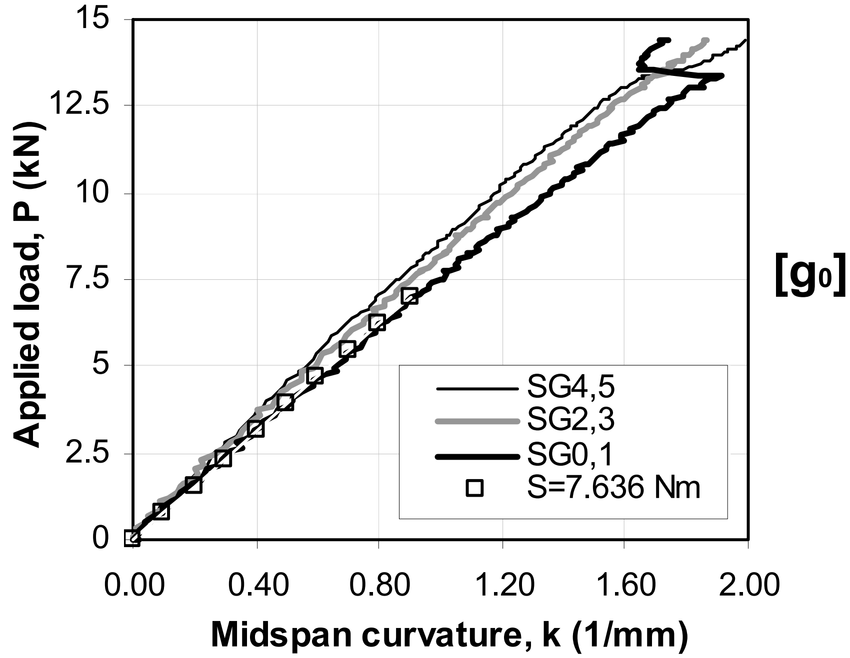
- Progressively stiffer response if **dg/dD>0** (size effect in inelasticity)
- Cauchy stiffness decreases with damage: **dK/dD = -K0 <0**

But, $\mathbf{K_{0gr} > K_{0cl}}$



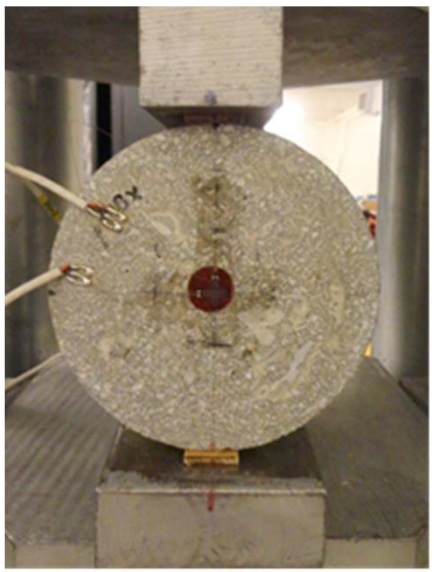
STRAIN GAGE MEASUREMENTS

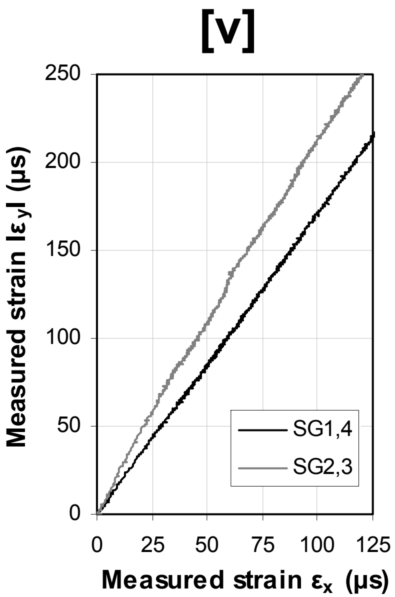


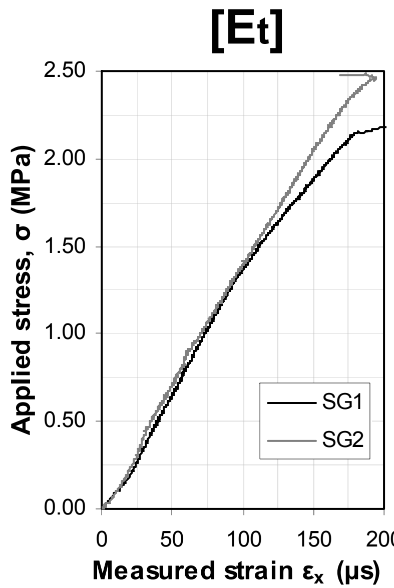


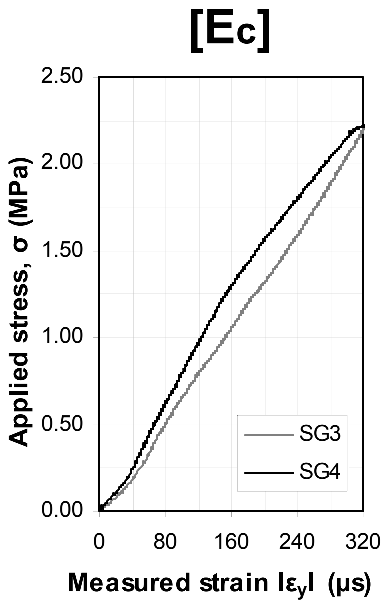
For g>0, $S_{exp}>S_{cl}$ or $k_{exp}<k_{cl}$

(P=Sk)

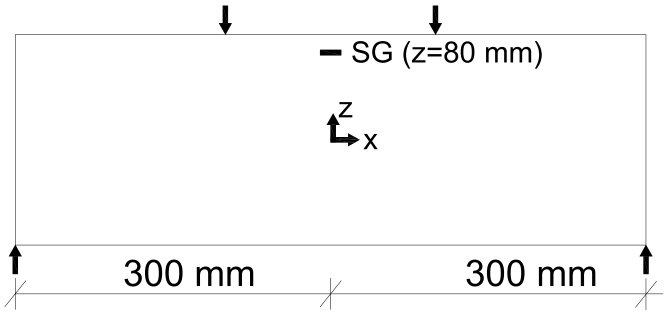




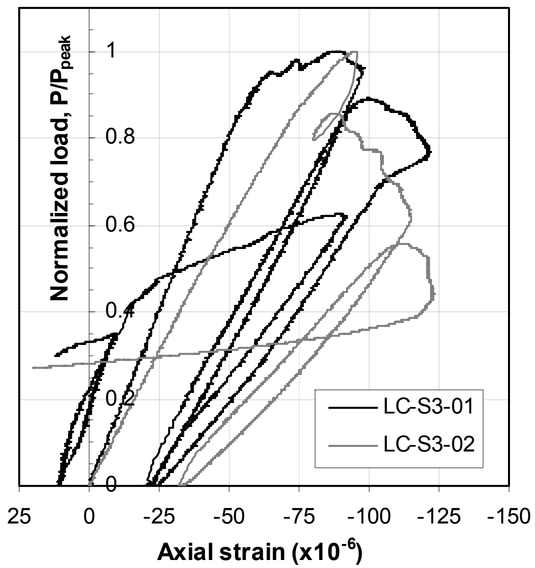


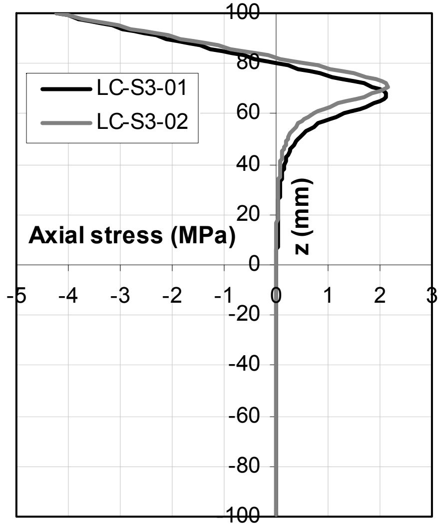


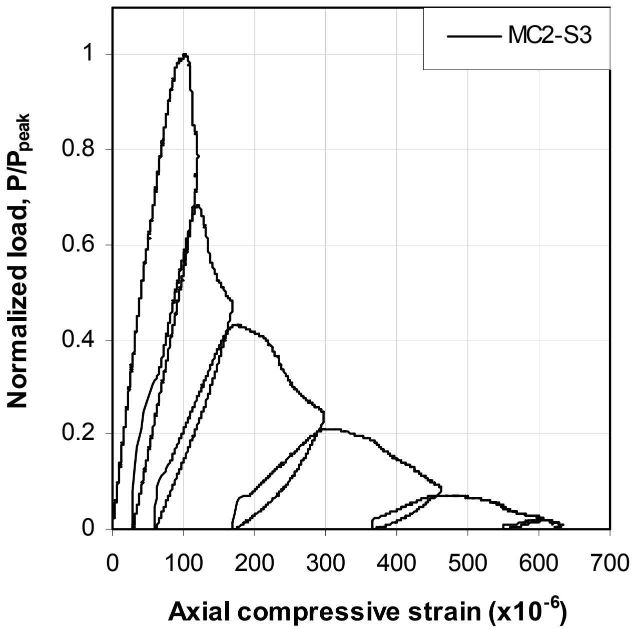
STRAIN GAGE MEASUREMENTS



Damage characterization







Plastic strains: verification of progressively stiffer response in inelasticity

$$\sigma(\varepsilon, \varepsilon_{,xx}) = \left(1 - D(\varepsilon)\right)E(\varepsilon - g^2\varepsilon_{,xx})$$

Size effect in elasticity (**g>0**)

Size effect in inelasticity (**g=g(D**) with **dg/dD>0**)

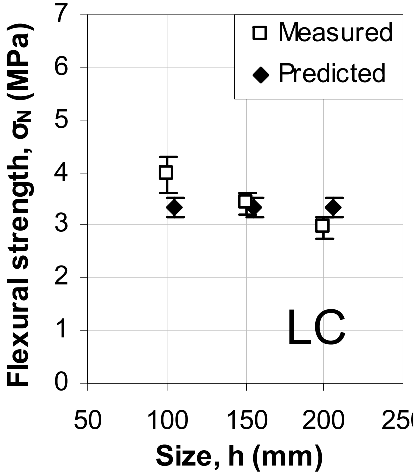
Size effect on strength

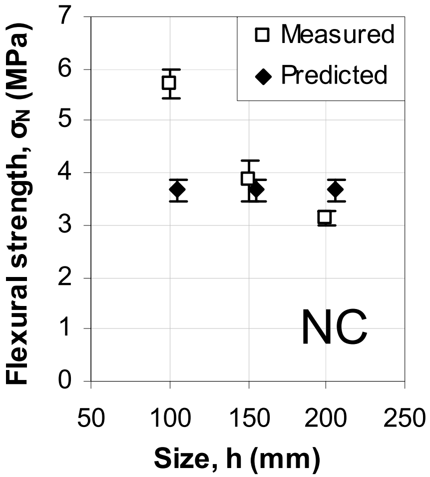
Sources/Explanations of size effect on strength

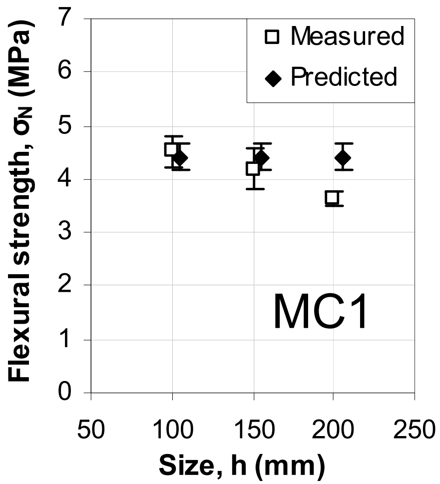
1. Fracture mechanics size effect
2. Statistical
3. Stress redistribution at meso-scale (Lattice models)
4. Multi-fractal size effect

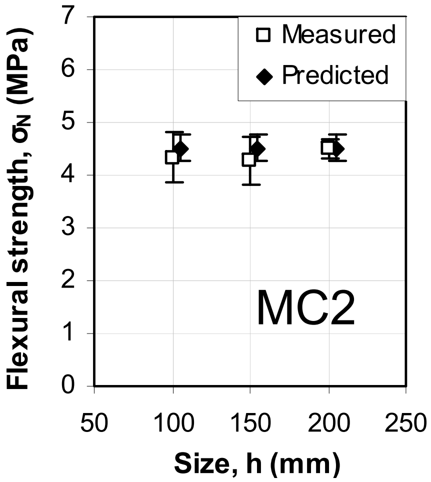
1. Diffusion phenomena
2. Hydration heat – microcracking during cooling
3. Wall effect due to inhomogeneous boundary layer

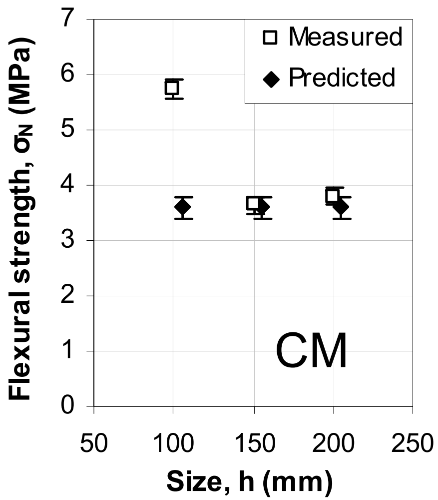
SIZE EFFECT ON FLEXURAL STRENGTH









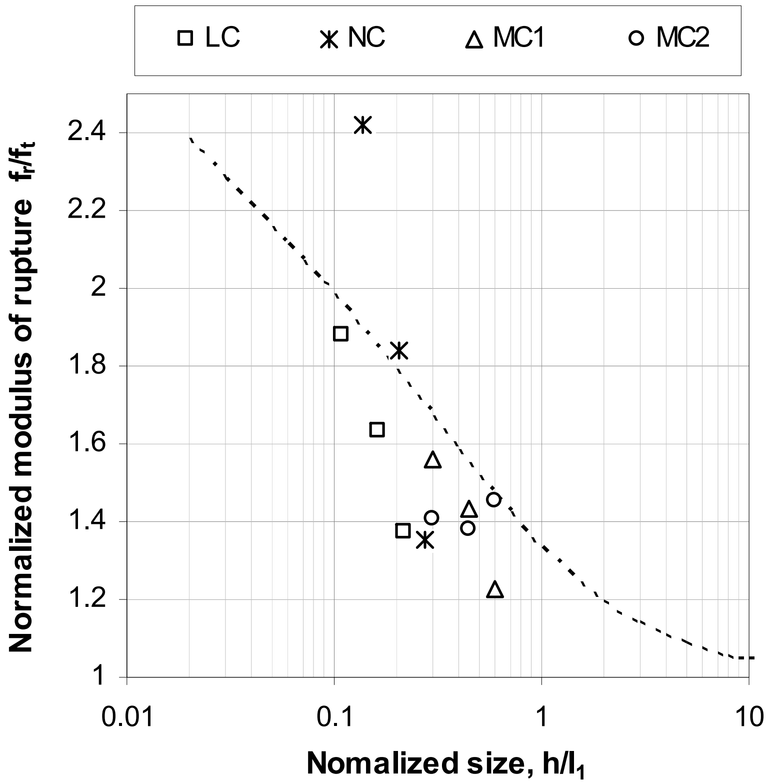


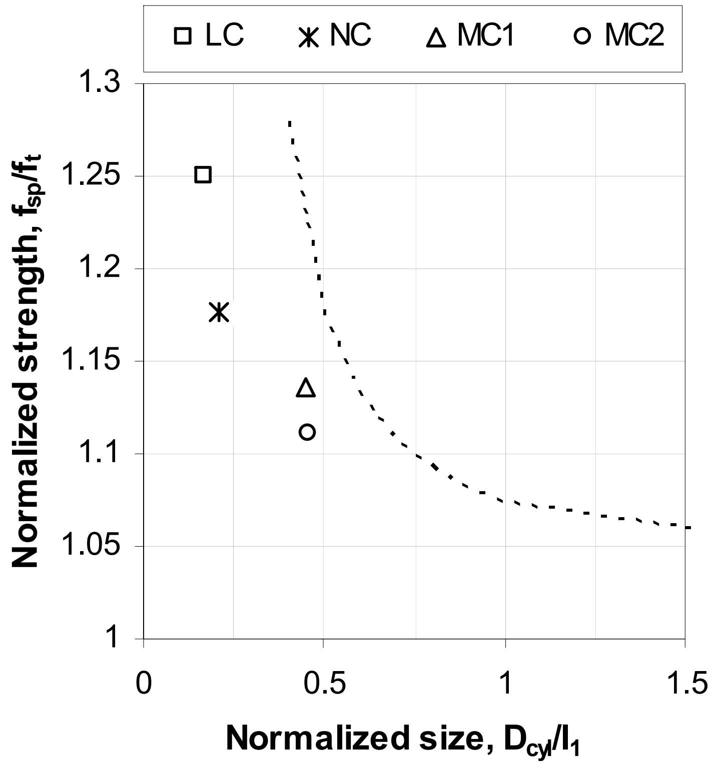
<u>Statistical size effect</u>

$$\sigma_N \propto h^{-1/8}$$

Observed size effect on flexural strength cannot be attributed to statistical sources

SIZE EFFECT ON FLEXURAL STRENGTH





Limited scale range , **1 : 1.5 : 2**,

for examining size effect on strength

CONCLUDING REMARKS

1. Experimental quantification of the internal length assumed by simplified dipolar elasticity

2. Concrete can be seen as a model material for studying size effects in elasticity

3. Physical correlation between the internal length and material's microstructure

4. Experimental verification of key assumptions

5. Use of SG's and continuous strain gradient models

6. Further experimental work is needed

7. Size effect on strength

Introducing size effect on strength in the proposed model for the case of concrete

- Refinement of the model
  Include effect of distributed damage to the Timoshenko model

- Size dependent damage characterization (Van Vliet & Van Mier)

- Damage characterization not based on Cauchy strains alone: $D(\varepsilon,\varepsilon_{,x})$

- Strain gradient lattice models predicting size effect on strength ?

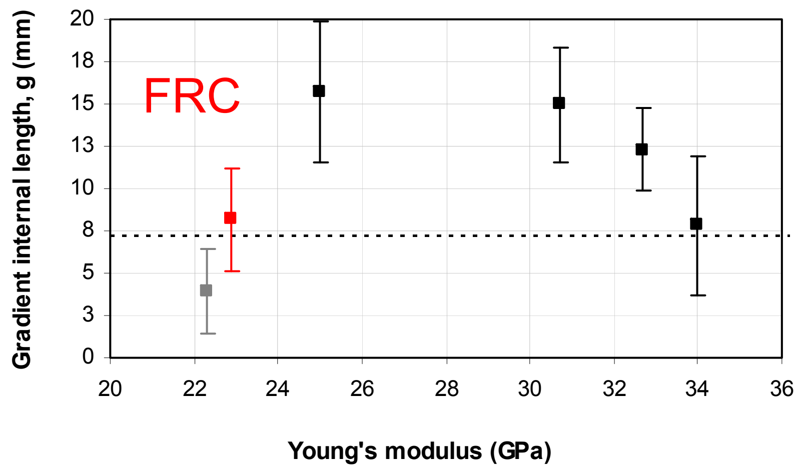
Fiber Reinforced Concrete (FRC)

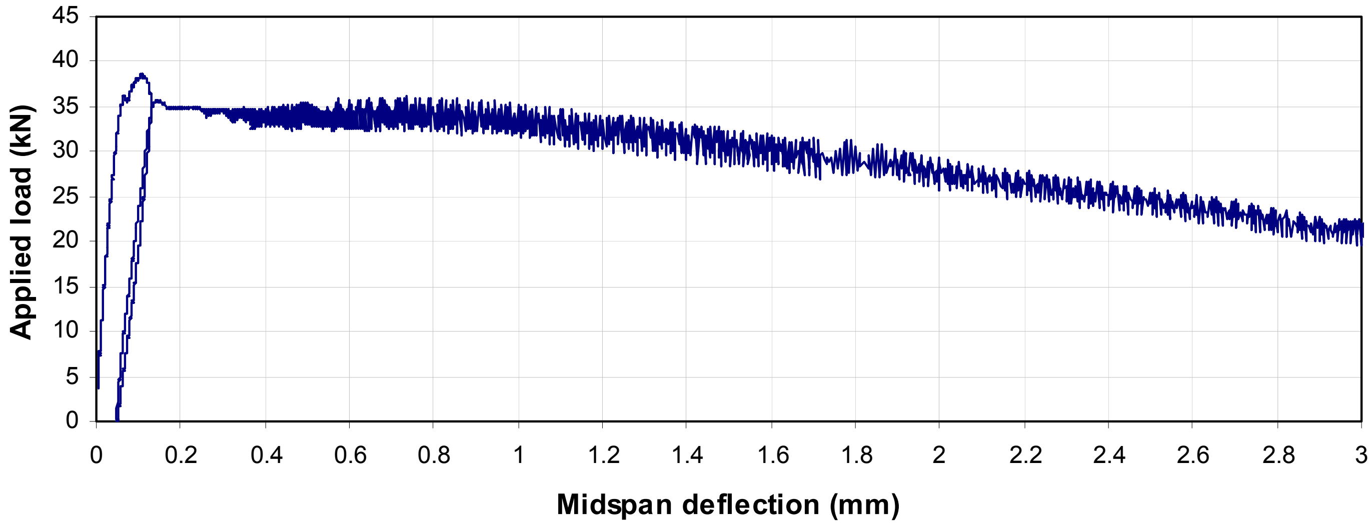
<u>4-point bending experiments</u>

g_0 of FRC = 8 mm

g_0 of matrix = 4 mm

same order of magnitude as predicted by the simplified 2D case





Very ductile softening response – $\mathbf{b_t}$ parameter of stress-strain in the range of **1.5**

For FRC, **n→0**. Therefore, $\mathbf{g(D)=g_0}$ (the thermodynamic limit of **dg/dD=0** is recovered)

Micro-inertia length and dynamic loading:

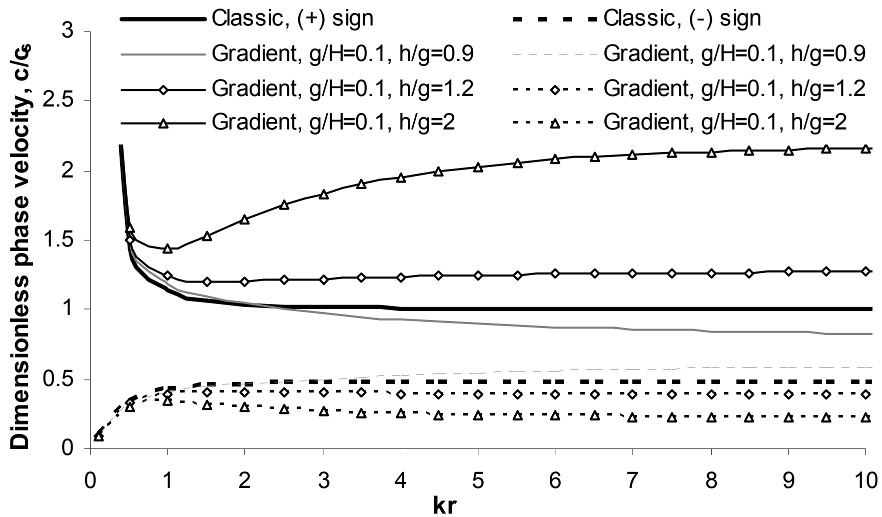
Hamilton's principle:

$$\delta\int_{t_0}^{t_1}\left(U_{tot} - W - K\right)dt = 0$$

2 Eqs. of motion and 2 gradient length: stiffness related **g** and inertia related **h**

$$\left(1+\frac{A}{I}g^2 - g^2\frac{d^2}{dx^2}\right)\frac{d\overline{M}}{dx} - \left(1-g^2\frac{d^2}{dx^2}\right)\overline{Q} = \left(1+h^2\frac{A}{I} - h^2\frac{d^2}{dx^2}\right)m_I\ddot{\psi}$$

$$\left(1-g^2\frac{d^2}{dx^2}\right)\frac{d\overline{Q}}{dx} = -q + \left(1-h^2\frac{d^2}{dx^2}\right)m_A\ddot{w}$$



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PhD presentation on

"Size Effects in Semi-brittle Materials and Gradient Theories with Application to Concrete"

By

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