Risk Problems Identifying Optimal Pollution Level

George E. Halkos and Dimitra C. Kitsou

Abstract The evaluation of the Benefit Area (BA) is essential in Environmental Economics. The Associated Risk, for evaluating BA, is based on various factors. The uncertainty in the model fitting can be reduced by choosing the appropriate approximations for the marginal abatement (MAC) and marginal damage (MD) cost functions. The target of this paper is to identify analytically and empirically the optimal pollution level in the case of quadratic MD cost and linear MAC functions, extending the work of Halkos and Kitsos (2005).

1 Introduction

Rationality in the formulation and applicability of environmental policies depends on careful consideration of their consequences for nature and society. For this reason it is important to quantify the costs and benefits in the most accurate way. But the validity of any cost benefit analysis (hereafter CBA) is ambiguous as the results may have large uncertainties. Uncertainty is present in all environmental problems and this underscores the need for thoughtful policy design and evaluation. We may have uncertainty in the underlying physical or ecological processes, as well as in the economic consequences of the change in environmental quality.

As uncertainty may be due to the lack of appropriate abatement and damage cost data, we apply here a method of calibrating hypothetical damage cost estimates relying on individual country abatement cost functions. In this way a "calibrated" Benefit Area (BA^c) is estimated.

The intersection of MSC and MD defines the optimal pollution level I with coordinates (z_0, k_0), $I(z_0, k_0)$. The value of z_0 describes the optimal damage reduction while k_0 corresponds to the optimal cost to that. The area in $\mathbb{R}^2$ covered by the

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MAC and MD and the axis of cost is defined as the Benefit Area. In Figure 1, where MAC and MD are linear is the triangle (ABI) and represents the maximum of the net benefit that is created by the activities that trying to reduce the pollution. Specifically, we try to identify the optimal pollution level under the assumptions of linear marginal abatement and quadratic damage cost functions. As far as the parameters are concerned the first two are linear while the third is a non–linear function. That is, we consider another case of the possible approximations of the two cost curves improving the work in [10] extending the number of different model approximations of abatement and damage cost functions and thus the assumed correct model eliminates uncertainty about curve fitting. The target of this paper is to develop the appropriate theory in this specific case.

2 Determining the Optimal Level of Pollution

Economic theory suggests that the optimal pollution level occurs when the marginal damage cost equals the marginal abatement cost. Graphically the optimal pollution level is presented in Figure 1 where the marginal abatement $(\mathrm{MAC} = g(z))$ and the marginal damage $(\mathrm{MD} = \varphi(z))$ are represented as typical mathematical cost functions. The point of intersection of the two curves, $I = I(z_0, k_0)$, reflects the optimal level of pollution with k_0 corresponding to the optimum cost (benefit) and z_0 to the optimum damage restriction. It is assumed (and we shall investigate the validity of this assumption subsequently) that the curves have an intersection and the area created by these curves (region AIB) is what we define as Benefit Area (see [20], among others).

Consider Figure 1. Let A and B be the points of the intersection of the linear curves $\mathrm{MD} = \varphi(z) = \alpha + \beta z$ and $\mathrm{MAC} = \beta_0 + \beta_1 z$ with the "Y–axis". We are restricted to positive values. For these points $A = A(0, \alpha)$ and $B = B(0, \beta_0)$ the values of $a = \alpha$ and $b = \beta_0$ are the constant terms of the assumed curves that represent MD and MAC respectively.

Let us now assume that

$$\mathrm{MAC}(z) = g(z) = \beta_0 + \beta_1 z,\ \ \beta_1 \neq 0 \quad \text{and} \quad \mathrm{MD}(z) = \varphi(z) = \alpha z^2 + \beta z + \gamma,\ \ \alpha > 0.$$

The intersections of MD and MAC with the Y–axis are $b = \mathrm{MAC}(0) = \beta_0$ and $a = \mathrm{MD}(0) = \gamma$, see Figures 2, 3 and 4 bellow. To ensure that an intersection between MAC and MD occurs we need the restriction $0 < \beta_0 < \gamma$. yboxAssuming $\alpha > 0$ three cases can be distinguished, through the determinant of $\varphi(z)$, say D, $D = \beta^2 - 4\alpha\gamma$; (a) $D = 0$ (see Figure 2), (b) $D > 0$ (see Figure 3) while the case $D < 0$ is without economic interest (due to the complex–valued roots). Cases (a) and (b) are discussed below, while for the dual $\alpha < 0$ see Case (c). See also for details [19].

Case (a): $\alpha > 0,\ D = \beta^2 - 4\alpha\gamma = 0$. In this case there is a double real root for $\mathrm{MD}(z)$, say $\rho = \rho_1 = \rho_2 = -\frac{\beta}{2\alpha}$. We need $\rho > 0$ and hence $\beta < 0$. To identify the optimal pollution level point $I(z_0, k_0)$ the evaluation of point z_0 is the one for which

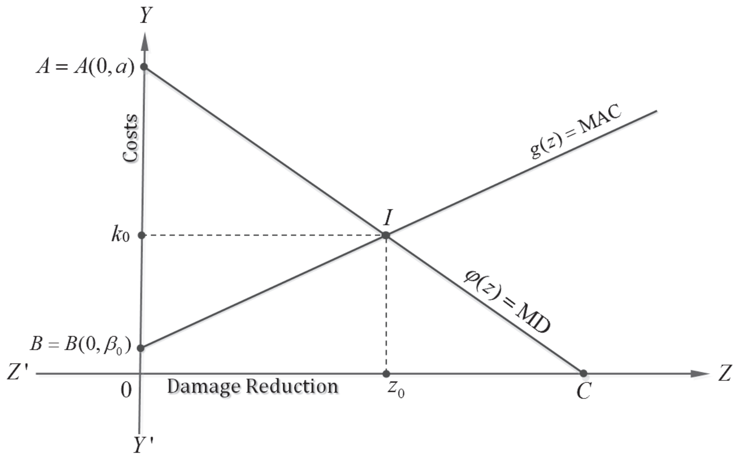

Fig. 1 Graphical presentation of the optimal pollution level (general case).

$$\mathrm{MD}(z_0) = \varphi(z_0) \Leftrightarrow g(z_0) = \mathrm{MAC}(z_0) \Leftrightarrow \alpha z_0^2 + \beta z_0 + \gamma = \beta_0 + \beta_1 z_0 \Leftrightarrow$$
$$\alpha z_0^2 + (\beta - \beta_1) z_0 + (\gamma - \beta_0) = 0. \tag{1}$$

Relation (1) provides the unique (double) solution when $D_1 = (\beta - \beta_1)^2 - 4\alpha(\gamma - \beta_0) = 0$ which is equivalent to

$$z_0 = -\frac{\beta - \beta_1}{2\alpha} = \frac{\beta_1 - \beta}{2\alpha}. \tag{2}$$

As z_0 is positive and $\alpha > 0$ we conclude that $\beta_1 > \beta$. So for the conditions are: $\alpha > 0$, $\beta_1 > \beta$, $0 < \beta_0 < \gamma$ we can easily calculate

$$k_0 = \mathrm{MAC}(z_0) = \beta_0 + \beta_1 \frac{\beta_1 - \beta}{2\alpha} > 0, \tag{3}$$

and therefore $I(z_0, k_0)$ is well defined. The corresponding Benefit Area ($\mathrm{BA_{QL}}$) in this case is

$$\begin{aligned}\mathrm{BA_{QL}} =& (\mathrm{ABI}) = \int_0^{z_0} \varphi(z) - g(z) dz = \int_0^{z_0} \alpha z^2 + (\beta - \beta_1) z + (\gamma - \beta_0) dz = \\ & \left[\tfrac{\alpha}{3} z^3 + \tfrac{1}{2}(\beta - \beta_1) z^2 + (\gamma - \beta_0) z\right]_{z=0}^{z_0} = \\ & \tfrac{\alpha}{3} z_0^3 + \tfrac{1}{2}(\beta - \beta_1) z_0^2 + (\gamma - \beta_0) z_0.\end{aligned} \tag{4}$$

Case (b): $\alpha > 0, D = \beta^2 - 4\alpha\gamma > 0$. For the two roots ρ_1, ρ_2, we have $|\rho_1| \neq |\rho_2|$, $\varphi(\rho_1) = \varphi(\rho_2) = 0$ and we suppose $0 < \rho_1 < \rho_2$, see Figure 3. The fact that $D > 0$

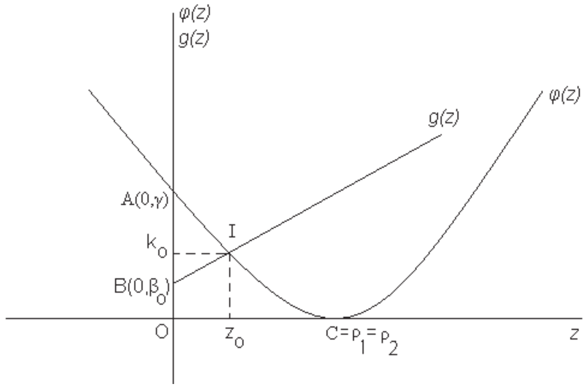

Fig. 2 $C = C(-\frac{\beta}{2\alpha}, 0)$, $\alpha > 0$.

is equivalent to $0 < a\gamma < (\beta/2)^2$, while the minimum value of the MD function is $\varphi(-\beta/(2\alpha)) = (4\alpha\gamma - \beta^2)/(4\alpha)$.

Proposition 1. *The order* $0 < \rho_1 < \rho_2$ *for the roots and the value which provides the minimum is true under the relation*

$$\beta < 0 < \alpha\gamma < (\frac{\beta}{2})^2. \tag{5}$$

Proof. The order of the roots $0 < \rho_1 < \rho_2$ is equivalent to the set of relations:

$$D > 0, \quad \alpha\varphi(-\frac{\beta}{2\alpha}) < 0, \quad \alpha\varphi(0) > 0, \quad 0 < \frac{\rho_1 + \rho_2}{2}. \tag{6}$$

The first is valid, as we have assumed $D > 0$. For the imposed second relation from (6) we have $\alpha\varphi(-\frac{\beta}{2\alpha}) < 0 \Leftrightarrow \alpha\frac{4\alpha\gamma-\beta^2}{4\alpha} < 0 \Leftrightarrow D > 0$, which holds. As both the roots are positive $\rho_1, \rho_2 > 0$, then the product $\rho_1\rho_2 > 0$ and therefore $\frac{\gamma}{\alpha} > 0 \Leftrightarrow \alpha\gamma > 0$. The third relation $\alpha\varphi(0) = \alpha\gamma > 0$, in (6) is true already and $0 < \frac{\rho_1+\rho_2}{2} \Rightarrow 0 < -\frac{\beta}{2\alpha}$ equivalent to $\beta < 0$. Therefore we get $\beta < 0 < \alpha\gamma < (\frac{\beta}{2})^2$.

We can then identify the point of intersection $I(z_0, k_0)$, $z_0 : \mathrm{MAC}(z_0) = \mathrm{MD}(z_0)$ as before. Therefore under (5) and $\beta_1 > \beta_0$ we evaluate k_0 as in (3) and the Benefit Area $\mathrm{BA}_{\mathrm{QL}}$ can be evaluated as in (4).

Case (c): $\alpha < 0, D = \beta^2 - 4\alpha\gamma > 0$. Let us now consider the case $\alpha < 0$. Under this assumption the restriction $D = 0$ is not considered, as the values of $\varphi(z)$ have to be negative.

Under the assumption of Case (c), the value $\varphi(-\frac{\beta}{2\alpha}) = \frac{4\alpha\gamma-\beta^2}{4\alpha}$ corresponds to the maximum value of $\varphi(z)$. We consider the situation where $\rho_1 < 0 < -\frac{\beta}{2\alpha} < \rho_2$ (see Figure 4) while the case $0 < \rho_1 < -\frac{\beta}{2\alpha} < \rho_2$ has no particular interest (it can be also considered as in Case (b), see Fig. 3).

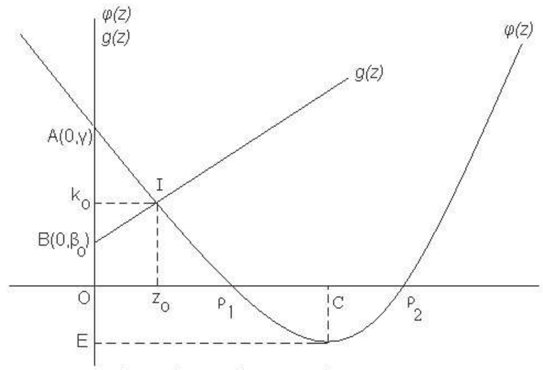

Fig. 3 $C = C(-\frac{\beta}{2\alpha}, 0)$, $E = E(0, \varphi(-\frac{\beta}{2\alpha}))$, $\varphi(-\frac{\beta}{2\alpha}) = \min \varphi(z)$, $\alpha > 0$.

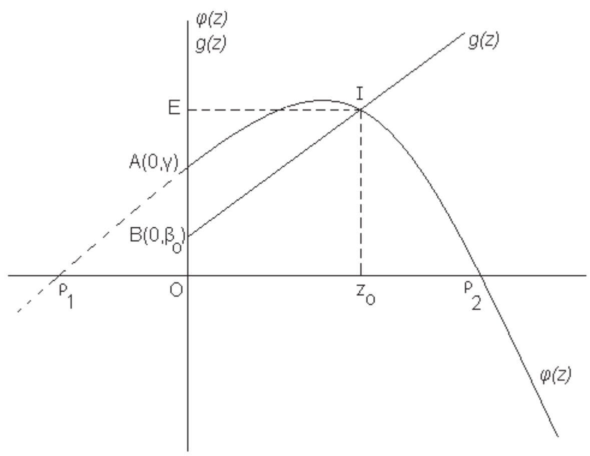

Fig. 4 $C = C(-\frac{\beta}{2\alpha}, 0)$, $E = E(0, \varphi(-\frac{\beta}{2\alpha}))$, $\varphi(-\frac{\beta}{2\alpha}) = \min \varphi(z)$, $\alpha < 0$.

Proposition 2. *For the Case (c) as above we have:* $\rho_1 < 0 < -\frac{\beta}{2\alpha} < \rho_2$ *when* $\alpha\gamma < 0$.

Proof. The imposed assumption is equivalent to $\alpha\varphi(0) < 0 \Leftrightarrow \alpha\gamma < 0$ as $\rho_1\rho_2 < 0$, $\alpha\varphi(-\frac{\beta}{2\alpha}) < 0 \Leftrightarrow \alpha\gamma < (\frac{\beta}{2})^2$. Therefore the imposed restrictions are $\alpha\gamma < 0 < (\frac{\beta}{2})^2$ (compare with (5)). Actually, $\alpha\gamma < 0$.

Case (c) requires that $\beta_0 < \gamma$ and $\beta_1 > 0$. To calculate z_0 we proceed as in (1) and z_0 is evaluated as in (2). Therefore, with $\alpha < 0$ we have $\beta_1 - \beta < 0$, i.e. $\beta_1 < \beta$. Thus for $\beta_1 < \beta$, $\alpha\gamma < 0$, the BA as in (4) is still valid.

3 An Empirical Application

In the empirical application regression analysis is adopted to evaluate the estimate of the involved parameters. The available data for different European countries are used.

The abatement cost function measures the cost of reducing tonnes of emissions of a pollutant, like sulphur (S), and differs from country to country depending on the local costs of implementing best practice abatement techniques as well as on the existing power generation technology. For abating sulphur emissions various control methods exist with different cost and applicability levels, see [5, 6, 7, 8].

Given the generic engineering capital and operating control cost functions for each efficient abatement technology, total and marginal costs of different levels of pollutant's reduction at each individual source and at the national (country) level can be constructed. According to [5, 6, 12], the cost of an emission abatement option is given by its total annualized cost (TAC). For every European country a least cost curve is derived by finding the technology on each pollution source with the lowest marginal cost per tonne of pollutant removed in the country and the amount of pollutant removed by that method on that pollution source.

For analytical purposes, it is important to approximate the cost curves of each country by adopting a functional form extending the mathematical models described above to stochastic models, [11].

The calculation of the damage function $\varphi(z)$ is necessary. For the followed procedure see [18, 10] and [17]. The only information available is to "calibrate" the damage function, on the assumption that national authorities act independently (as Nash partners in a non-cooperative game with the rest of the world) taking as given deposits originating in the rest of the world, see [17].

The results are presented in Table 1 where Eff, as in [10], is the efficiency of the benefit area, in comparison with the maximum evaluated from the sample of countries under investigation and can be estimated using as measure of efficiency the expression:

$$\mathrm{Eff} = \left(\frac{\mathrm{BA}}{\max \mathrm{BA}} \right) \times 100.$$

4 Conclusions and Policy Implications

The typical approach defining the optimal pollution level has been to equate the marginal damage cost (of an extra unit of pollution) with the corresponding marginal abatement cost. An efficient level of emissions maximizes the net benefit, that is, the difference between abatement and damage costs. Therefore the identification of this efficient level shows the level of benefits maximization, which is the resulting output level if external costs (damages) are fully internalized.

Table 1 Coefficient estimates in the case of quadratic MD and MAC functions.

Countries	c_0	c_1	c_2	b_0	b_1	b_2
Albania	0.7071	0.01888	0.0001397	-3.3818	0.015	0.0048
Austria	8.57143	0.055012	0.0001145	3.274	-0.221	0.004
Belgium	2.2424	0.03869	0.0001688	0.497	-0.124	0.003
Former Czech.	37.794	0.100323	0.000059	11.241	0.2358	0.00018
Denmark	10.	0.1923	0.0060811	-2.49	0.099	0.0053
Finland	4.021	0.0781	0.0001459	2.343	-0.098	0.0046
France	33.158	0.277352	0.000197	42.374	-0.053	0.0018
Greece	3.7373	0.034133	0.0000491	-1.614	0.342	0.0006
Hungary	5.101	0.031488	0.0000417	2.506	0.216	0.0004
Italy	21.01	0.030036	0.0000191	12.5	0.36	0.0003
Luxembourg	0.421	0.3161	0.0272381	-0.7272	0.01	0.09234
Netherlands	8.353	0.19513	0.0035144	-6.18	0.41	0.0009
Norway	1.421	0.07852	0.0001701	0.94	-0.244	0.0164
Poland	6.212	0.023153	0.000071	-8.023	0.324	0.00009
Romania	9.091	0.011364	0.00006237	5.502	0.19	0.0001
Spain	11.7	0.007288	0.00497419	10.21	-0.021	0.00014
Sweden	2.4	0.06423	0.0000932	4.074	-0.252	0.004
Switzerland	2.4	0.56027	0.002803	5.7543	-1.6289	0.11203
Turkey	14.9	0.01781	0.00001223	8.0622	0.011	0.00036
UK	19.1	0.06879	0.0000467	15.54	0.0264	0.0003

Table 2 Calculated "calibrated" Benefit Areas (BA^c).

	Linear–Quadratic					
Countries	D	z_0	$g(z_0)$	$G(z_0)$	BA	Eff
Albania	0.0785	29.594	-3.38	-52.05	81.24	3.5872
Austria	0.1609	84.649	3.3	294.1	628.6	27.756
Belgium	0.0474	63.406	0.5	37.2	182.8	8.0715
Former Czech.	0.0378	160.988	11.24	5119.8	2264.6	100
Denmark	0.2735	58.138	-2.5	369.72	536.7	23.698
Finland	0.0619	46.182	2.4	154.72	114.3	5.0453
France	0.0428	149.22	42.4	7726.3	309.2	13.65
Greece	0.1076	16.83	-1.62	22.23	45.5	2.0095
Hungary	0.0381	13.66	2.51	54.72	17.9	0.7901
Italy	0.1191	25.22	12.5	431.19	108.1	4.7726
Luxembourg	0.5178	5.56	-0.73	1.4	5.8	0.2572
Netherlands	0.0985	54.98	-6.18	329.7	424.4	18.741
Norway	0.1356	21.056	0.94	16.75	30.6	1.3508
Poland	0.0956	46.67	-8.03	-18.57	333.7	14.734
Romania	0.0333	19.87	5.5	147.1	35.8	1.5803
Spain	0.0016	245.43	10.2	2563.2	527.8	23.305
Sweden	0.0732	73.35	4.1	147.1	201.7	8.9075
Switzerland	3.2893	17.87	5.56	55.8	76.5	3.378
Turkey	0.0099	147.82	8.1	1698.5	698.65	30.851
UK	0.0061	200.5	15.6	4452.1	759.9	33.551

In this paper the corresponding optimal cost and benefit points were evaluated analytically. We shown that the optimal pollution level can be evaluated only under certain conditions. From the empirical findings is clear that the evaluation of the "calibrated" Benefit Area, as it was developed, provides an index to compare the different policies adopted from different countries In this way a comparison of different policies can be performed. Certainly the policy with the maximum Benefit Area is the best, and the one with the minimum is the worst. Clearly the index BA^c provides a new measure for comparing the adopted policies.

It is clear that due to the model selection, the regression fit of the model, the undergoing errors and the propagation create a Risk associated with the value of the Benefit Area. This Associated Risk is that we try to reduce, choosing the best model, and collecting the appropriately data (more than 10set of the observation when two variables are involved and more than 15 when three variables are involved or the model is non-linear).

Policy makers may have multiple objectives with efficiency and sustainability being high priorities. Environmental policies should consider that economic development is not uniform across regions and may differ significantly, [14]. At the same time reforming economic policies to cope with EU enlargement may face problems and this may in turn affect their economic efficiencies, [13].

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