Stochastic Inventory Routing Problem with Transshipment Recourse Action

Evangelia Chrysochoou
PhD Student, Department of Mechanical Engineering, University of Thessaly, Volos, Thessaly, Greece, email: echryso@certh.gr

Prof. Athanasios Ziliaskopoulos
Professor, Department of Mechanical Engineering, University of Thessaly, Volos, Thessaly, Greece, email: ziliasko@gmail.com

Major multinationals have recognized that optimizing their global supply chains can yield substantial impact on their bottom line. Vendor Managed Inventory (VMI) systems seem to be at the core of most global supply chains serving this goal. The concept behind this business model is that the supplier monitors the inventories of each retailer and determines the replenishment policy, guaranteeing that no stock out will occur. Inventory Routing Problem (IRP) constitutes the backbone of the VMI model. IRP aims to jointly optimize transportation and inventory decisions. While the IRP accounts for almost 30 years of research it is still an open and active area, especially its stochastic counterpart. In this paper a two –stage stochastic programming model with recourse action is introduced for the IRP with stochastic demand. Transshipment is treated as a recourse action when extra demand is revealed and stock out situation occurs. An exact L –Shaped algorithm is developed to solve the problem; computational experiments demonstrate the efficiency of the model while proving that the L –Shaped method converge in a finite number of steps.

Keywords: Inventory Routing Problem; Stochastic Programming with recourses; L –Shaped decomposition

1 Introduction

Vendor Managed Inventory (VMI) systems seem to be one of the most tractable business models in global logistics and supply chain operations nowadays. This is increasingly the case for the electronics and automotive industry that manufacture parts in Asia and assemble them in Europe. This is a deviation of the traditional production models where typically parts manufactured in one facility are assembled on a next door plan. Now work-in-progress seems to spread along the whole supply chain, often stretching from China to Hungary or Czech Republic. Recently many of these global supply chains started using Piraeus, Greece as the port of entry into the European Union with warehouses and transshipment facilities anywhere from Greece to Czech Republic along the European Railway Network that is used for land transport. Most of these parts are assembled in five (5) major plants in Central Europe, operating with Just – In – Time production procedures, using the VMI principles, as typically one manufacturer provides assembly services for most electronics companies. 3PL entities involved in these global supply chains realized the need to adjust their operations to meet the requirement of their clients in this regard, which motivated this research. The overall aim is the performance of more efficient resource utilization.

The main algorithmic component of the VMI systems is the Inventory Routing Problem (IRP), which is one of the most interesting extensions of traditional vehicle routing problems. It combines the decision making process of inventory management and distribution – transportation of goods. The decision maker in such a model has to make three decisions; the amount to be transported; the frequency of shipments and the distribution plan. IRP in real life is stochastic since demand uncertainty is the undisputed reality of actual problems. The basic difference between the Stochastic and the Deterministic IRP is the level of realism and the difficulty of solving instances given that the data are given in a probabilistic sense.

In a two stage stochastic program a long term anticipatory decision must be made prior to the full information of the random parameter of the problem and short terms decisions are available as recourse actions once the uncertainty has been revealed. The overall aim is to make "*here and now*" a decision which minimizes the total expected cost associated with both: the long term and the short term decisions (Carøe and Tind, 1998). However a key issue in the solution of an IRP is how to model long term effects of short term planning decisions.

Although the IRP is a long term problem, almost all proposed solution schemes solve only a short term version of the problem for reasons of computational tractability. Besides the planning horizon modelled typically in number of days, key features that distinguish different solution approaches include:

the manner it is determined, the customers that are included in short term problem, and whether the customer demand is treated deterministic or stochastic.

With respect to the existing literature the contribution of this paper can be summarized as follows:

(I) It introduces a formulation for the SIRP with recourse using transshipment as a recourse action.

(II) It introduces new valid inequalities that enhance the computational process of optimal transported quantities under the maximum level policy.

The paper is organized as followed: In §2 a literature is reviewed, where milestones related to the developments of IRP solutions are discussed. In §3 the framework of stochastic programming approach is analysed. The proposed mathematical model of our approach and the L – Shaped method are presented in §4. Computational results are discussed in §5. Finally conclusions and further steps are given in §6.

2 Literature Review

IRP was introduced 30 years ago by the seminal paper of Bell et al., (1983) which studied the case with stochastic demand accounting only for transportation costs. Federgruen and Zipkin (1984) extended the vehicle routing problem (VRP) model of Fisher and Jaikumar (1981) accounting for the shortage and inventory cost. Later Blumenfeld et al. (1985) analyzed the trade off among inventory, production and distribution costs on the freight transportation networks. Anily and Federgruen (1990) introduced the first clustering algorithm for the IRP.

Baita et al. (1998) provided the first comprehensive literature review dedicated on this subject, where the main solution approaches were classified as being either in the frequency or in the time domain. Frequency domain approaches and more specifically the aggregate ones set the basis for the Economic Quantity Order model which is commonly used for inventory management. Time domain approaches work in a closed loop manner, in a sense that the decisions taken in a one period affect the decisions of the next one. A major drawback of the EOQ models is that it provides headways which are not integer. Hall (1985) suggested rounding up EOQ values to the nearest feasible frequency; however, it was proved by Speranza and Ukovich (1994a) that this decision raises the overall cost by almost 20%. Speranza and Ukovich (1994b) proposed a mixed integer programming model, which assigns items to frequencies for the one origin to one destination case, by using trucks of a given capacity. Following this approach, Bertazzi et al. (1997) studied the more general case with several destinations, and proposed several decomposition heuristics which solves the problem in three phases. Few years later Bertazzi, Palettas and Speranza (2002) introduced a practical VMI policy, called deterministic order – up – to – level (OU) policy for the IRP. Based on the proposed policy Arhetti (2007) developed the first exact algorithm using a branch and cut scheme for the single vehicle. Very recently Coelho and Laporte (2014) and Adulyasak et al. (2014) have solved multivehicle versions of IRP in a branch and cut fashion under OU and maximum level (ML) policies. Solyali and Sural (2011) (based on the work of Arhetti et al., 2007) proposed a strong formulation for the inventory replenishment part of the IRP, which provides linear programming bounds and significantly better computational results. Transshipment has been studied for the purposes of inventory management since the seminal paper of Allen (1958). However to the best of our knowledge, transshipment was introduced in the context of inventory – routing problem only recently by Coelho (2012). Coelho have included the concept of transshipment in the branch and cut algorithm of Arhetti et al. (2007) and proposed also an adaptive large scale neighbourhood search (ALNS) for large scale instances, which was proved to be a quite powerful meta – heuristic.

Reviewing the work that has been published recently related to the IRP under uncertainty, we shall refer to the development of four domains; the framework of finite rolling horizon, the Marcov decision process for the infinite horizon consideration the robust IRP, the dynamic stochastic IRP.

Related to this paper is the work by Campbell et al. (1998) that set the basis for the rolling horizon framework, especially as used by Bard et al. (1998) in their decomposition scheme, and Jalliet et al.(2002) that optimally incorporated the long term delivery costs to short term planning horizon. Kleywegt et al. (2002, 2004) formulated the stochastic IRP as a Marcov Decision Process (MDP) over an infinite horizon. Adelman (2004) introduced a price – directed approach to deal with the stochastic counterpart of the IRP where the future cost is approximated by the duals. In contrast to the MDP Hvattum, et al. (2009) presented a framework based on scenario tree and proposed efficient heuristics that set the basis for our stochastic programming approach. Solyali et al.(2012) proposed two mixed integer programming formulations of the robust version of the problem, which produce policies leading to feasible solution and optimal cost for any realization of the demand.

Bertazzi et al. (2013) as well as Coelho (2012) rely on a dynamic programming formulation that allows the design of a hybrid rollout formulation aiming to find good quality solution. To the best of our knowledge this is the first approach that uses the rollout approach to solve the stochastic IRP. Finally, we refer to Andersson et al. (2010) and Coelho et al. (2014) for a comprehensive overview of the VMI problem and solution methods.

Our paper builds on Arhetti et al. (2007) and Coehlo's (2012) approach by treating transshipment as a recourse action, while the OU to level policy is not applied for the first stage decisions of IRP. Instead, new valid inequalities are proposed in order to determine the optimal quantities to be delivered for the ML policy. This approach was motivated by the fact that in the context of a deterministic model all parameters are known in the beginning of the process; thus a vendor can take advantage of the fact that knows the total demand of each stock keeping venue in advance and can transport quantities in an early stage in order to fulfill the future known demand. However, the amounts that a vendor is able to transport are bounded by the amounts that are made available at each stage.

3 Stage Stochastic Programming Framework

Following the typical notation of stochastic programming approaches (Kall and Wallace, 1994; Birge and Louveaux, 1997) , a two stage stochastic programming model can be stated as follows:

$$min_x\, c^T x + E_\omega Q(x,\omega) \tag{1}$$

Subject to:

$$Ax = b \tag{2}$$

$$x > 0 \tag{3}$$

Where:

$$Q(x,\omega) = min_y\, d_\omega^T y \tag{4}$$

Subject to:

$$T_\omega x + W_\omega y = h_\omega \tag{5}$$

$$y > 0 \tag{6}$$

The objective function consists of two components: the first one is related to the first stage decision process and the second one to the expected cost of the second stage decision process. Constraints (2) and (3) formulate the feasible solution space of first stage decision process. E_ω , corresponding to the expectation of the second stage decision process, and ω denotes the possible outcomes of each scenario with respect to the probability space *(Ω, P*). The objective function of the second stage decision process (4) is related to the respected decision variables and expresses the additional cost required performing recourse actions. The first component of left side equality (5) includes the parts of the first stage that are affected by the realization of ω and the second component includes the recourse actions that are foreseen to be implemented at the second stage to incorporate uncertainty. Finally, the right hand side of equality (5) represents the variable parameters of the first stage process that have to be met. By considering only discrete probability distribution functions P, the expectation of the second stage decision process can be stated by the following

$$E_\omega Q(x,\omega) = \sum_{\omega\in\Omega} p(\omega) Q(x,\omega) \tag{7}$$

Based on (7) it is convenient to formulate a large LP that forms the following deterministic equivalent problem.

$$min_x\, c^T + \sum_{\omega\in\Omega} p(\omega) d_\omega^T y_\omega \tag{8}$$

Subject to:

$$Ax = b \tag{9}$$

$$T_\omega x + W_\omega y = h_\omega \tag{10}$$

$$x > 0, y_\omega > 0 \tag{11}$$

In order to solve the stochastic programming model with recourse, first the decision maker implements the first stage decisions *x;* then the system will be subject to the random process denoted by (Ω, P) which results in an outcome $\omega \in \Omega$. Finally the decision maker will execute the recourse actions of the second stage decisions *y* accordingly. Using a Benders decomposition approach the process is divided into four (4) steps. In the first step, the initial master problem of the first stage decisions is solved. Next, for each scenario $\omega \in \Omega$ of possible outcomes the sub problems of the second stage decisions are computed. Step three is used to perform convergence tests, which are satisfied when the desired accuracy is achieved. Finally, in the forth step the master problem is solved with a column generation scheme by adding the proper produced optimality cuts from the dual prices of the second stage decision process. Based on the linear programming duality all second stage decision variables are projected out to the master problem in a cutting plane scheme. The master problem which constitutes the first stage IRP problem is solved and provides these solutions to the second stage decision process. If the second stage problem is feasible and bounded, an optimality cut is added to the master problem; otherwise a feasibility cut is added. The process is repeated until an optimal solution is found or the optimality gap is achieved.

4 Mathematical Formulation

In order to illustrate this 2 – Stage programming approach easier, initially the first stage decision process of the model is presented. Then the process of recourse actions of transshipment is presented. Finaly the stochastic program and its deterministic equivalent are discussed.

4.1 First Stage Decision Process – IRP model

We consider an IRP where a supplier denoted by node 1 distributes to N-1 retailers over a finite discrete time T, using a single vehicle with capacity C. Traditionally, the problem is defined on an undirected graph *G=(V,A)* where *{1}* is the vertex representing the supplier and vertices *V' = {2,3,…,N}* represent the set of stock keeping venues (they will be called retailers from thereafter as it is common in the literature); $A = \{(i,j): i \neq j, i,j \epsilon V\}$ is the set of arcs . Inventory holding cost occurs for both supplier and retailers and is denoted as $h_i : i\epsilon V$ per period and each vertex has an inventory capacity $C_i : i\epsilon V$. The length of the discrete planning horizon is H where $t\epsilon T = \{1, \dots, H\}$. In the beginning of the planning horizon the decision maker knows that: (1) each period the quantities r^t is made available to the supplier in order to fulfil the request of his retailers; (2) the initial inventory levels of both supplier and retailers are known $\{I_1^0, I_i^0, i\epsilon V'\}$; and (3) the demand of each retailer and at each period, denoted $d_i^t, i\epsilon V'$, are known. A single vehicle can perform one route at each period with capacity *C*, and a routing cost $c_{i,j}$is associated with arc (i, j)ϵA. Throughout the paper, we assume that since the supplier has the information about the demand of his retailers in advance, he can transport the quantities q_i^t to meet the demand of period t as well as of subsequent periods. However, the available quantities r^t shall be added to the total available quantities at period t, as they can be used for deliveries to retailers at the same period t as well as the subsequent periods. The binary decision variables $x_{i,j}^t$ is equal to 1, if the vehicle is delivering products at period t and the retailer j is served exactly after retailer i. Also, we define the binary variable y_i^t which is equal to 1 if the inventory i is served at period t. Additionally the non negative variable q_i^t express the quantities to be delivered to retailer i at period t, and u_i^t the quantities that the vehicle carries after delivering to the retailer i. The objective function minimizes the total transportation and inventory cost for the whole planning horizon, while meeting the demand of each retailer. Thus it is formed by the parts: the total inventory cost $\sum_{t\epsilon T}\sum_{i\epsilon V} h_i I_i^t$ and the total transportation cost $\sum_{t\epsilon T}\sum_{i\epsilon V}\sum_{\substack{i\epsilon V\\ i\neq j}} c_{ij}x_{i,j}^t$.

$$minimize \sum_{t\epsilon T}\sum_{i\epsilon V} h_i I_i^t + \sum_{t\epsilon T}\sum_{i\epsilon V}\sum_{\substack{i\epsilon V\\ i\neq j}} c_{ij}x_{i,j}^t \tag{12}$$

Subject to:

$$I_1^t \geq 0\ \forall t \in T \tag{13}$$

$$I_1^t \geq I_1^{t-1} + r^t - \sum_{i\epsilon V'} q_i^t\ \forall t \in T \tag{14}$$

$$I_i^t \geq 0\ \forall t \in T, \forall i \in V' \tag{15}$$

$$I_i^t \geq I_i^{t-1} + q_i^t - d_i^t\ \ \forall t \in T, \forall i \in V' \tag{16}$$

$$I_i^t \leq C_i\ \forall t \in T, \forall i \in V' \tag{17}$$

$$\sum_{i\epsilon V'} q_i^t \leq C\ \forall t \in T \tag{18}$$

$$\sum_{t\epsilon T} q_i^t \geq \sum_{t\epsilon T} d_i^t - I_i^0, \forall i \in V' \tag{19}$$

$$q_i^t \leq y_i^t \sum_{j=t}^{H} d_i^j, \forall i \in V', \forall t \in T \tag{20}$$

$$\sum_{j=1}^{t} q_i^j \leq I_1^0 + \sum_{j=1}^{t} r^j\ \ , \forall t \in T, , \forall i \in V' \tag{21}$$

$$\sum_{t\epsilon T}\sum_{j\epsilon V'} x_{1j}^t \leq H \tag{22}$$

$$\sum_{j\epsilon V'} x_{1j}^t \leq y_1^t\ \forall t \in T \tag{23}$$

$$\sum_{j\epsilon V'} x_{ij}^t = \sum_{j\epsilon V'} x_{ji}^t\ \forall t \in T, \forall i \in V' \tag{24}$$

$$\sum_{j\epsilon V'} x_{ij}^t + \sum_{j\epsilon V'} x_{ji}^t = 2y_i^t\ \ \forall t \in T, \forall i \in V' \tag{25}$$

$$x_{i,j}^t \leq y_i^t \forall t \in T, \forall i,j \in V' \tag{26}$$

$$x_{i,j}^t \leq y_j^t \forall t \in T, \forall i,j \in V' \tag{27}$$

$$C(1 - x_{i,j}^t) + u_i^t \geq u_j^t + q_j^t\ \ \ \forall i,j \in V' : i \neq j, t \in T \tag{28}$$

$$q_i^t \leq u_i^t\ \ \ \forall i,j \in V', t \in T \tag{29}$$

$$u_i^t \leq y_i^t * C\ \ \ \forall i \in V', t \in T \tag{30}$$

$$y_i^t \leq y_1^t \forall t \in T, \forall i \in V' \tag{31}$$

$$q_i^t, u_i^t \geq 0 \forall i \in V', t \in T \tag{32}$$
$$x_{i,j}^t \in \{0,1\} \forall i, j \in V': i \neq j, t \in T \tag{33}$$
$$y_i^t \in \{0,1\} \forall i \in V, t \in T \tag{34}$$

Constraints (13) and (14) are related to the inventory level at the supplier's site. The first one expresses the fact that the inventory level at the supplier cannot be negative in any period, thus avoiding a stock out situation. The second one defines the inventory level of the supplier at the end of period t by the inventory level at the end of period t-1, minus the total quantities to be transported at period t, plus the quantities r^tthat are made available at time t. Constraint (15) makes sure that no stock out for any of the retailers takes place. Constraint (16) forces the inventory level at each retailer at the end of period t to be equal to the inventory level at the end of period t-1, plus the quantities that are made available at period t minus the demand at period t. Constraint (17) ensures that the inventory level of each retailer cannot exceed its capacity. Constraint (18) – (21) define the quantities delivered, aiming to secure the ML policy. More specifically, Constraint (18) secures that for each period the quantities to distribute cannot exceed the capacity of the vehicle. Constraint (19) ensures that the total quantities to be transported to each retailer are equal to the total demand over the whole planning horizon minus the starting inventory level. Constraint (20) expresses the fact that the quantities to be transported to each retailer at period t can be less or equal to the demand requested at period t and subsequent periods, when the retailer is served at period t. Constraint (21) ensures that the transported quantities at period t cannot exceed the supplier's staring inventory level plus the quantity made available since period t. Constraints (22) – (31) serve the routing counterpart of the problem. More specifically, constraint (22) ensures that the total number of routes cannot exceed the number of periods of the planning horizon; however, it is not necessary to perform a route for each period. Constraint (23) ensures that if a route is performed at time t it will start from the supplier and will visit only one retailer. Constraints (24) and (25) secure the flow of the route among intermediate retailers. Constraints (26) and (27) define the relationship of the two indices and the three indexed variables of the routing constraint and it states that when a retailer is served at period t, he will be an origin or a destination of a valid path. Constraints (28) – (29) is the well known sub tour elimination constrains based on the Miller-Tucker-Zemlin (MTZ) constraint formulation as suggested in Anken et al. (2012); this is achieved by introducing extra variables u_i^t that express the quantities that are in the vehicle until the inventory of retailer i is reached. Constraint (31) secures that if a route is performed at period t , then there will be intermediate points in the route. Constraints (32) – (34) enforce integrality and non - negativity conditions.

4.2 Second Stage Decision Process – Transshipment model

As mentioned in the introduction of the paper the above presented deterministic IRP model is needed to determine the original (first stage) total cost of inventory management and transportation of goods. However in the stochastic IRP the aim is to find the distribution plan that minimizes the sum of our first stage plan and the expected recourse action costs. Assuming that we face shortage situations, the extra needed demand is served by lateral transshipments from either neighbouring retailers or from the supplier. Thus the second stage problem can be stated as an transportation problem, where the supply of the lateral transshipment is assigned to the supplier or the retailers with extra products in their inventory and minimum transportation cost. The cost of the assignment is calculated as a proportion of the initial transportation cost from a node $i \rightarrow j$, let it be $a * c_{ij}, a < 1$.Thansshipment cost are both distance and volume – dependent because this is sometimes how outsourced carriers define the terms of their contracts.Let w_{ij}^tbe the amount of product delivered directly from iϵV to retailer $j \in V'$ at period t using outsourced carrier. The objectivefunction of the second stage model is defined aiming to minimize the transshipment cost thus:

$$minimize \sum_{t \epsilon T} \sum_{i,j} a * c_{ij} {w^t}_{i,j} \tag{35}$$

Subject to:

$$I_i^o = St\_Inv(i) \forall i \in V \tag{36}$$
$$I_i^t = I_i^{t-1} + q_i^t - d_i^t(\omega) \ \forall t \in T, \forall i \in V' \tag{37}$$
$$I_i^t + \sum_{j \epsilon V'} w_{ji}^t \geq 0 \ \forall t \in T, \forall i \in V' \tag{38}$$

Equation (36) secures that the initial values of inventories are the same as the ones at the first stage. Eqution (37) ensures that the revealed demand has to be satisfied from the new inventory level. Finaly inventory level and transshipments amounts should be non negative secured by the equation (38).

4.3 Two stage stochastic IRP with transshipment as a recourses

Let us assume that ξ represents the random vector that belongs to the $\Xi \in \mathbb{R}^k$, and also assume that $\mathcal{F}$ is the family of events of subsets of Ξ;P is the probability measure defined at $\mathcal{F}$, where $A \subset \Xi$, $A \in \mathcal{F}$, and $P(A)$ are known. The random vector includes all necessary parameters of the problem that are assumed to be random parameters thus the demand of each retailer for each time period $d_i^t(\omega), \forall i \in V'$ $\xi^T = \{d_2^t(\omega), d_3^t(\omega), \ldots, d_N^t(\omega)\}$. Therefore the objective function of two stage stochastic programming model can be stated as follows:

$$min \sum_{t\epsilon T} \sum_{i\epsilon V} h_i I_i^t + \sum_{t\epsilon T} \sum_{i\epsilon V} \sum_{\substack{j\epsilon V \\ i\neq j}} c_{ij} x_{i,j}^t + E_\xi \left\{ \sum_{t\epsilon T} \sum_{\substack{i\epsilon V \\ j\epsilon V'}} \alpha * c_{i,j} w_{ij}^t(\omega) \right\}$$

(39)

In order to merge the two deterministic models we have to identify which constraints of the first stage are affected by the stochasticity of the demand. This way, we can determine $h(\omega)$ of the right hand side of the second stage model and also T (ω), which are the coefficient of decision variables that are affected by the random demand. The equalities that serve these needs are the ones that determine the level of inventory amounts for the set of retailers, such as (13) – (16).

Therefore the above mentioned constrains have to be modified in order to account for the integration of the two models. More specifically those constraints can be stated as follows:

$$I_i^t - \sum_{\substack{k\in V' \\ k\neq i}} w_{ik}^t(\omega) + \sum_{\substack{k\in V \\ k\neq i}} w_{ki}^t(\omega) \geq 0 \,, \forall t \in T, \forall i \in V \qquad (40)$$

$$I_i^t = I_i^{t-1} - \sum_{\substack{k\in B \\ k\neq i}} w_{ik}^t(\omega) + \sum_{\substack{k\in A \\ k\neq i}} w_{\kappa i}^t(\omega) + q_i^t - d_i^t(\omega) \; \forall t \in T \,, \forall i \in V' \qquad (41)$$

$$\sum_{t\in T} q_i^t + I_i^0 - \sum_{\substack{k\in B \\ k\neq i}} z_{ik}^t(\omega) + \sum_{\substack{k\in A \\ k\neq i}} z_{\kappa i}^t(\omega) = \sum_{t\in T} d_i^t(\omega) \,, \forall i \in V' \qquad (42)$$

Constraint (40) ensures that the level of inventory should be positive, which is valid for both first stage transported amounts as well as the second stage transshipped amounts either inbound or outbound for each inventory item. Constraint (41) includes the transshipped amounts of the second stage, in the close loop relationship of the level of inventories within the same time period. Constraint (46) equates the transshipment amounts to the total transported quantities. When a finite number of scenarios are considered, the objective functions can be transformed and the stochastic model is stated by the deterministic equivalent. Thus the deterministic equivalent problem has the following objective function.

$$\min \sum_{t\epsilon T} \sum_{i\epsilon V} h_i I_i^t + \sum_{t\epsilon T} \sum_{i\epsilon V} \sum_{\substack{j\epsilon V \\ i\neq j}} c_{ij} x_{i,j}^t + \sum_{l=1}^{r} p_l \left\{ \sum_{t\epsilon T} \sum_{\substack{i\in A\subset V \\ j\in B\subset V'}} \alpha * c_{i,j} v_{ij}^t(\omega^l) \right\} \qquad (43)$$

4.4 L – Shaped Method

The L – shaped algorithm we have applied to solve the two - stage SIRP is a modification of Benders decomposition algorithm. We split the process to the master problem that constitutes the first stage decision process and the sub problem in a way to represent second stage decisions that constitute the transshipment recourse actions. Initially the first IRP model is solved and then for each possible scenario it is checked if it is feasible; otherwise a feasibility cut is added. Since the process adds to the problem all the feasibility cuts, the optimality cuts are next computed and added to the master problem until a lower bound equal to the upper bound or a predefined tolerance is reached.

Algorithm: The L – shaped algorithm for stochastic inventory with transshipment

for t=0
  compute the IRP plan by solving the first stage decision process (Obj. Function: (12) s.t. (13) - (34))
  for each scenario $\omega \in \Omega$ **do** indentify if feasibility cuts are required
    compute the dual multiplier and produce the feasibility cut
    add feasibility cut to the master problem
  endfor
  for each scenario $\omega \in \Omega$ **solve** the sub problem (Obj. Function: (35) st. (36) – (38))
    compute optimal value and optimal dual value
  endfor
compute UB and execute convergence test
If $\frac{UB-LB}{1-|LB|} \leq \mathbf{0.001}$ **then**
Stop required accuracy achieved
Return $\bar{x}^v$
Endif

15: **solve** the master problem Objective function (43)
16 : **add** Optimality cuts
17: Compute optimal values and LB
18: return to 11
19: **endfor**

5 Computational Results

The algorithm described above was coded in C++ using IBM Concert Technology and CPEX 12.4 with 2 threads. All computations were executed in an Intel Atom 1.83 GHz with a 2 GB RAM, a typical laptop, in less than 2 hours CPU time. Validation of the performance of the proposed alternative valid inequalities can be found in Chrysochoou & Ziliaskopoulos (2015). The computational results demonstrate that the decision of taking into account the forthcoming time period demand to estimate the quantities to be delivered improves the optimal value by an average 15%. In order to validate the results of the stochastic model, we assume that the recourse action in each period is independent and varies in the range $[d_{it}(1-e), d_{it}(1+e)]$ where d_{it} is the demand of the initial instances. The value of e is set to be *0, 2* which represent the maximum demand variation. We also assume that the distribution of the probability of the demand is discrete and produces a finite number of scenarios, where the initial demand represents the scenario with the greater probability. The number of scenarios is set to be 3 and 5 and the maximum computing time is set to 2 hours of CPU time.

Table 1. Computational results of the L - Shaped approach.

		High Inventory cost				Low Inventory cost			
Scenario	Customers	CPU (s)	#Optim	Gap	Bender Cuts	CPU (s)	#Optim	Gap	Bender Cuts
p=3	5	213	10 // 10	0.0	15	219	10 // 10	0.0	18
	10	257	10 // 10	0.0	58	264	10 // 10	0.0	68
	15	468	8 // 10	0.0	136	479	9 // 10	0.0	154
	20	1235	9 // 10	0.2	686	1277	7 // 10	0.6	755
p=5	5	276	10 // 10	0.0	45	282	10 // 10	0.0	54
	10	318	10 // 10	0.0	254	330	10 // 10	0.0	279
	15	674	8 // 10	5.6	508	690	8 // 10	7.4	589
	20	5076	9 // 10	9.78	1749	5223	8 // 10	13.0	1959

In Table 1 appear the results of certain indicative experiments; the number of instances solved, the optimality gap as well as the computational time are shown. These initial results demonstrate the potential of the approach to efficiently evaluate more complex scenarios as well as the possibility of incorporating brunch and cut schema in the first stage problem.

6 Conclusions

In this paper, we considered the vendor – managed inventory problem, where a supplier manages the inventory level of the retailers using transshipment as a recourse action when demand uncertainty leads to shortages. The problem is motivated by the need of a major electronics multinational with production both in Asia and Europe and various warehouses throughout their global supply chain. We proposed an L – shaped algorithm that efficiently solves the stochastic inventory routing problem using transshipment as a recourse action, which seems to be a realistic strategy for the electronics and automotive industry that stretch their production line from China to Europe. Finally, though the L shaped method was not exhaustively tested computationally, it was demonstrated that presents the potential to be applied for more complex real life problems.

Acknowledgements

This research has been co – financed by the European Union (European Social Fund – NSF) & Greek national funds through the Operational Program " Education and Lifelong Learning" of National Strategic Reference

Framework(NSRF) Research Funding Program:HeracleitusII. Investigation in knowledge society through the European Social Fund.

References

Adelman, D. 2004. A Price-Directed Approach to Stochastic Inventory/Routing. *Operations Research* **52**(4) 499-514.

Adulyasak, Y., J.-F. and Cordeau, R.Jans. 2014. Formulations and Branch-and-Cut Algorithms for Multivehicle Production and Inventory Routing Problems.*INFORMS Journal on Computing* **26** (1) 103 – 120.

Aksen, D. et al.2012. Selective and periodic inventory routing problem for waste vegetable oil collection. *Optimization Letters* **6**(6) 1063–1080.

Allen, S. 1958. Residribution of total stock over several user locations. *Naval Research Logistics Quarterly* **5** (4) 337 – 345.

Andersson, H. et al. 2010. Industrial aspects and literature survey: Combined inventory management and routing. *Computers & Operations Research* **37**(9) 1515–1536.

Anily, S. and A. Federgruen. 1990. One warehouse multiple retailer systems with vehicle routing costs. *Management Science* **36** 92 – 114.

Archetti, C. et al. 2007. A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, **41**(3) 382–391.

Baita, F. et al. 1998. Dynamic routing-and inventory problems: A review. *Transportation Research Part A: Policy and Practice* **32**(8) 585–598.

Bard, J. et al. 1998. A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science* **32** 189 – 203.

Bell, W. J. et al.1983. Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces* 13(6) : 4–23.

Bertazzi, L.et al. 2013. A stochastic inventory routing problem with stock-out. *Transportation Research Part C: Emerging Technologies*, **27** 89-107.

Bertazzi, L., M.G. Speranza, and W. Ukovich. 1997. Minimization of logistic costs with given frequencies. *Transportation Research* **31**: 327 – 340 .

Birge, J. R. and F. Louveaux. 1997. *Introduction to Stochastic Programming*. Springer Series in Operations Research. Springer.

Blumenfeld, D. E., et al.1985. Analyzing trade-offs between transportation, inventory and production costs on freight networks. *Transportation Research* **19** :361– 380.

Campbell, AM, Clarke L, Kleywegt AJ, Savelsbergh MWP. 1998. The inventory routing problem.,Crainic TG, Laporte G, eds. *Fleet Management and Logistics* (Springer, Boston), 95–113.

Carøe, C. C. and J.Tind. 1998. L- Shaped decomposition of two – stage stochastic programs with integer recourse. *Mathematical Programming*, **83** (1-3) : 451 -464.

Chrysochoou E., A. Ziliaskopoulos 2015. An exact method for the stochastic inventory routing problem with transshipment. *Proc 94th Annual Meeting Transportation Research Board* 11 -15 January. Whashigton D.C.

Coelho, L. C. and G. Laporte. 2013. The exact solution of several classes of inventory routing problems. *Computers & Operations Research*, **40**(2) : 558–565.

Coelho, L. C.2012.Flexibility and Concistency in Inventory Routing , PhD Thesis. , Graduate in Administration – HEC Montréal.

Coelho, L. C., J.-F. Cordeau, and G. Laporte .2014. Thirty years of Inventory Routing. *Transportation Science* **48**(1) 1-19.

Federgruen, A. and P. H. Zipkin. 1984. A combined vehicle-routing and inventory allocation problem. *Operations Research*, **32**(5) :1019–1037.

Fisher, M. L., and R. Jaikumar. 1981. A generalized assignment heuristic for vehicle routing. *Networks*, **11**(2) :109–124.

Hall, R. W. 1985. Determining vehicle dispatch frequency when shipping frequency differs among suppliers. *Transportation Research* **19** (B): 421 – 431.

Kall, P. and S.W. Wallace. 1994. *Stochastic Programming* John Wiley & Sons, Inc., New York.

Kleywegt, A. J., V. S. Nori, and M. W. P. Savelsbergh. 2002. The stochastic inventory routing problem with direct deliveries. *Transportation Science*, **36**(1) : 94–118.

Kleywegt, A. J., V. S. Nori, and M. W. P. Savelsbergh. 2004. Dynamic programming approximations for a stochastic inventory routing problem. *Transportation Science* **38**(1): 42–70.

Bertazzi,L., G. Paletta, and M. G. Speranza. 2002. Deterministic order-up-to level policies in an inventory routing problem. *Transportation Science*, **36**(1) : 119 – 132,

Solyalı, O., J.-F. Cordeau, and G. Laporte.2012. Robust inventory routing under demand uncertainty. *Transportation Science*, **46**(3) : 327–340.

Solyalı, O. and H. Süral. 2011. A branch-and-cut algorithm using a strong formulation and an a priori tour based heuristic for an inventory-routing problem. *Transportation Science*, **45**(3) : 335–345.

Speranza, M. G, and W.Ukovich. 1994a. Minimizing transportation and inventory costs for several products on a single link. *Operations Research* **42**: 879 – 894.

Speranza, M. G. and W.Ukovich. 1994b.Analysis and integration of optimization odels for logistic systems. *International Journal of Production Economics*. **35**: (1-3) 183 -190.