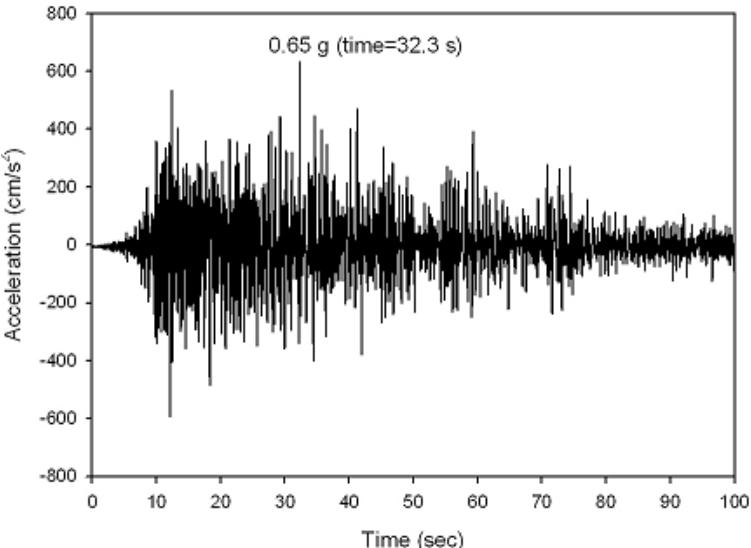


# Reliability-Based Analysis and Design of Isolation Systems for Large Scale Building Models



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# Motivation



Chilean earthquake, February 27, 2010 (Magnitude 8.8)

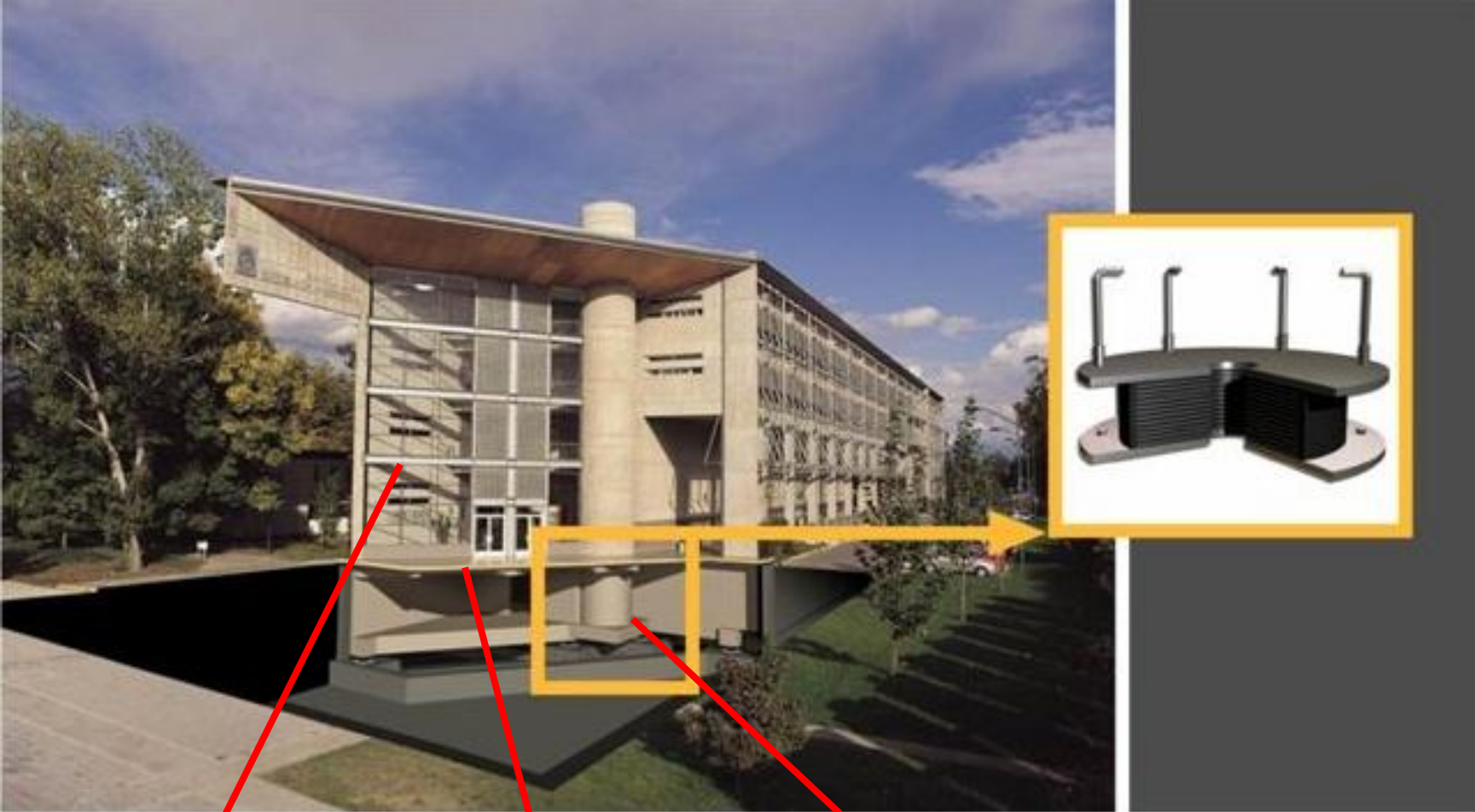


# Outline

- **Introduction**
- **Structural Model**
- **Isolation System Modeling**
- **Stochastic Excitation Model**
- **Reliability Measures**
- **Example Problem**
- **Conclusions**

# Introduction

# Base-isolated structural system



Superstructure

Base

Isolators

# Isolation systems

## Isolation elements

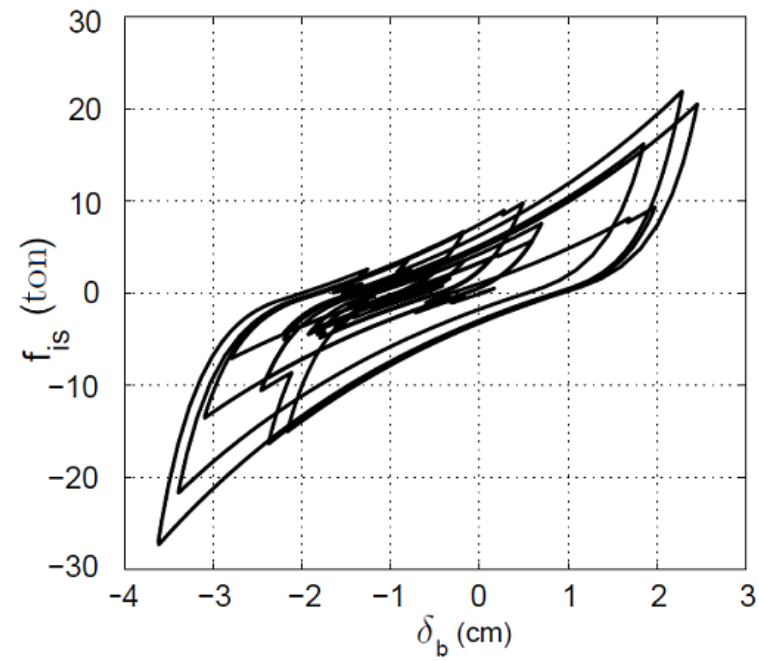
- Lead rubber bearings or high-damping rubber bearings
- Nonlinear fluid dampers
- Sliding friction bearings
- Friction pendulums



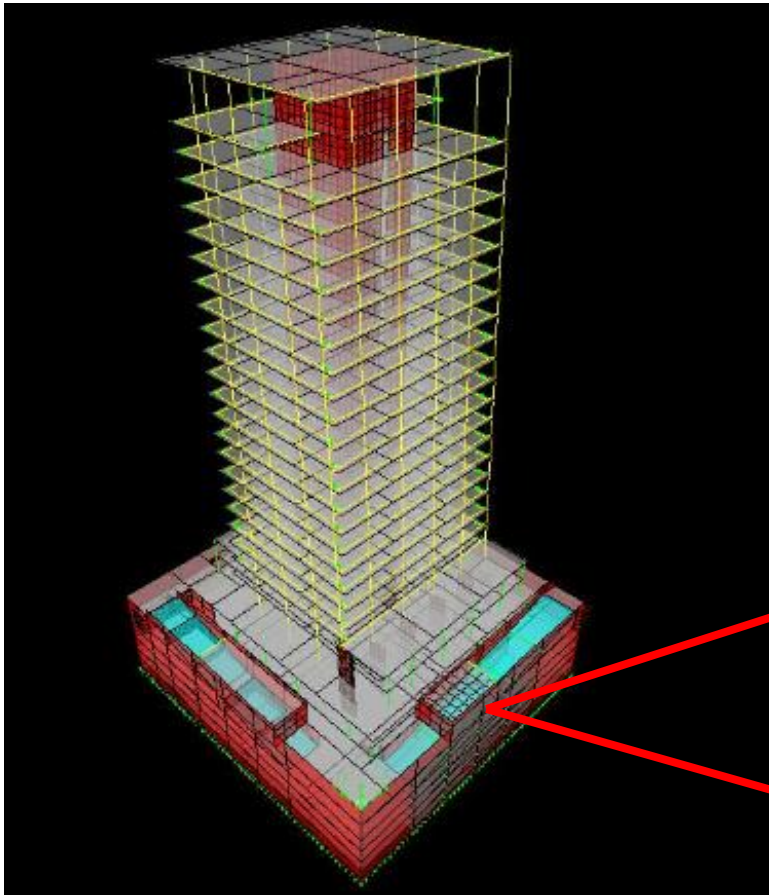
**Net effect**  
**reduce the energy dissipation demand of the**  
**superstructure**



**Typical displacement-restoring force curve (rubber bearing)**



**In general the design of isolation systems is performed (at the design stage) using deterministic analyses (code based)**



## **Objective**

**Propose a general framework for the analysis and design of isolation systems from a reliability point of view**



# Structural Model

# Equation of motion (superstructure FEM model)

$$[M_s]\{\ddot{u}_s(t)\} + [C_s]\{\dot{u}_s(t)\} + [K_s]\{u_s(t)\} = -[M_s][G_s](\{\ddot{u}_b(t)\} + \{\ddot{u}_g(t)\})$$

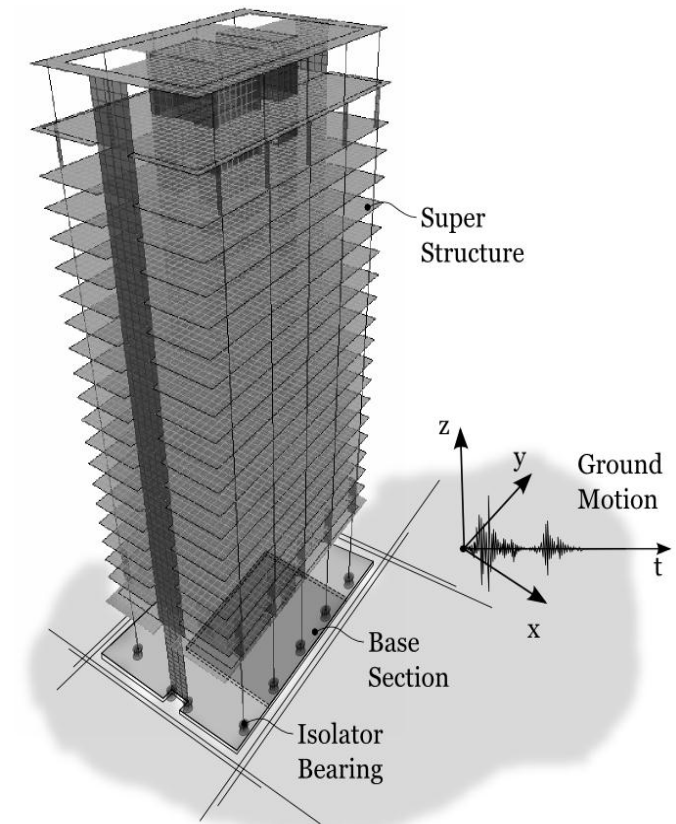
$\{u_s(t)\}$ : vector of absolute displacements

$[M_s]$ ,  $[C_s]$ ,  $[K_s]$ : mass, damping, stiffness matrices

$\{u_b(t)\}$ : vector of base displacements

$[G_s]$ : matrix of earthquake influence coefficients

$\{\ddot{u}_g(t)\}$ : vector of excitation components



# Equation of motion (base)

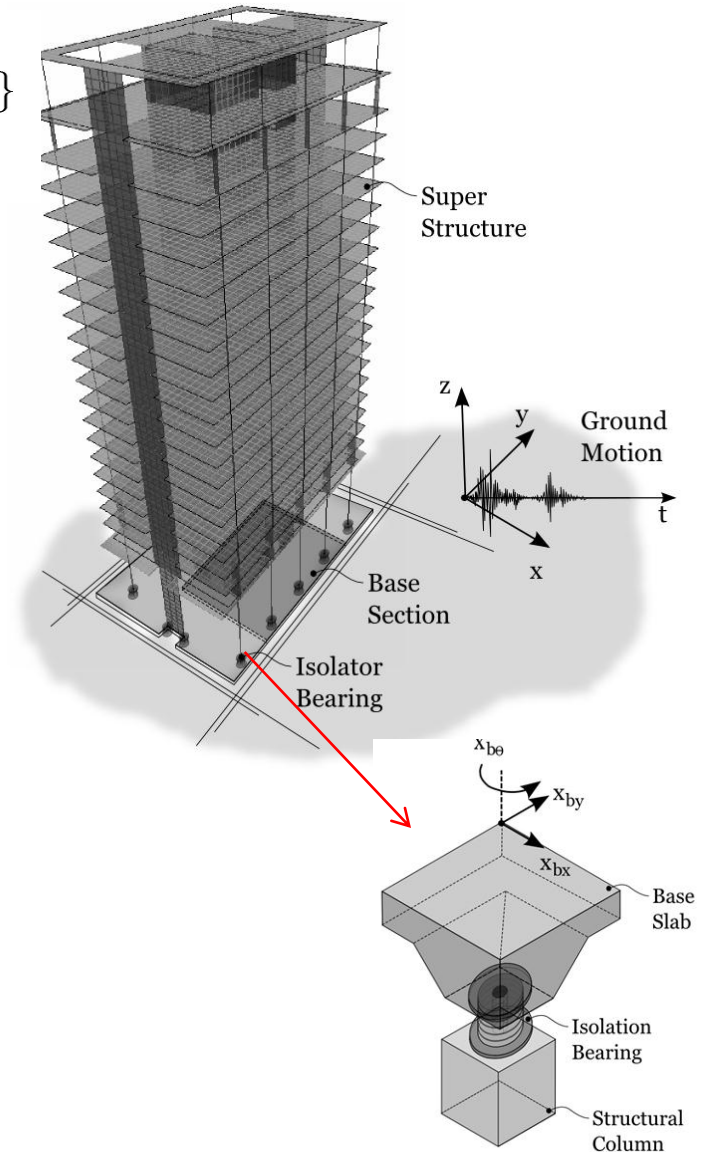
$$([G_s]^T [M_s] [G_s] + [M_b]) (\{\ddot{u}_b(t)\} + \{\ddot{u}_g(t)\}) + [G_s]^T [M_s] \{\ddot{u}_s(t)\} + [C_b] \{\dot{u}_b(t)\} + [K_b] \{u_b(t)\} + \{f_{is}(t)\} = \{0\}$$

$[M_b]$ : mass matrix of the rigid base

$[C_b]$ : damping matrix of the viscous isolation components

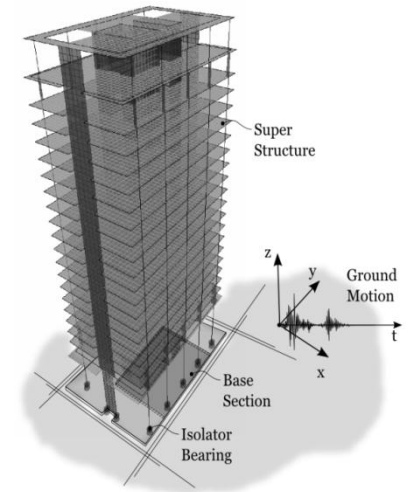
$[K_b]$ : stiffness matrix of the elastic isolation components

$\{f_{is}(t)\}$ : vector of non-linear isolation forces



# Combined equation of motion

$$\begin{bmatrix} [M_s] & [M_s][G_s] \\ [G_s]^T[M_s] & [M_b] + [G_s]^T[M_s][G_s] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}_s(t)\} \\ \{\ddot{u}_b(t)\} \end{Bmatrix} + \begin{bmatrix} [C_s] & [0] \\ [0] & [C_b] \end{bmatrix} \begin{Bmatrix} \{\dot{u}_s(t)\} \\ \{\dot{u}_b(t)\} \end{Bmatrix} + \begin{bmatrix} [K_s] & [0] \\ [0] & [K_b] \end{bmatrix} \begin{Bmatrix} \{u_s(t)\} \\ \{u_b(t)\} \end{Bmatrix} = - \begin{bmatrix} [M_s][G_s] \\ [M_b] + [G_s]^T[M_s][G_s] \end{bmatrix} \{\ddot{u}_g(t)\} - \begin{Bmatrix} \{0\} \\ \{f_{is}(t)\} \end{Bmatrix}$$



# Solution equation of motion

Newmark method (second-order)

Crank-Nicolson method (second-order)

Runge-Kutta method (fourth-order)

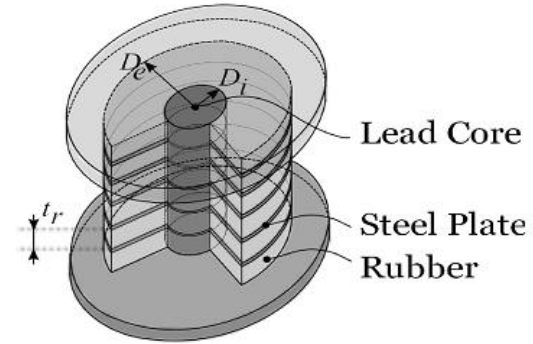
Model reduction techniques:

Static condensation-Guyan reduction

Component mode synthesis techniques

# Isolation System Modeling

# Standard approach (mathematical model)



Schematic representation of a rubber bearing

(Buoc-Wen model)

$$U^y \dot{z}(t) = \dot{x}_b(t) [\alpha - z^n(t) (\gamma \operatorname{sgn}(\dot{x}_b(t)) + \beta \operatorname{sgn}(z(t)))]$$

$z(t)$ : dimensionless hysteretic variable

$\alpha, \beta, \gamma$ : dimensionless quantities

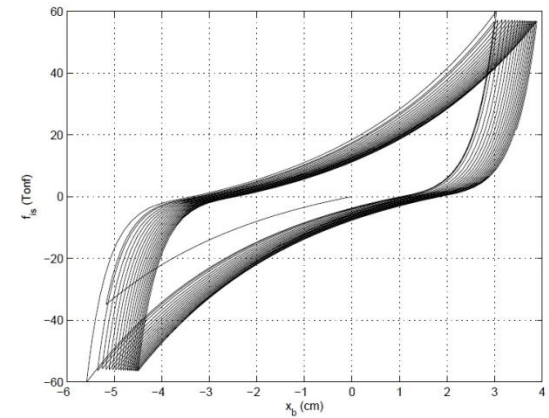
$U^y$ : yield displacement

$x_b(t), \dot{x}_b(t)$ : base displacement and velocity

$\operatorname{sgn}(\cdot)$ : sign function

## Force activated

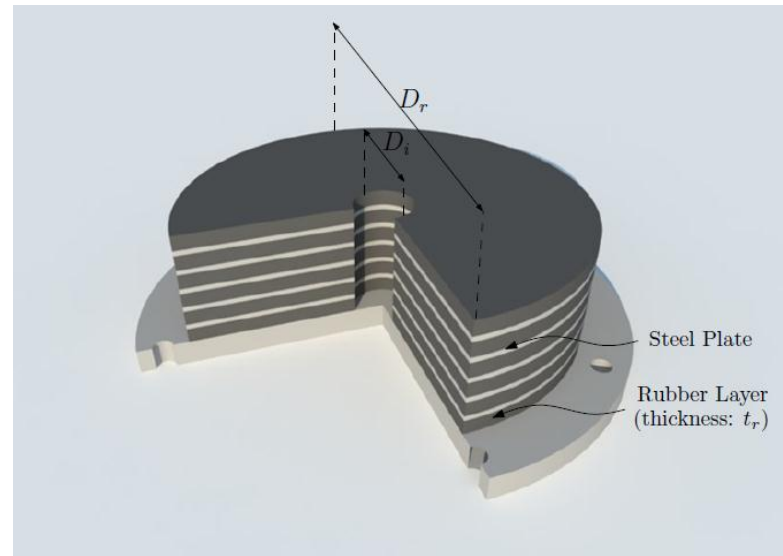
$$f_{is}(t) = k_p x_b(t) + c_v \dot{x}_b(t) + (k_e - k_p) U^y z(t)$$



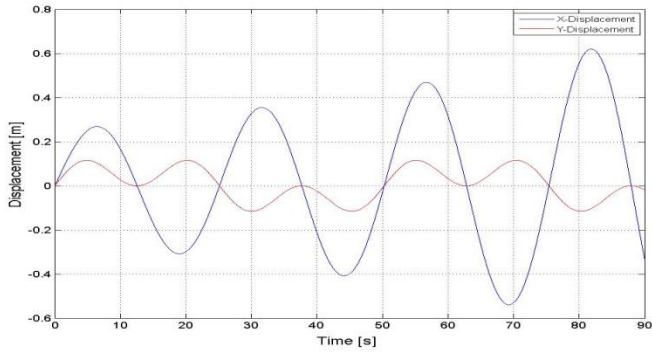
Displacement-restoring force curve

# Alternative approach

(experimental data)



**From observation of a series of test (using horizontal bidirectional loading) it has been proposed to compose the restoring force in terms of a force directed to the origin and another force approximately opposite to the direction of the movement (Yamamoto, 2012)**

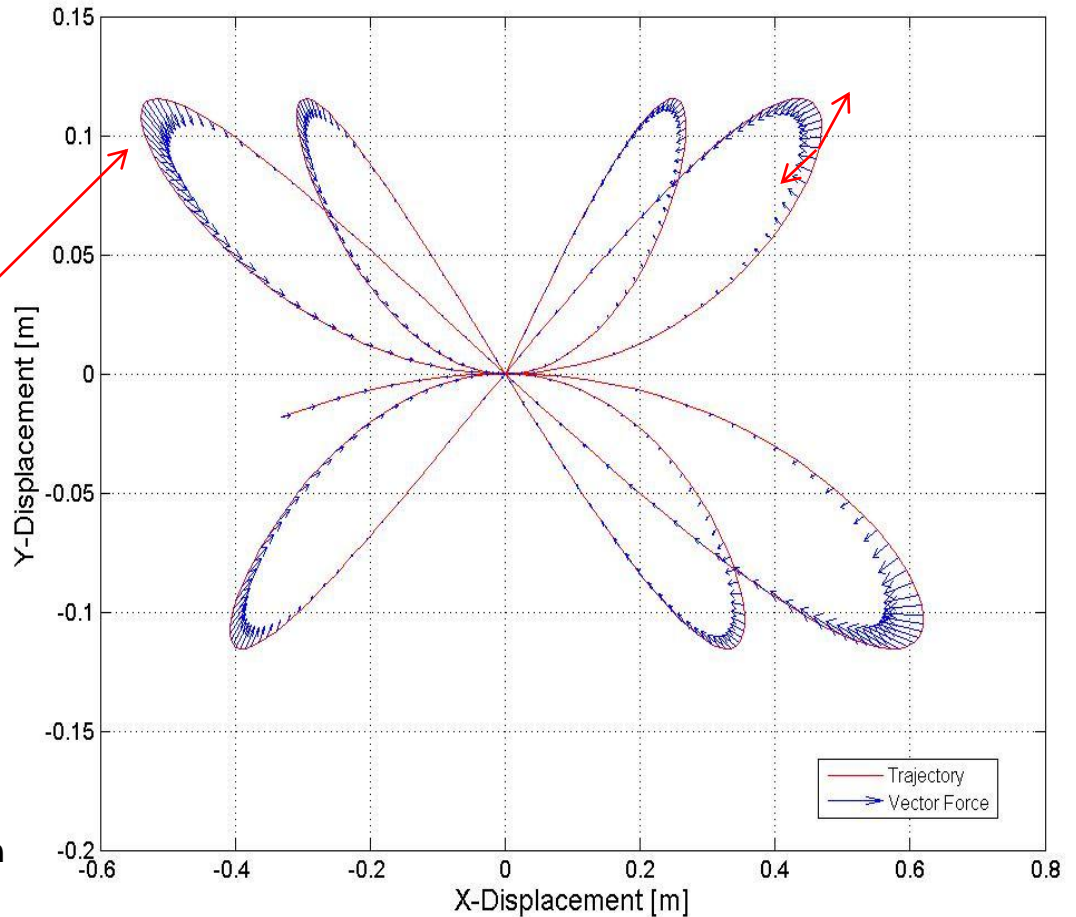


**loading displacements**

upper flange plate



**displacement pattern**



**Restoring forces along the path of the loading**

**direction do not lead straight to the origin**

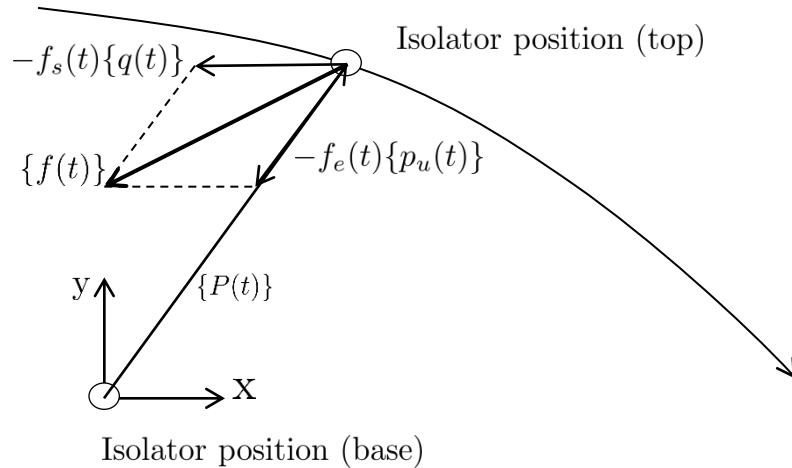


**internal twisting shear deformation occurs in the bearing**



# Analytical Model

Trajectory of displacement



$\{p(t)\}$ : displacement vector (trajectory of the isolator)

$\{p_u(t)\}$ : unit directional vector

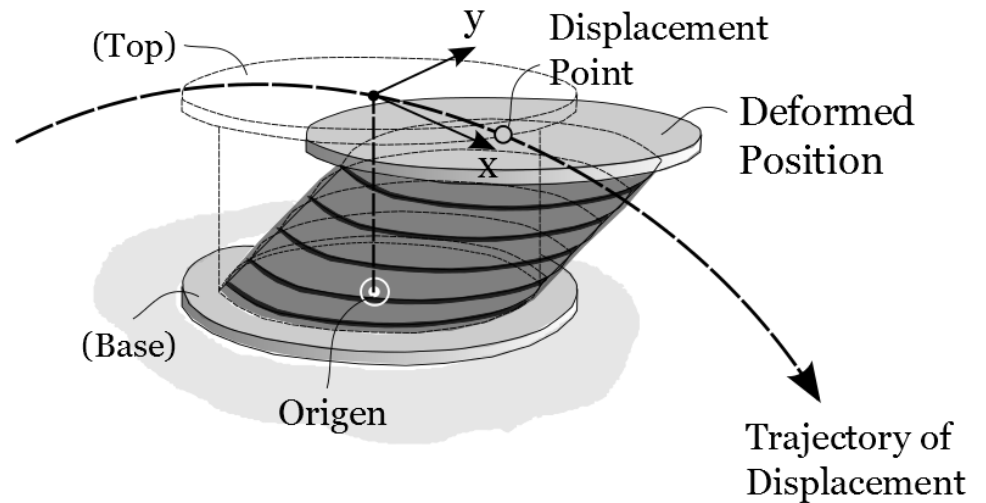
$\{q(t)\}$ : direction of the movement

$f_e(t)$ : elastic component of the force

$f_s(t)$ : elastoplastic component of the force

restoring force:

$$\{f(t)\} = -\{p_u(t)\}f_e(t) - \{q(t)\}f_s(t)$$



# Direction of the movement

$$\{\dot{q}(t)\} = \frac{1}{\alpha} \|\{\dot{p}(t)\}\| [\{\dot{p}_u(t)\} - \|\{q(t)\}\|^n \{q_u(t)\}]$$

$$\{p(0)\} = \{0\}, \{q(0)\} = \{0\}$$

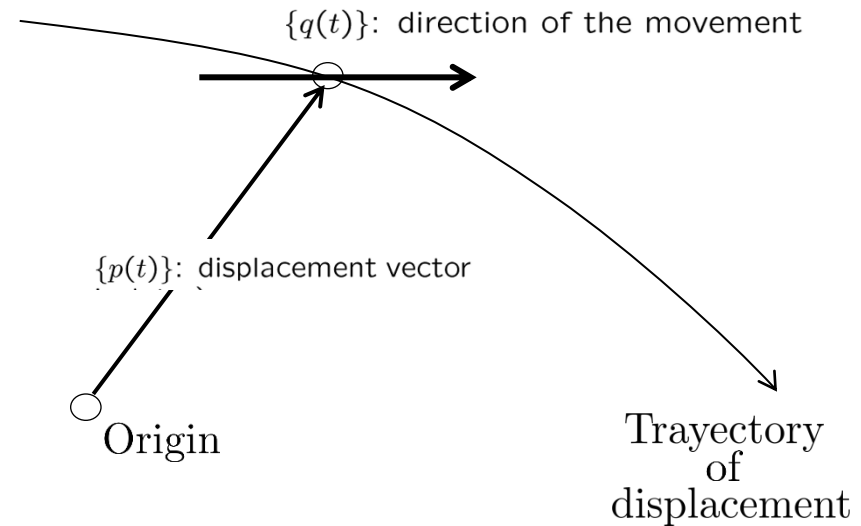
$\{\dot{p}(t)\}$ : velocity vector

$\{\dot{p}_u(t)\}$ : unit directional vector of the velocity vector

$\{q_u(t)\}$ : unit directional vector of the movement

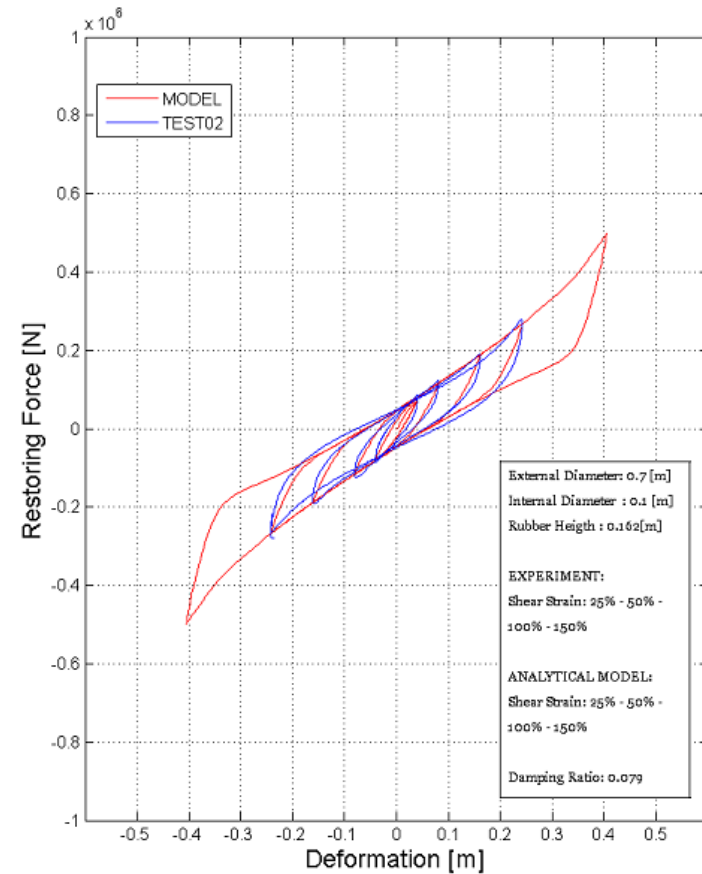
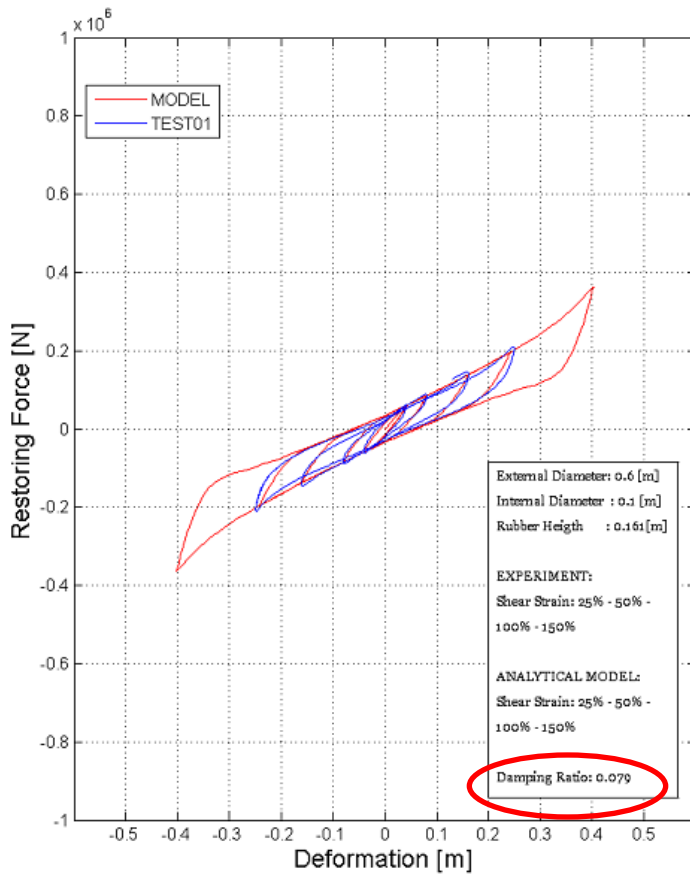
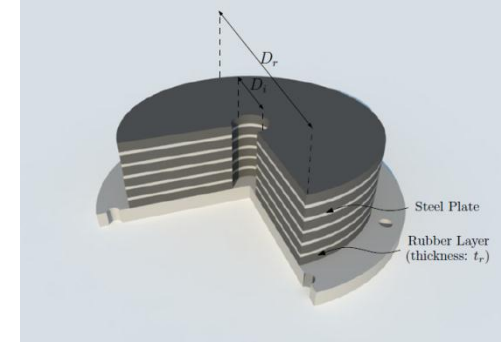
$\|\cdot\|$ : Euclidean norm

$\alpha, n$ : positive constants that relate to the yield displacement and smoothness of yielding



# Validation

## Small size rubber bearings



Equivalent viscous damping ratio

# Medium size rubber bearings

