Stochastic Excitation Model



Power Spectrum

Stationary or non-stationary



$$\Rightarrow \quad \ddot{x}_g(t) \approx \sum_{i=1}^N A_i B(\omega_i, t) Cos(\omega_i t + \theta_i)$$
(Superposition of harmonic waves)

 θ_i : independent, identically distributed uniform random variables [0,2 π]

 $B(\omega,t):~$ Deterministic function of time

Gaussian Processes (second-order processes)

(Karhunen-Loève expansion)

$$\ddot{x}_g(t) \approx \ddot{x}_g^0(t) + \sum_{i=1}^N \ddot{x}_i(t) z_i$$

 z_i : independent, identically distributed standard Gaussian random variables.

K-L components

<u>Continuous Filters</u> (Filtered and modulated white noise processes)



Mathematical characterization:

$$\begin{split} \ddot{x}_g(t) &= M(v_1(t), v_2(t)) \\ L_1(v_1(t)) &\equiv \omega(t)h(t) \\ L_2(v_1(t), v_2(t)) &\equiv 0 \end{split} \qquad M, L_1, L_2 : \text{Linear operators} \end{split}$$

<u>Source-Based Models:</u> Point-Source Model (characterized by the moment magnitude M and epicentral distance r)

(Boore (1997, 2003) Anderson (1984), Atkinson (2000))



High-Frequency components and Long-Period components

Reliability Measures

Failure Events:

Failure event related to the base drift:

$$F_{\text{base drift}}(\{z\}) = 1 - \max_{t \in [0,T]} \frac{|u_b(t,\{z\})|}{u_b^*} \le 0$$

 u_b^* : critical threshold level

Failure event related to the superstructure drift:

$$F_{\text{superstructure drift}}(\{z\}) = 1 - \left(\max_{t \in [0,T]} \frac{|\Delta u_s(t,\{z\})|}{\Delta u^*}\right) \leq C$$

 Δu^* : critical threshold level

Failure event related to the superstructure absolute acceleration:

$$F_{\text{acceleration}}(\{z\}) = 1 - \left(\max_{t \in [0,T]} \frac{|\ddot{u}_s^{(\text{absolute})}(t,\{z\})|}{\ddot{u}^*}\right) \le 0$$

 $\ddot{u}^*\!\!:$ acceptable level of response



Reliability Problem:

$$P_F = \int_{\Omega_F(\{z\}) \le 0} p(\{z\}) d\{z\}$$

 $\Omega_{F(\{z\})}$: Failure domain

- high dimensional uncertain parameter spaces
- small failure probabilities for practical problems (10⁻³ 10⁻⁶)



Simulation methods



Advanced simulation procedures

• Subset simulation

Markov chains Splitting trajectories Hybrid techniques



- Auxiliary domain method
 - Horseracing algorithm
 - Importance Sampling
- Bridge Importance Sampling
 - Line sampling

Example Problems

Example No.1 : Three dimensional building model

PLAN VIEW

FEM model : 40,000 DOFs

Isolation system: 52 rubber bearings



Fundamental period Fixed: 1.02s Isolated: 1.65s



A) Reliability-Based Characterization of the Response

Base drift

Failure event related to the base drift:

$$F_{\text{base drift}}(\{z\}) = 1 - \max_{t \in [0,T]} \frac{|u_b(t,\{z\})|}{u_b^*} \le 0$$

 $u_b^* = 25 \text{ cm}$









Superstructure drift

Failure event related to the superstructure drift:

 $F_{\text{superstructure drift}}(\{z\}) = 1 - \left(\max_{t \in [0,T]} \frac{|\Delta u_s(t,\{z\})|}{\Delta u^*}\right) \leq 0$

 $\Delta u^* = 0.2\%$ of the story height



Iso-probability curves





Superstructure absolute acceleration

Failure event related to the superstructure absolute acceleration:

$$\begin{split} F_{\text{acceleration}}(\{z\}) &= 1 - \left(\max_{t \in [0,T]} \frac{|\ddot{u}_s^{(\text{absolute})}(t,\{z\})|}{\ddot{u}^*} \right) \leq 0 \\ \ddot{u}^* &= 40\% \text{ g} \end{split}$$



Iso-probability curves





Observations:

• introduction of additional stiffness in the isolation system (control the base displacement) makes the superstructure response ineffective

• flexibility of the isolation system (may induce undesirable effects on the base isolation system) has a positive impact on the reliability of the superstructure

conflicting objectives

B) Reliability-Based Design Formulations

Formulation I: Cost minimization problem

Minimize $C(\{D_r, H_r, D_l\})$

subject to

$$P_{F_j}(\{D_r, H_r, D_l\}) \le P_{F_j}^*, \ j = 1, ..., n_c$$

 $C(\{D_r, H_r, D_l\}) = H(c_r[\frac{D_r}{2}]^2 - [\frac{D_l}{2}]^2 + c_l[\frac{D_l}{2}]^2)$



Probability of failure in terms of the base drift

Lead Core Steel Plate Rubber



design space

Probability of failure in terms of the base drift





design space

Formulation II: Reliability maximization problem

Minimize $P_F(\{D_r, H_r, D_l\})$

subject to

 $C(\{D_r, H_r, D_l\}) \le C^*$, $j = 1, ..., n_c$



Probability of failure in terms of the base drift

Lead Core Steel Plate Rubber



design space

Probability of failure in terms of the base drift

Lead Core Steel Plate Rubber



Conclusions

- The modeling procedure (for the nonlinear behavior of the rubber bearings) is able to accurately simulate the test results for both unidirectional and bidirectional loading (simple and numerically stable model).
- The protection of the superstructure (drifts and accelerations) and the control of base displacements are conflicting goals (performance of the superstructure as well as the base isolation system should be considered simultaneously)
- Near-fault ground motions with large velocity pulses can change the reliability of base-isolated systems significantly.
- The proposed tools allow to analyze and design real base-isolated systems from a reliability (probabilistic) point of view.