Stochastic Inventory Routing Problem with Transshipment Recourse Action

Evangelia Chrysochoou, PhD Candidate ,
Research Associate CERTH/HIT
Prof. Athanasios Ziliaskopoulos, Supervisor,
Chairman & CEO at Greek Railways

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Introduction

Vendor Managed Inventory (VMI) systems seems to be at the core of most global supply chains .

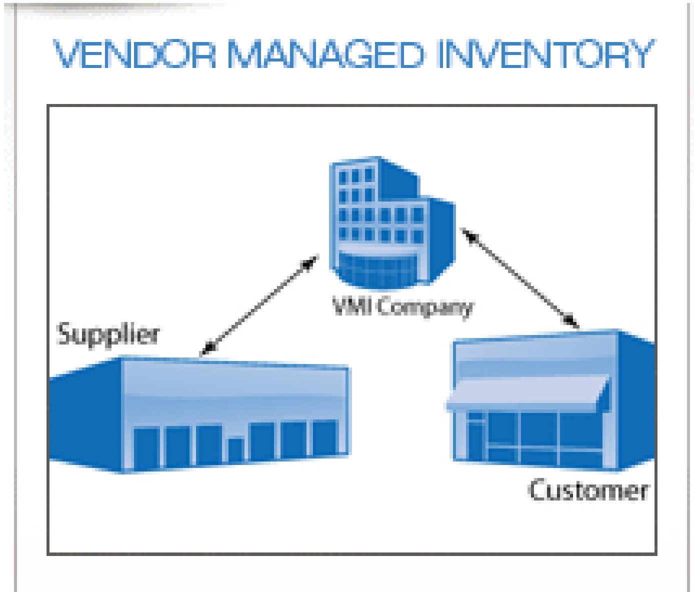
Concept: The replenishment & the distribution making process is centralized at supplier level. Supplier acts as central decision maker. This policy leads to an overall reduction of logistic cost.

Advantage: More efficient resource utilization. Often described as a win – win situation.

Inventory Routing Problem (IRP) constitutes the backbone of the VMI systems.

Decision to be taken are:

1. **When** to deliver to each customer
2. **How much** to deliver to each customer each time it is served
3. **How to route** the vehicles so as to minimize the total cost.



Motivation

- Need of major electronics multinationals with production both in Asia and Europe and various warehouses throughout their global supply chain management for more **efficient** resource utilization .
- Need to adjust their operations to meet the requirements of their clients.
- Need to account the uncertainty of demand .

State of the art

The problem was first introduced by **Bell et al.(1983)** and **Federgruen & Zipkin (1984) .**

To the best of our knowledge there are two seminal papers regarding literature review on the IRP **Andersson et al.(2010)** related to business models and classification of problems and **Coelho et al. (2014)** related to methods and algorithms.

On the other hand **Geisen, Mahmassani and Jaillet (2009)** and **Rabah and Mahmassani (2002)** provide an excellent reference for applications of VMI policies with stochastic demand.

Bertazzi ,Paletta and Speranza(2002) introduced a practical VMI policy the deterministic Order – Up – to level policy.

Arhetti et al. (2007) developed the first exact method based on the OU-Policy.

Coelho & Laporte (2012) introduce the transshipment cost within IRP and developed an exact method as well as an ALNS metaheuristic for large scale instances.

State of the art

IRP under uncertainty

- Campbell et al. (1998) set the basis for the rolling horizon framework
- Kleywegt et al. (2002, 2004) formulated the stochastic IRP as a Marcov Decision Process (MDP) over an infinite horizon,
- Solyali et al.(2012) proposed two mixed integer programming formulations of the robust version of the problem, which produce policies leading to feasible solution and optimal cost for any realization of the demand
- Bertazzi et al. (2013) as well as Coelho (2012) rely on a dynamic programming formulation that allows the design of a hybrid rollout formulation aiming to find good quality solution.

Contribution

- Introduce a stochastic programming model for the IRP and propose an L – Shaped algorithm that efficiently solves the SIRP using transshipment as recourse action.
- Introduce new valid inequalities for the first stage decision process which accounting forthcoming time period demand to determine the delivered quantities

Methodology

Stochastic Programming Model

$$\min_x c^T x + E_\omega Q(x,\omega)$$
$$\textit{Subject to:} Ax = b$$
$$x > 0$$
$$\textit{Where:}$$
$$Q(x,\omega) = \min_y d_\omega^T y$$
$$\textit{Subject to:} T_\omega x + W_\omega y = h_\omega$$
$$y > 0$$

Deterministic Equivalent Model

$$\min_x c^T x + \sum_{\omega \in \Omega} p(\omega) d_\omega^T y_\omega$$
$$\textit{Subject to: } Ax = b$$
$$T_\omega x + W_\omega y = h_\omega$$
$$x > 0, y_\omega > 0$$

First Stage Decision Process:
Inventory Routing Problem

$$\min_x c^T x$$
$$\textit{Subject to:} Ax = b,$$
$$x \geq 0$$

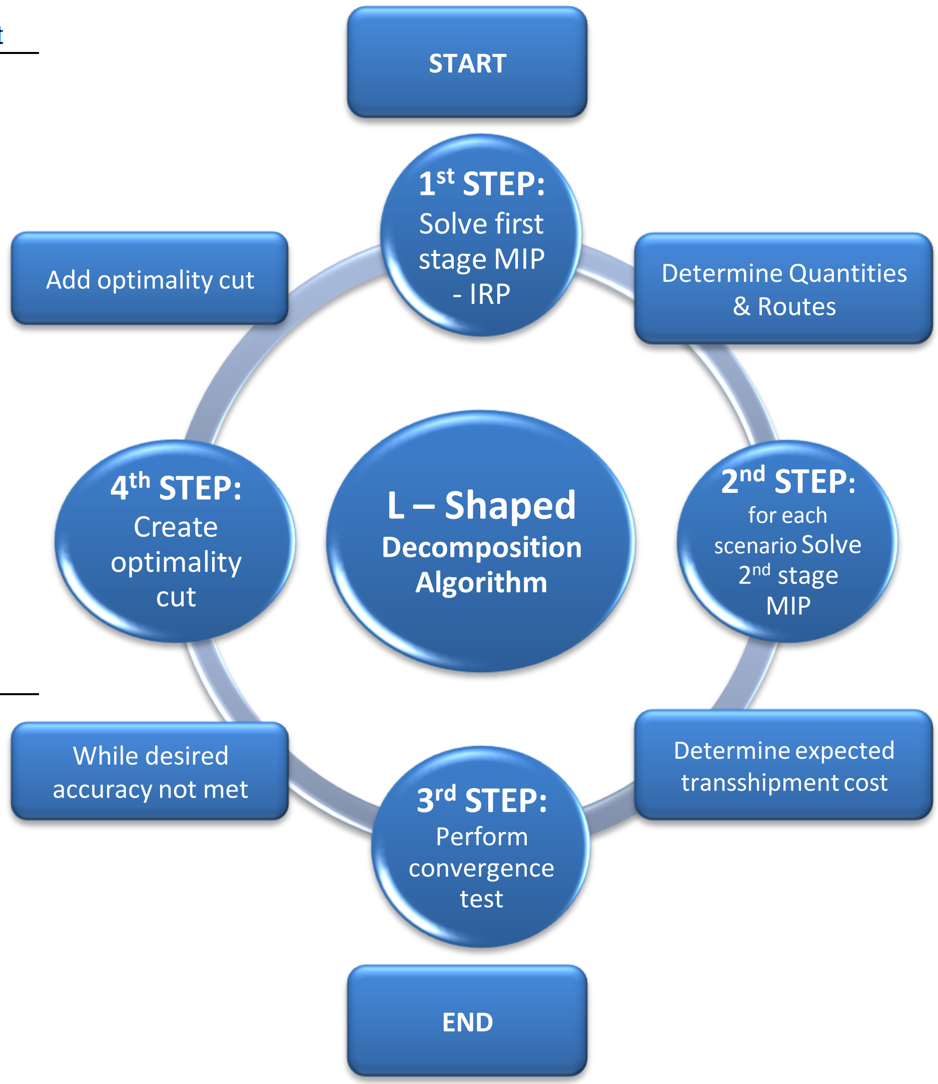
Second Stage Decision Process:
Transshipment Recourse Actions

$$Q(x,\omega) = \min_y d_\omega^T y$$
$$\textit{Subject to:} T_\omega x + W_\omega y = h_\omega$$
$$y > 0$$

Methodology

Algorithm: The L – shaped algorithm for stochastic inventory with transshipment

```
1: for t=0
2:    compute the IRP plan by solving the first stage decision process
(Obj. Function: (12) s.t. (13) - (34))
3:   for each scenario ω ∈ Ω do indentify if feasibility cuts are required
4:         compute the dual multiplier and produce the feasibility cut
5:         add feasibility cut to the master problem
6:   endfor
7:   for each scenario ω ∈ Ω solve the sub problem
(Obj. Function: (35) st. (36) – (38)
8:         compute optimal value and optimal dual value
9:   endfor
10:  compute UB and execute convergence test
11: If (UB−LB)/(1−|LB|) ≤ 0.001 then
12: Stop required accuracy achieved
13: Return x̄^v
14: Endif
15: solve the master problem Objective function (43)
16 : add Optimality cuts
17: Compute optimal values and LB
18: return to 11
19: endfor
```



IRP without OU Policy

$$minimize \sum_{t \in T} \sum_{i \in V} h_i I_i^t + \sum_{t \in T} \sum_{i \in V} \sum_{\substack{i \in V \\ i \neq j}} c_{ij} x_{i,j}^t$$

$$I_1^t \geq 0 \; \forall t \in T$$

$$I_1^t \geq I_1^{t-1} + r^t - \sum_{i \in V'} q_i^t \; \forall t \in T$$

$$I_i^t \geq 0 \; \forall t \in T, \forall i \in V'$$

$$I_i^t \geq I_i^{t-1} + q_i^t - d_i^t \quad \forall t \in T, \forall i \in V'$$

$$I_i^t \leq C_i \; \forall t \in T, \forall i \in V'$$

$$\sum_{i \in V'} q_i^t \leq C \; \forall t \in T$$

$$\sum_{t \in T} q_i^t \geq \sum_{t \in T} d_i^t - I_i^0, \forall i \in V'$$

$$q_i^t \leq y_i^t \sum_{j=t}^{H} d_i^j, \forall i \in V', \forall t \in T$$

$$\sum_{j=1}^{t} q_i^j \leq I_1^0 + \sum_{j=1}^{t} r^j, \forall t \in T,, \forall i \in V'$$

$$\sum_{t \in T} \sum_{j \in V'} x_{1j}^t \leq H$$

$$\sum_{j \in V'} x_{1j}^t \leq y_1^t \; \forall t \in T$$

$$\sum_{j \in V'} x_{ij}^t + \sum_{j \in V'} x_{ji}^t = 2 y_i^t \quad \forall t \in T, \forall i \in V'$$

$$x_{i,j}^t \leq y_i^t \forall t \in T, \forall i,j \in V'$$

$$x_{i,j}^t \leq y_j^t \forall t \in T, \forall i,j \in V'$$

$$C(1 - x_{i,j}^t) + u_i^t \geq u_j^t + q_j^t \quad \forall i,j \in V' : i \neq j, t \in T$$

$$q_i^t \leq u_i^t \quad \forall i,j \in V', t \in T$$

$$u_i^t \leq y_i^t * C \quad \forall i \in V', t \in T$$

$$y_i^t \leq y_1^t \forall t \in T, \forall i \in V'$$

$$q_i^t, u_i^t \geq 0 \forall i \in V', t \in T$$

$$x_{i,j}^t \in \{0,1\} \; \forall i,j \in V' : i \neq j, t \in T$$

$$y_i^t \in \{0,1\} \; \forall i \in V, t \in T$$

SIRP with recourse action of transshipment

$$\min \sum_{t\in T}\sum_{i\in V} h_i I_i^t + \sum_{t\in T}\sum_{i\in V}\sum_{\substack{j\in V\\ i\neq j}} c_{ij} x_{i,j}^t + E_\xi\left\{\sum_{t\in T}\sum_{\substack{i\in V\\ j\in V}} \alpha * c_{i,j} w_{ij}^t(\omega)\right\}$$

$$I_1^t \geq 0 \; \forall t\in T$$

$$I_1^t \geq I_1^{t-1} + r^t - \sum_{i\in V'} q_i^t \; \forall t \in T$$

$$I_i^t \geq 0 \; \forall t\in T, \forall i \in V'$$

$$I_i^t \geq I_i^{t-1} + q_i^t - d_i^t \quad \forall t\in T, \forall i\in V'$$

$$I_i^t \leq C_i \; \forall t\in T, \forall i\in V'$$

$$\sum_{i\in V'} q_i^t \leq C \; \forall t\in T$$

$$\sum_{t\in T} q_i^t \geq \sum_{t\in T} d_i^t - I_i^0, \forall i\in V'$$

$$q_i^t \leq y_i^t \sum_{j=t}^{H} d_i^j, \forall i\in V', \forall t\in T$$

$$\sum_{j=1}^{t} q_i^j \leq I_1^0 + \sum_{j=1}^{t} r^j, \forall t\in T,, \forall i\in V'$$

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$$C(1 - x_{i,j}^t) + u_i^t \geq u_j^t + q_j^t \quad \forall i,j\in V' : i\neq j, t\in T$$

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$$q_i^t, u_i^t \geq 0 \forall i\in V', t\in T$$

$$x_{i,j}^t \in \{0,1\} \; \forall i,j\in V' : i\neq j, t\in T$$

$$y_i^t \in \{0,1\} \; \forall i\in V, t\in T$$

$$I_i^t - \sum_{\substack{k\in V'\\ k\neq i}} w_{ik}^t(\omega) + \sum_{\substack{k\in V\\ k\neq i}} w_{ki}^t(\omega) \geq 0, \forall t\in T, \forall i\in V$$

$$I_i^t = I_i^{t-1} - \sum_{\substack{k\in B\\ k\neq i}} w_{ik}^t(\omega) + \sum_{\substack{k\in A\\ k\neq i}} w_{ki}^t(\omega) + q_i^t - d_i^t(\omega) \; \forall t\in T, \forall i\in V'$$

$$I_1^t = I_1^{t-1} + r^t - \sum_{\substack{i\in V\\ i>1}} w_{1i}^t(\omega) - \sum_{\substack{i\in V\\ i>1}} q_i^t \forall t\in T$$

Deterministic Equivalent

$$\min \sum_{t\in T}\sum_{i\in V} h_i I_i^t + \sum_{t\in T}\sum_{i\in V}\sum_{\substack{j\in V\\ i\neq j}} c_{ij} x_{i,j}^t + \sum_{l=1}^{r} p_l \left\{\sum_{t\in T}\sum_{\substack{i\in A\subset V\\ j\in B\subset V}} \alpha * c_{i,j} v_{ij}^t(\omega^l)\right\}$$

$$I_1^t \geq 0 \; \forall t \in T$$

$$I_1^t \geq I_1^{t-1} + r^t - \sum_{i\in V'} q_i^t \; \forall t \in T$$

$$I_i^t \geq 0 \; \forall t \in T, \forall i \in V'$$

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$$I_i^t \leq C_i \; \forall t \in T, \forall i \in V'$$

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$$q_i^t \leq y_i^t \sum_{j=t}^{H} d_i^j, \forall i \in V', \forall t \in T$$

$$\sum_{j=1}^{t} q_i^j \leq I_1^0 + \sum_{j=1}^{t} r^j \;, \forall t \in T,, \forall i \in V'$$

$$I_i^t - \sum_{\substack{k\in V'\\ k\neq i}} w_{ik}^t(\omega) + \sum_{\substack{k\in V\\ k\neq i}} w_{ki}^t(\omega) \geq 0 \;, \forall t \in T, \forall i \in V$$

$$I_i^t = I_i^{t-1} - \sum_{\substack{k\in B\\ k\neq i}} w_{ik}^t(\omega) + \sum_{\substack{k\in A\\ k\neq i}} w_{ki}^t(\omega) + q_i^t - d_i^t(\omega) \; \forall t \in T, \forall i \in V'$$

$$I_1^t = I_1^{t-1} + r^t - \sum_{\substack{i\in V\\ i>1}} w_{1i}^t(\omega) - \sum_{\substack{i\in V\\ i>1}} q_i^t \forall t \in T$$

$$\sum_{t\in T}\sum_{j\in V'} x_{1j}^t \leq H$$

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$$y_i^t \in \{0,1\} \forall i \in V, t \in T$$

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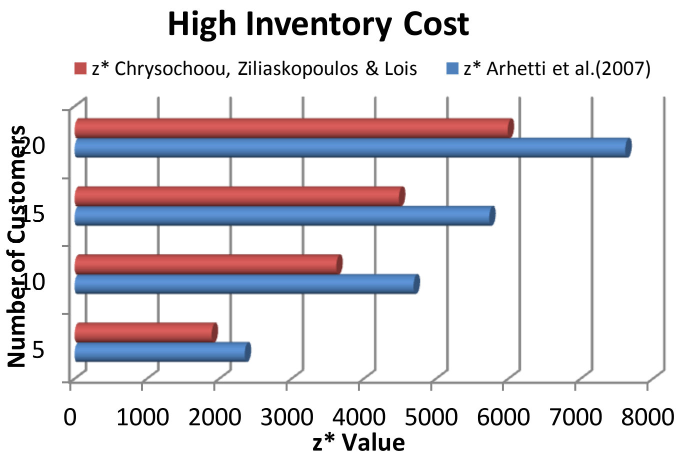
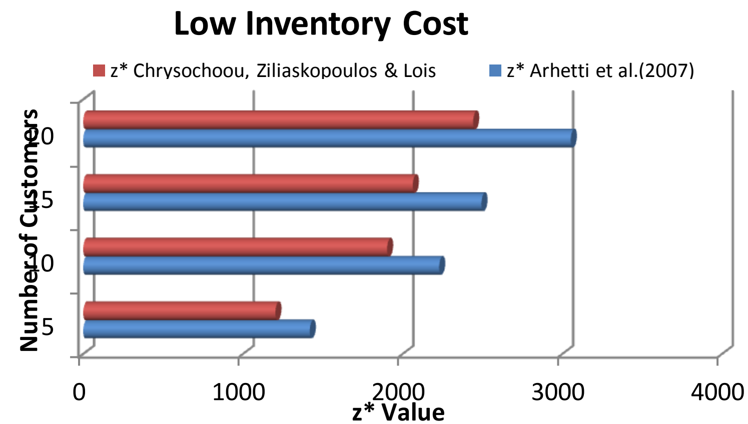
Results

Algorithm was coded in ILOG Optimization Studio CPLEX 12.4.

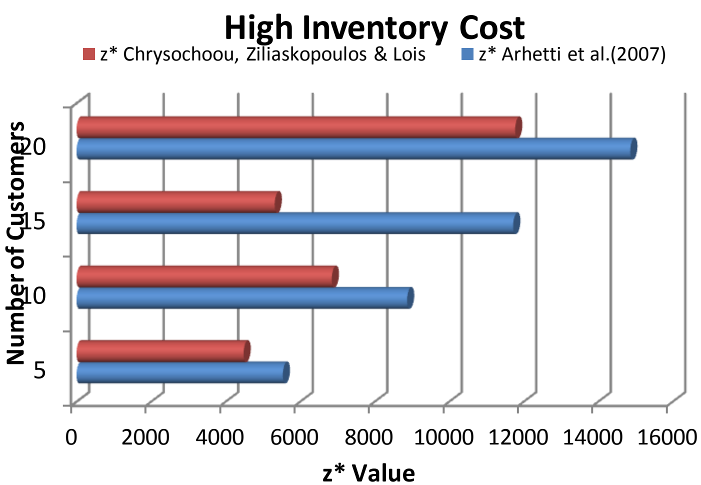
Benchmark instances of Arhetti et al. (2007) were used to evaluate the proposed valid inequalities.

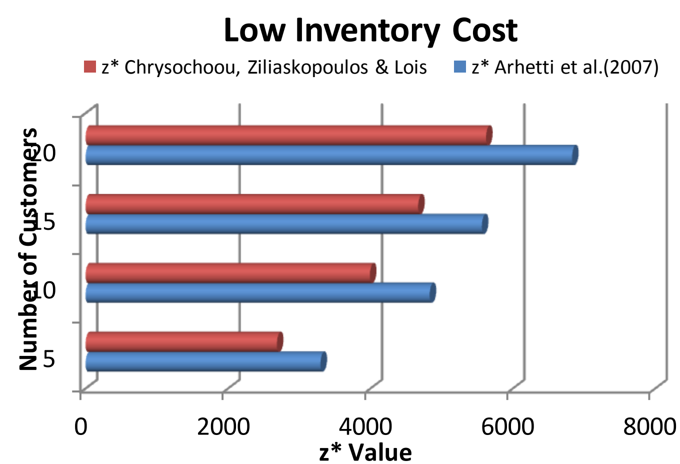
H = 3 periods





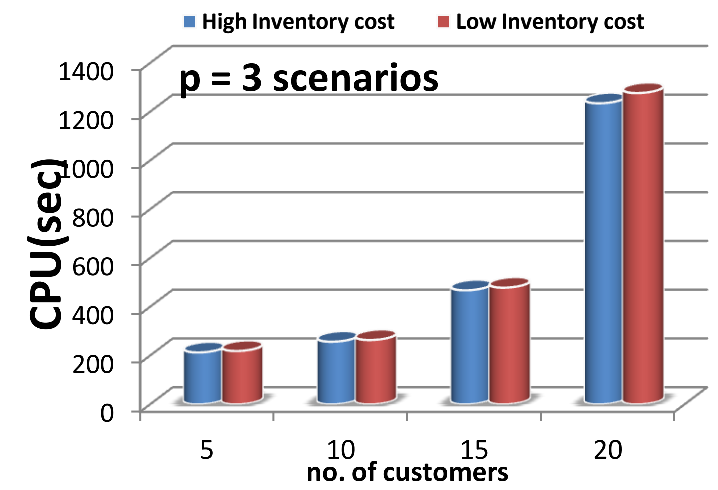
H = 6 periods

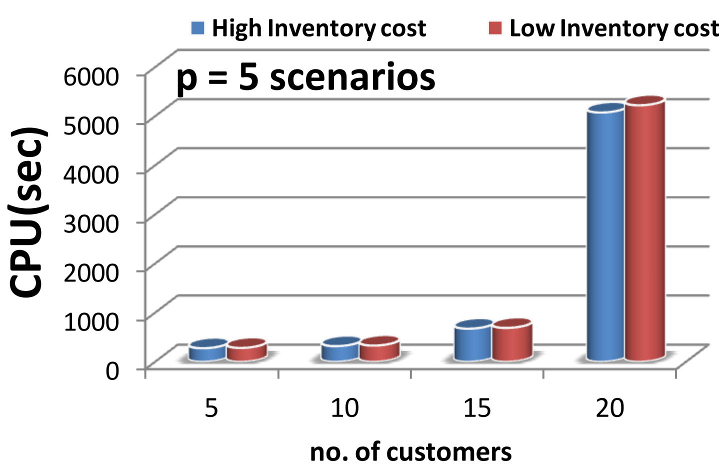




Results

Computational results of L - Shaped

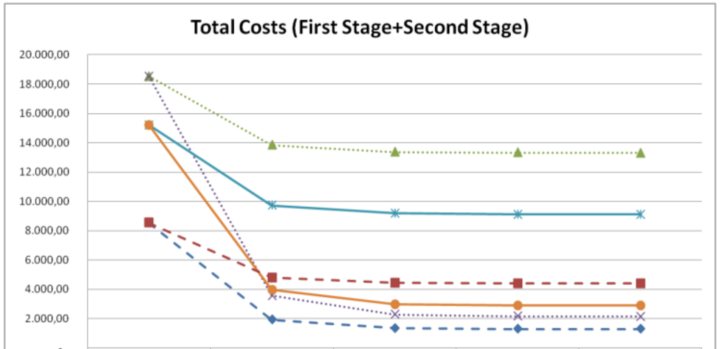




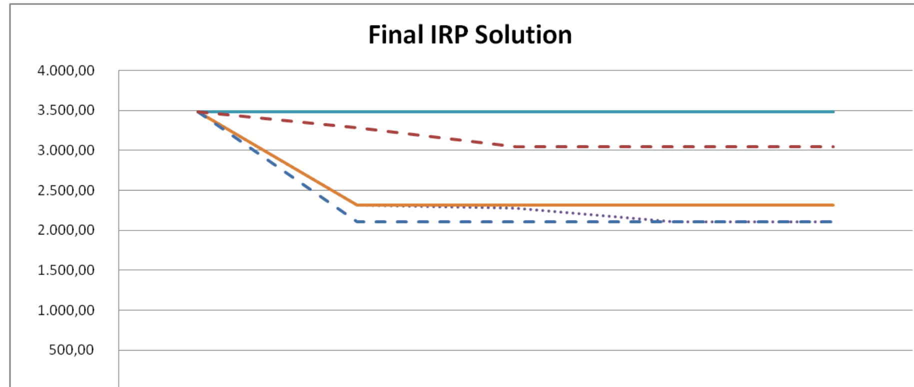
- Transshipment significantly improves the overall performance of vendor managed inventory supply chain.

- Relaxation of the Order – Up to level policy in coherence to the decision of accounting forthcoming demand to determine the quantity of shipments demonstrate savings of 15% on an average.

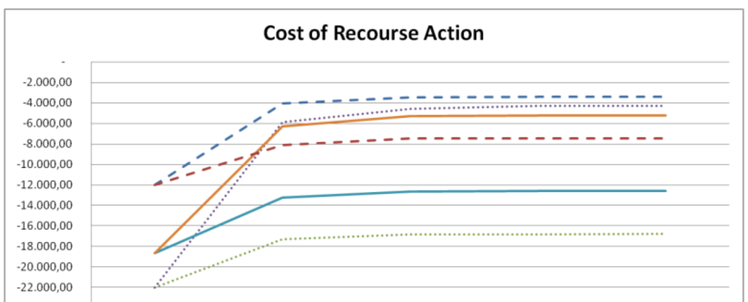
Evaluation of Transshipment costs



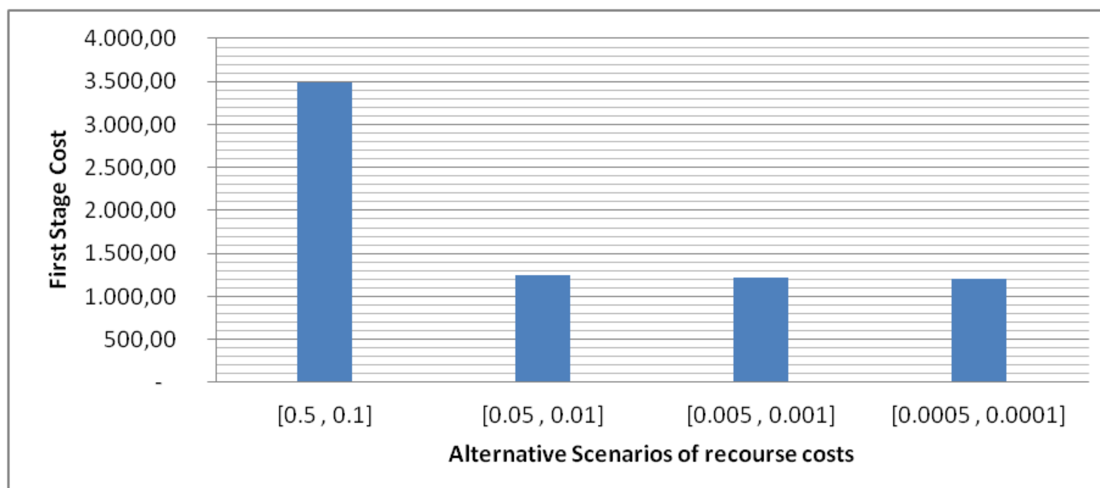
	0,1	0,01	0,001	0,0001	0,00001
60%	18.530,49	13.837,71	13.368,43	13.321,51	13.316,81
40%	18.530,49	3.570,38	2.289,76	2.165,25	2.152,99
70%	15.199,95	9.725,05	9.177,56	9.122,81	9.117,34
30%	15.199,95	3.956,18	2.993,14	2.896,84	2.887,21
90%	8.538,03	1.935,00	1.357,68	1.302,15	1.296,40
10%	8.538,03	4.808,05	4.440,52	4.404,59	4.401,00



	0,1	0,01	0,001	0,0001	0,00001
60%	3.481,97	3.481,97	3.481,97	3.481,97	3.481,97
40%	3.481,97	2.311,24	2.277,24	2.109,89	2.109,89
70%	3.481,97	3.481,97	3.481,97	3.481,97	3.481,97
30%	3.481,97	2.311,24	2.311,24	2.311,24	2.311,24
90%	3.481,97	2.109,90	2.109,90	2.109,90	2.109,90
10%	3.481,97	3.282,19	3.048,00	3.048,00	3.048,00

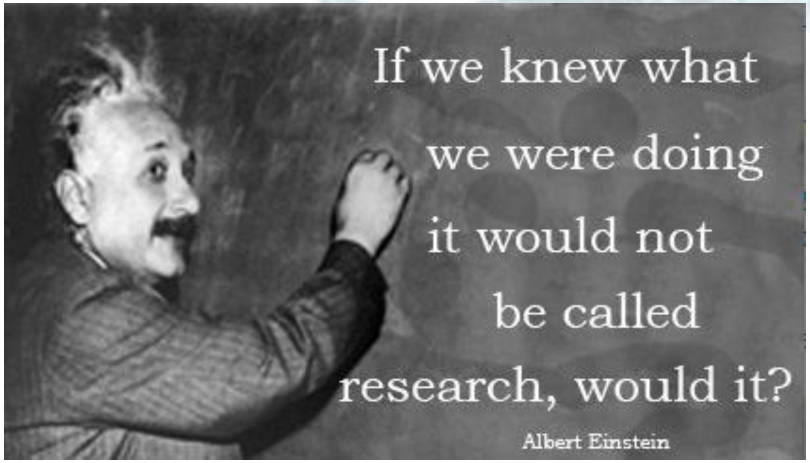


	0,1	0,01	0,001	0,0001	0,00001
60%	-22.012,46	-17.319,68	-16.850,40	-16.803,48	-16.798,78
40%	-22.012,46	-5.881,62	-4.567,00	-4.275,14	-4.262,88
70%	-18.681,92	-13.207,02	-12.659,53	-12.604,78	-12.599,31
30%	-18.681,92	-6.267,42	-5.304,38	-5.208,08	-5.198,45
90%	-12.020,00	-4.044,90	-3.467,58	-3.412,05	-3.406,30
10%	-12.020,00	-8.090,24	-7.488,52	-7.452,59	-7.449,00



Conclusion

- Nowadays of **unstable global economic conditions** the demand of products become highly uncertain in many business areas.
- **Sustainability** of business depend on the ability to handle market uncertainties.
- Research should focus on development of models and methods that fit the industries needs of **robust flexible plans to handle the uncertainties.**



Thank you for your attention

Contact Details:
Evangelia Chrysochoou
email: echryso@certh.gr

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