PhD presentation on

“Size Effects in Semi-brittle Materials and Gradient Theories with Application to Concrete”

by

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INTRODUCTION

Microcracking: “branching” and “crack face bridging”

Images reproduced from the book “Fracture Processes of Concrete” by Van Mier
INTRODUCTION

Complexity of the observed behavior of microcracking

Mode I fracture

Images reproduced from the book “Fracture Processes of Concrete” by Van Mier
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Mode I cracking in compression

Images reproduced from the book “Fracture Processes of Concrete” by Van Mier
Lattice models: Modeling the full details of the microstructure

most important: particle structure

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INTRODUCTION

Ability to predict the complex behavior of microcracking

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INTRODUCTION

Dipolar elasticity

Strain gradient elasticity: \( \sigma_{xx} = (1 - g^2 \nabla^2) E \varepsilon_{xx} \) (1D, \( v=0 \))

Hooke’s law extended to the 2\(^{nd}\) spatial derivative of the Cauchy strain

Non-local models:

A length scale parameter is needed for modeling materials such as concrete exhibiting size effect

Stress-strain softening law:

\[
\frac{\sigma}{f_i} = \frac{\beta_i (\varepsilon / \varepsilon_i)}{\beta_i - 1 + (\varepsilon / \varepsilon_i)^{\beta_i}}
\]

Size effect in elasticity:

Stiffer static response than that predicted by classical elasticity if \( g>0 \)

What is a physical interpretation of the internal length?
HOMOGENIZATION ESTIMATES OF THE INTERNAL LENGTH

The simplest case of composite material (2D case of circular inclusions)

**Input parameters**

Matrix and inclusion elasticity properties
Composition value, $c = a/b$

**Effective material properties of composite material**

$$\mu = f_1(\mu_m, \nu_m, \mu_i, \nu_i, c)$$
$$\nu = f_2(\mu_m, \nu_m, \mu_i, \nu_i, c)$$

**Estimation of the internal length**

$$U_{cl} = U_{gr}$$
HOMOGENIZATION ESTIMATES OF THE INTERNAL LENGTH

![Graph showing the relationship between composition and gradient internal length to inclusion radius ratio](image)

- Loading case 1
- Loading case 2

Graph parameters:
- $\mu / \mu_n = 2$
- $\mu / \mu_n = 2.5$
- $\mu / \mu_n = 5$
- $\mu / \mu_n = 10$
- $\mu / \mu_n = 15$
- $\mu / \mu_n \to \infty$

Composition, $c$

Gradient internal length to inclusion radius ratio, $\xi / \alpha$
**Structural Analysis Using a Dipolar Elastic Timoshenko Beam**

Timoshenko kinematics

- \( u_x = z \, \psi(x) \)
- \( u_y = 0 \)
- \( u_z = w(x) \)

Cauchy stresses

- \( \sigma_{xx} = E \varepsilon_{xx} \)
- \( \sigma_{xz} = kG \gamma_{xz} \)

**Total Strain Energy:**

\[
U_{\text{tot}} = U_{\text{cl}} + U_{\text{gr}}
\]

\[
U_{\text{cl}} = \frac{1}{2} \iiint_V \left( \sigma_{xx} \varepsilon_{xx} + 2 \sigma_{xz} \varepsilon_{xz} \right) \, dx \, dy \, dz
\]

\[
U_{\text{gr}} = \frac{1}{2} g^2 \iiint_V \left( \frac{\partial \bar{\sigma}_{xx}}{\partial x} \frac{\partial \varepsilon_{xx}}{\partial x} + 2 \frac{\partial \bar{\sigma}_{xz}}{\partial x} \frac{\partial \varepsilon_{xz}}{\partial x} + \frac{\partial \bar{\sigma}_{xx}}{\partial z} \frac{\partial \varepsilon_{xx}}{\partial z} \right) \, dx \, dy \, dz
\]

**Non-zero Strains:**

- \( \varepsilon_{xx} = \frac{\partial u_x}{\partial x} = z \frac{d\psi}{dx} \)
- \( \gamma_{xz} = 2 \varepsilon_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = \frac{dw}{dx} + \psi \)

Term that should not be omitted
Structural Analysis Using a Dipolar Elastic Timoshenko Beam

\[
\delta U_{\text{tot}} = \int_0^L \left[ \delta \psi \left( -EI \left( 1 - g^2 \frac{d^2}{dx^2} \right) \frac{d^2 \psi}{dx^2} + kAG \left( 1 - g^2 \frac{d^2}{dx^2} \right) \left( \frac{dw}{dx} + \psi \right) - EAg^2 \frac{d^2 \psi}{dx^2} \right) \right] \, dx

+ \delta w \left( -kAG \left( 1 - g^2 \frac{d^2}{dx^2} \right) \left( \frac{d^2 w}{dx^2} + \frac{d \psi}{dx} \right) \right)
\]

\[
+ \int_0^L \left[ \delta \psi \left( EI \left( 1 - g^2 \frac{d^2}{dx^2} \right) \frac{d \psi}{dx} + kAGg^2 \left( \frac{d^2 w}{dx^2} + \frac{d \psi}{dx} \right) + EAg^2 \frac{d \psi}{dx} \right) \right] \, dx

+ \left[ \delta w \left( kAG \left( 1 - g^2 \frac{d^2}{dx^2} \right) \left( \frac{dw}{dx} + \psi \right) \right) \right]_0^L

+ \left[ \delta \psi' \left( g^2 EI \frac{d^2 \psi}{dx^2} \right) \right]_0^L

+ \left[ \delta w' \left( kAGg^2 \left( \frac{d^2 w}{dx^2} + \frac{d \psi}{dx} \right) \right) \right]_0^L
\]

4 independent BC

\( w, w', \psi, \psi' \)

(bold are the classical elasticity variables)

4 energetically conjugate quantities

\( Q, Y, M, m \)

(Y, m: double shear or bi-force and double moment or bi-moment)

\[
\delta W = \int_0^L q \delta w \, dx + \left[ Q \delta w \right]_0^L + \left[ M \delta \psi \right]_0^L + \left[ Y \delta w' \right]_0^L + [m \delta \psi']_0^L
\]
Equilibrium equations

\[
\left(1 + \frac{A}{I} g^2 - g^2 \frac{d^2}{dx^2}\right) \frac{d\bar{M}}{dx} = \left(1 - g^2 \frac{d^2}{dx^2}\right) \bar{Q} \\
\left(1 - g^2 \frac{d^2}{dx^2}\right) \frac{d\bar{Q}}{dx} = -q
\]

Boundary conditions

\[
\left[\left[Y - \left(g^2 \frac{d\bar{Q}}{dx}\right)\right] \delta w \right]_{0}^{L} = 0 \\
\left[\left[Q - \left(1 - g^2 \frac{d^2}{dx^2}\right) \bar{Q}\right] \delta w \right]_{0}^{L} = 0 \\
\left[\left[M - \left(1 - g^2 \frac{d^2}{dx^2}\right) \bar{M} + \frac{A}{I} g^2 \bar{M} + g^2 \frac{d\bar{Q}}{dx}\right] \delta \psi \right]_{0}^{L} = 0 \\
\left[\left[m - \left(g^2 \frac{d\bar{M}}{dx}\right)\right] \delta \psi' \right]_{0}^{L} = 0
\]

Closed-form solution

\[
w(x) = \left\{ \begin{array}{l}
\frac{c_3 - d_3}{2} x - \frac{d_4}{2} x^2 + c_4 + d_2 \ell^3 e^{-x/\ell} - d_1 \ell^3 e^{x/\ell} + c_1 g^2 e^{x/g} + c_2 g^2 e^{-x/g} \\
+ \frac{q}{24EI} \left(\frac{\ell}{g}\right)^2 x^4 - \frac{q}{2kAG} x^2 - \frac{c_3 kAG}{6EI} \left(\frac{\ell}{g}\right)^2 x^3
\end{array} \right\}
\]

\[
\psi(x) = -\frac{q}{6EI} \left(\frac{\ell}{g}\right)^2 x^3 + \frac{kAG}{2EI} \left(\frac{\ell}{g}\right)^2 c_3 x^2 + d_1 \ell^2 e^{x/\ell} + d_2 \ell^2 e^{-x/\ell} + d_3 + d_4 x
\]

8 constants to be determined from 8 B.C.’s
(4 at each end)
STRUCTURAL ANALYSIS USING A DIPOLAR ELASTIC TIMOSHENKO BEAM

**Comparison with FEM results**

+ Effect of $g/h$

**Clamped end:** Boundary layer effect as in BE beam

**Free end:** zero axial stains
NON-LOCAL DAMAGE MODEL

Gibb’s energy:  \[ G = \frac{1}{2} \tau : C : \tau + \frac{1}{2} \lambda : B : \lambda - A^c \]

\[ \lambda = g^2 \nabla \tau \quad B = (1 / g^2) C \]

Microcracking adds flexibility:  \[ C = C^0 + C^c \]

Opening and closing of microcracks:

\[ C^c = P^+ : \overline{C^c} : P^+ \quad \text{(positive projections)} \]

\[ \varepsilon \approx C^0 : \tau + P^+ (\overline{C^c} : \tau^+) \]

\[ \nabla \varepsilon \approx C^0 : \nabla \tau + P^+ (\overline{C^c} : (\nabla \tau)^+) \quad \text{Minimization problem solution for a given state of stress} \]

\[ \overline{C^c} = \overline{C^c}_{IT} + \overline{C^c}_{IC} \quad \text{Active microcracks: } \varepsilon^c_{IT} \geq 0 \quad \varepsilon^c_{IC} \leq 0 \]
Non-Local Damage Model

Damage rules: \( \dot{\mathbf{C}}_{I_T}^e = \dot{\mu} \mathbf{R}_{I_T} (\tau) \) and \( \dot{\mathbf{C}}_{I_c}^e = \dot{\mu} \mathbf{R}_{I_c} (\tau) \)

Irreversible character of damage: \( \dot{\mu} \geq 0 \)

Internal length is assumed to be a function of damage: \( g = g(\mu) \)

Energy density dissipation:

\[
d = \left\{ \begin{array}{l}
\frac{1}{2} \tau^+ : \mathbf{R}_{I_T} : \tau^+ + \frac{1}{2} \tau^- : \mathbf{R}_{I_c} : \tau^- + \frac{1}{2} g^2 (\nabla \tau)^+ : \mathbf{R}_{I_T} : (\nabla \tau)^+ \\
+ g \frac{dg}{d\mu} (\nabla \tau)^+ : \dot{\mathbf{C}}^c : (\nabla \tau)^+
\end{array} \right\} \dot{\mu} - \dot{\mathbf{A}}^c \geq 0
\]

\( \mathbf{R}_{I_T}, \mathbf{R}_{I_c}, \mathbf{C}_{I_T}^e, \mathbf{C}_{I_c}^e \) positive definite and \( \dot{\mu} \geq 0 \Rightarrow \frac{dg}{d\mu} \geq 0 \)

Associated damage surface:

\[
\Phi = \frac{1}{2} \tau^+ : \tau^+ + \frac{1}{2} c \tau^- : \tau^- - \frac{\pi}{2} t^2 (\mu)
\]
4-point bending: an assumed stress-strain law is sufficient input information for damage characterization

Definition of damage

\[ \tau_i = E_{0i} \varepsilon_i, \quad \text{for} \quad \varepsilon_i \leq \varepsilon_{0i} \]
\[ \tau_i = (1 - D_i) E_{0i} \varepsilon_i = \frac{E_{0i} \varepsilon_i}{1 + E_{0i} \mu_i}, \quad \text{for} \quad \varepsilon_i > \varepsilon_{0i} \]

Assumed stress-strain law:

\[ \tau_i = f_i \frac{\beta_i (\varepsilon / \varepsilon_i)}{\beta_i - 1 + (\varepsilon / \varepsilon_i)^{\beta_i}} \]

Damage functions:

\[ D_i = 0 \quad \text{for} \quad \varepsilon < \varepsilon_{0i} \]
\[ D_i = 1 - \frac{\beta_i - 1 + (\varepsilon_{0i} / \varepsilon_i)^{\beta_i}}{\beta_i - 1 + (\varepsilon / \varepsilon_i)^{\beta_i}} \quad \text{for} \quad \varepsilon \geq \varepsilon_{0i} \]

Young’s modulus and isotropy:

\[ E_{0i} = \frac{\beta_i f_i}{(\beta_i - 1 + (\varepsilon_{0i} / \varepsilon_i)^{\beta_i}) \varepsilon_i} \]
\[ E_t = \frac{\beta_t f_t (\beta_c - 1 + (\varepsilon_{0c} / \varepsilon_c)^{\beta_c})}{\beta_c f_c (\beta_t - 1 + (\varepsilon_{0t} / \varepsilon_t)^{\beta_t})} \]
\[ \varepsilon_c = \frac{\beta_t f_t (\beta_c - 1 + (\varepsilon_{0c} / \varepsilon_c)^{\beta_c})}{\beta_c f_c (\beta_t - 1 + (\varepsilon_{0t} / \varepsilon_t)^{\beta_t})} \]
APPLICATION TO 4-POINT BENDING

Linear strain distribution: \( \varepsilon_{xx} = \varepsilon_m + kZ \)

Input: a given value for curvature, \( k \)

Find: \( \varepsilon_m \) that satisfies equilibrium (stress-strain law)

Output: \( M, k \) point on the response diagram (size independent)

Input: \( k-\delta \) kinematic relation (boundary value problem)

Final output: Force vs. midspan deflection diagram (size dependent)

![Graph showing predicted midspan curvature vs. predicted bending moment capacity]

![Graph showing predicted midspan deflection vs. predicted applied load]
Non-local predictions:

The local $M$ vs. $k$ is transformed to $M$ vs. $k_{\text{grad}}$ (scaling of curvature-elasticity)

The gradient solution of the boundary value problem is used to obtain the kinematic relation, $\delta_{\text{grad}}/k_{\text{grad}} = f(g/L)$

This relation is not constant but evolves with damage since $g=g(D)$

Internal length evolution law: $g = g_0 e^{nD}$
APPLICATION TO 4-POINT BENDING

Objectivity of the model:

Acoustic tensor positive definite

Good agreement with AE findings:

- Initiation of softening ($D > 0$) and therefore microcracking

- Intense localization of AE energy release rate – macro-crack formation ($D > 0.95$)

Objective damage characterization:

(for both $g = 0$ and $g > 0$)
EXPERIMENTAL PROGRAM

3D geometrical similar specimens
### MATERIALS

<table>
<thead>
<tr>
<th>Mix</th>
<th>Quantities (Kg/m³)</th>
<th>Dry density (Kg/m³)</th>
<th>Slump (cm)</th>
<th>Air-content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cement (a)</td>
<td>Aggregates (b)</td>
<td>Water (w/c ratio)</td>
<td>Additives (c)</td>
</tr>
<tr>
<td>CM</td>
<td>450</td>
<td>1350</td>
<td>293 (0.65)</td>
<td>3.6</td>
</tr>
<tr>
<td>LC</td>
<td>208</td>
<td>1980</td>
<td>162 (0.78)</td>
<td>1.6</td>
</tr>
<tr>
<td>NC</td>
<td>276</td>
<td>2080</td>
<td>176 (0.64)</td>
<td>1.5</td>
</tr>
<tr>
<td>MC1</td>
<td>448</td>
<td>1720</td>
<td>204 (0.45)</td>
<td>4.0</td>
</tr>
<tr>
<td>MC2</td>
<td>447</td>
<td>1640</td>
<td>207 (0.46)</td>
<td>6.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sieve opening (mm)</th>
<th>% passing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LC</td>
</tr>
<tr>
<td>32</td>
<td>100</td>
</tr>
<tr>
<td>16</td>
<td>85.8</td>
</tr>
<tr>
<td>8</td>
<td>70.7</td>
</tr>
<tr>
<td>4</td>
<td>62.7</td>
</tr>
<tr>
<td>2</td>
<td>45.4</td>
</tr>
<tr>
<td>1</td>
<td>29.6</td>
</tr>
<tr>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>0.25</td>
<td>12.9</td>
</tr>
<tr>
<td>0.075</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Four concrete mixes $d_{\text{max}}=32$ mm

- Low-strength (LC)
- Normal-strength (NC)
- Medium-strength (MC1 and MC2)

Cement mortar $d_{\text{max}}=1$ mm
Classical elasticity properties used in the analysis

<table>
<thead>
<tr>
<th>Mix</th>
<th>LC</th>
<th>NC</th>
<th>MC1</th>
<th>MC2</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>E, GPa</td>
<td>25.0</td>
<td>30.70</td>
<td>32.7</td>
<td>34.0</td>
<td>22.3</td>
</tr>
<tr>
<td>ν</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Elasticity:** \( P = K \delta \) & \( P = S k \)

**Size effect in elasticity**

If: \( \frac{K_{\text{exp}}}{K_{\text{cl}}} > 1 \)
\[ S_{\text{exp}}/S_{\text{cl}} > 1 \]
Concrete contains different aggregate sizes in different volume fractions. Average inclusion size ranges between 10 to 20 mm.
## INELASTICITY: PEAK LOAD

### Uniaxial stress-strain law parameters

<table>
<thead>
<tr>
<th>Mix</th>
<th>Compression</th>
<th>Tension</th>
<th>Young’s modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_c$ (MPa)</td>
<td>$\varepsilon_c$ (x10^{-6})</td>
<td>$\varepsilon_{dc}$ (x10^{-6})</td>
</tr>
<tr>
<td>LC</td>
<td>15.9</td>
<td>1500</td>
<td>250</td>
</tr>
<tr>
<td>NC</td>
<td>20.5</td>
<td>1500</td>
<td>270</td>
</tr>
<tr>
<td>MC1</td>
<td>34.7</td>
<td>1500</td>
<td>430</td>
</tr>
<tr>
<td>MC2</td>
<td>38.0</td>
<td>1500</td>
<td>450</td>
</tr>
<tr>
<td>CM</td>
<td>32.4</td>
<td>1800</td>
<td>600</td>
</tr>
</tbody>
</table>

### Graphs

- **Cylinder midheight compressive strain** at $a$%
  - $a=55\%$
  - $a=40\%$

- **Strains at 0.5 x $f_{sp}$ (MPa)**
  - Tensile strain
  - Compressive strain

### Table

<table>
<thead>
<tr>
<th>Mix</th>
<th>Compressive strength, $f_c$ (MPa)</th>
<th>Splitting strength, $f_{sp}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC</td>
<td>15.9</td>
<td>2.0</td>
</tr>
<tr>
<td>NC</td>
<td>20.5</td>
<td>2.5</td>
</tr>
<tr>
<td>MC1</td>
<td>34.7</td>
<td>3.0</td>
</tr>
<tr>
<td>MC2</td>
<td>38.0</td>
<td>3.5</td>
</tr>
<tr>
<td>CM</td>
<td>32.4</td>
<td>4.0</td>
</tr>
</tbody>
</table>
**INELASTICITY: PEAK LOAD**

Model predictions correspond to a size independent flexural strength.

Scatter of predicted values corresponds to a +/- 5% deviation of the assumed material’s tensile strength.
**LC mix:**  \( g = 17e^{0.9D} \) (mm)

**MC2 mix:**  \( g = 8e^{2D} \) (mm)
\[
\delta_{pl} = \bar{\delta} - \bar{P} / (1 - \bar{D})K_0
\]
**EVOLUTION LAW OF GRADIENT INTERNAL LENGTH**

<table>
<thead>
<tr>
<th>Non-local parameters</th>
<th>Mix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LC</td>
</tr>
<tr>
<td>$g_0$ (mm)</td>
<td>17</td>
</tr>
<tr>
<td>$n$</td>
<td>0.90</td>
</tr>
</tbody>
</table>

- Progressively stiffer response if $dg/dD>0$ (size effect in inelasticity)

- Cauchy stiffness decreases with damage: $dK/dD = -K_0 < 0$

$n$ increases as brittleness increases in the mixes

---

![Graph showing normalized internal length vs. damage](image1)

![Graph showing midspan deflection vs. applied load](image2)

![Graph showing size S3 predictions vs. experimental results](image3)
For $g > 0$, $S_{\text{exp}} > S_{\text{cl}}$ or $k_{\text{exp}} < k_{\text{cl}}$ 

$(P=Sk)$
**STRAIN GAGE MEASUREMENTS**

Damage characterization

Plastic strains: verification of progressively stiffer response in inelasticity

\[ \sigma(\varepsilon, \varepsilon_{xx}) = (1 - D(\varepsilon))E(\varepsilon - g^2\varepsilon_{xx}) \]
SIZE EFFECTS

Size effect in elasticity \((g>0)\)

Size effect in inelasticity \((g=g(D)\) with \(dg/dD>0)\)

Size effect on strength

Sources/Explanations of size effect on strength

1. Fracture mechanics size effect
2. Statistical
3. Stress redistribution at meso-scale (Lattice models)
4. Multi-fractal size effect

1. Diffusion phenomena
2. Hydration heat – microcracking during cooling
3. Wall effect due to inhomogeneous boundary layer
SIZE EFFECT ON FLEXURAL STRENGTH

Statistical size effect

\[ \sigma_N \propto h^{-1/8} \]

Observed size effect on flexural strength cannot be attributed to statistical sources.
Limited scale range, 1 : 1.5 : 2, for examining size effect on strength
CONCLUDING REMARKS

1. Experimental quantification of the internal length assumed by simplified dipolar elasticity

2. Concrete can be seen as a model material for studying size effects in elasticity

3. Physical correlation between the internal length and material’s microstructure

4. Experimental verification of key assumptions

5. Use of SG’s and continuous strain gradient models

6. Further experimental work is needed

7. Size effect on strength
FUTURE WORK

Introducing size effect on strength in the proposed model for the case of concrete

• Refinement of the model
  Include effect of distributed damage to the Timoshenko model

• Size dependent damage characterization (Van Vliet & Van Mier)

• Damage characterization not based on Cauchy strains alone: \( D(\varepsilon, \varepsilon, x) \)

• Strain gradient lattice models predicting size effect on strength?
Fiber Reinforced Concrete (FRC)

4-point bending experiments

g₀ of FRC = 8 mm

g₀ of matrix = 4 mm

same order of magnitude as predicted by the simplified 2D case

Very ductile softening response – bᵣ parameter of stress-strain in the range of 1.5

For FRC, n→0. Therefore, g(D)=g₀ (the thermodynamic limit of dg/dD=0 is recovered)
FUTURE WORK

Micro-inertia length and dynamic loading:

Hamilton's principle:
\[
\delta \int_{t_0}^{t_f} \left( U_{tot} - W - K \right) dt = 0
\]

2 Eqs. of motion and 2 gradient length: stiffness related \( g \) and inertia related \( h \)

\[
\left( 1 + \frac{A}{I} g^2 - g^2 \frac{d^2}{dx^2} \right) \frac{d \bar{M}}{dx} - \left( 1 - g^2 \frac{d^2}{dx^2} \right) \bar{Q} = \left( 1 + h^2 \frac{A}{I} - h^2 \frac{d^2}{dx^2} \right) m_i \ddot{y}
\]

\[
\left( 1 - g^2 \frac{d^2}{dx^2} \right) \frac{d \bar{Q}}{dx} = -q + \left( 1 - h^2 \frac{d^2}{dx^2} \right) m_A \ddot{w}
\]
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&

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